Third, infants show greater facility in discriminating number than continuous magnitudes. For example, 6-month-old infants need a 1:4 ratio difference to detect a change in cumulative surface area and a 1:3 ratio difference to detect a change in cumulative perimeter, but only a 1:2 ratio difference to detect a change in number (Cordes & Brannon 2008; Starr & Brannon 2015). Furthermore, when a change in cumulative surface area is directly pitted against a change in number, infants prefer to look at the change in number (Libertus et al. 2014). This preference cannot be attributed to detecting a change in individual element sizes as Leibovich et al. argue (see sect. 3) because individual element size changed by the same ratio as change in number and change in cumulative surface area; that is, when a 1:3 ratio change in number was pitted against a 1:3 ratio change in cumulative surface area, the size of individual elements in both cases changed by a 1:3 ratio. When a 1:3 ratio change in number was pitted against a 1:5 ratio change in cumulative surface area, the size of the individual elements changed by 1:3 and 1:5 ratios, respectively. Despite the greater change in individual element size that accompanied the change in cumulative surface area, infants looked significantly longer at the change in number that was accompanied by a smaller change in individual element size. Therefore, changes in individual element size cannot explain why infants would attend more to a change in number than a change in cumulative surface area. The most parsimonious explanation for the observed findings is that infants are more sensitive to changes in number than continuous magnitudes and not vice versa as Leibovich et al. suggest.

Taken together, these three lines of research suggest that it is most parsimonious to assume that the concept of number is present early in development and that its acquisition does not rest on the acquisition of visual object individuation and experiences with correlations between number and continuous magnitude representations.

Right idea, wrong magnitude system

doi:10.1017/S0140525X16002156, e177

Stella F. Lourenco,[a] Lauren S. Aulet,[a] Vladislav Ayzenberg,[a] Chi-Ngai Cheung,[a] and Kevin J. Holmes[b]

[a]*Department of Psychology, Emory University, Atlanta, GA 30322;*
[b]*Department of Psychology, Colorado College, Colorado Springs, CO 80903.*
stella.lourenco@emory.edu **lauren.s.aulet@emory.edu**
vladislav.ayzenberg@emory.edu **ccheun4@emory.edu**
kjholmes@coloradocollege.edu **http://psychology.emory.edu/home/people/faculty/lourenco-stella.html**
https://sites.google.com/site/kjholmes05/

Abstract: Leibovich et al. claim that number representations are non-existent early in life and that the associations between number and continuous magnitudes reside in stimulus confounds. We challenge both claims – positing, instead, that number is represented independently of continuous magnitudes already in infancy, but is nonetheless more deeply connected to other magnitudes through adulthood than acknowledged by the "sense of magnitude" theory.

Leibovich et al. argue that it is time to reconsider mainstream "number sense" theories, and as an alternative, they propose a "sense of magnitude" theory based on two central claims. The first is that mental representations of number are non-existent early in human life because number is not represented independently of continuous magnitudes prior to experience with language or the development of executive control. The second is that the associations between number and continuous magnitudes reside in stimulus confounds, making it virtually impossible to examine the true nature of magnitude representations, even in adults. Here, we challenge both claims, positing instead that number may be represented independently early in development,

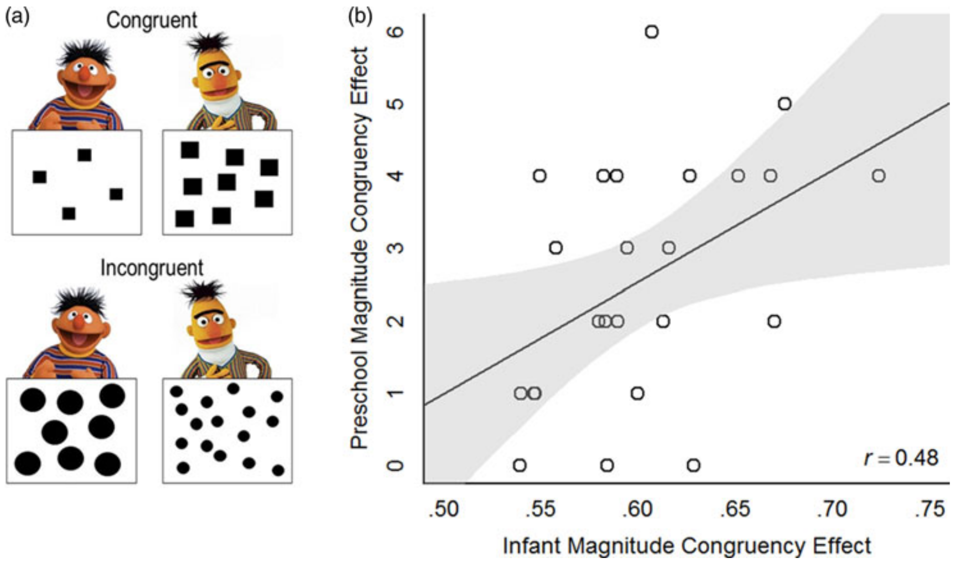

Figure 1. (Lourenco et al.). (A) Examples of congruent and incongruent trials in the numerical judgment task administered to preschool (3.5-year-old) children in the longitudinal study of Lourenco and Aulet (submitted). Children were asked to judge which character (Bert or Ernie) had the larger numerosity. On the congruent trials, the array with the larger numerosity was also larger in cumulative area and item size. On the incongruent trials, the array with the larger numerosity was smaller in cumulative area and item size. It is worth noting that mean accuracy on the incongruent trials was significantly above chance, confirming the use of number on this task while ensuring an assessment of the influence of area on number. (B) Scatterplot relating magnitude congruency effects across the two time points: infancy and preschool age. For the purpose of this commentary, we depict a subset of children ($N = 25$) whose scores were equal to or above 0.50 at time point 1 (infancy) and 0 at time point 2 (preschool), each of which indicates no association between magnitudes. Infant scores were computed as the proportion of looking time toward the novel test trials as a function of their total looking time (novel+familiar trials; see Lourenco & Longo [2010] for procedural details). Preschool scores were computed as the difference in errors between the two types of trials (incongruent−congruent). Children's performance at 9 months of age was significantly correlated with their performance at 3.5 years of age, suggesting that the association between magnitudes apparent in preverbal children remained relatively stable into preschool age. The scatterplot includes the best-fitting regression line and the 95% confidence interval (shaded area).

but nonetheless shows deep underlying connections with other magnitudes throughout life.

The idea that young infants lack number representations is based on a false premise, namely, that infants are unable to individuate objects. On the contrary, studies of object individuation suggest that by 4 months, infants are skilled at using motion, spatial separation, and featural cues such as shape and size to segment visual scenes, including distinguishing figure from ground, delineating the boundaries of connected or partially occluded objects, and tracking individual objects across time (Atkinson & Braddick 1992; Johnson & Aslin 1995; Kellman & Spelke 1983; Kestenbaum et al. 1987; Needham 1998; Slater et al. 1990; Valenza et al. 2006; Wilcox 1999). Although infants of this age certainly have difficulty with object *identification* (i.e., what an object is [Carey & Xu 2001; Leslie et al. 1998]), object *individuation* independent of identity is well within their capacity. Moreover, our own analysis of visual stimuli used to assess “number sense” in newborn infants challenges Leibovich et al.’s contention that object individuation is impossible because of newborns’ poor vision (see sect. 8, e.g., Fig. 6). We calculated spatial frequency (SF) for 50 visual displays using the parameters (e.g., viewing distance: 60 cm) specified by Izard et al. (2009). This analysis yielded a mean SF of 0.5 cycles/degree, well within the normal acuity of a newborn viewing a high-contrast image (Atkinson et al. 1974; Banks & Salapatek 1981; Brown & Yamamoto 1986). Even the most cluttered portions of these displays averaged 1.4 cycles/degree, a value lower than the upper limit of 1.8–2 cycles/degree of newborns’ visual acuity.

Leibovich et al. argue that even after infants come to individuate objects, they still cannot differentiate number from continuous magnitudes because this ability depends on linguistic experience (e.g., number words) and executive control (e.g., inhibition). However, this claim is challenged by recent longitudinal data from our lab showing that individual differences in the associations of various magnitudes (number, area, and duration) at 9 months of age predict the extent to which number remains associated with area in the same children at 3.5 years of age (see Fig. 1). The continuity over this period argues against language as a mechanism of differentiation because the children were mostly preverbal at the earliest time point and therefore had not learned any number words. The continuity was also not explained by inter-individual variability in inhibitory control (measured using the Day-Night Stroop task at 3.5 years), arguing against executive control as a mechanism of differentiation. Although we agree with Leibovich et al. (and others) that number words and inhibition may facilitate differentiation of number from other magnitudes (perhaps by drawing attention to individual stimulus items), our longitudinal data suggest that neither factor is necessary, because at least some differentiation is apparent in preverbal infants and cannot be accounted for by inhibition more generally.

Another reason to doubt Leibovich et al.’s claim that number does not dissociate from continuous magnitudes until relatively late in development is that it makes a dubious prediction: Early representations of number should be less precise than those of continuous magnitudes because of children’s earlier, and presumably greater, experience with the latter. Multiple studies examining discrimination sensitivity contradict this prediction. For example, Cordes and Brannon (2009; 2011) found that 6-month-olds require a larger difference between sets when discriminating cumulative area than when discriminating number. Similarly, Bonny and Lourenco (in preparation) found that 4- and 6-year-olds’ judgments based on cumulative area were no more accurate than those based on numerosity, regardless of the type of display used to assess accuracy (see Fig. 2) and even when stimuli in the number task included continuous magnitudes incongruent with number.

The crux of Leibovich et al.’s sense of magnitude theory rests on the observation that continuous magnitudes available within non-symbolic arrays influence numerosity judgments, even in adults with mature language and executive control. According to this argument, the association between number and continuous magnitudes is merely a by-product of visual cues confounded with number in non-symbolic stimuli. However, accumulating behavioral and neural data suggest far deeper connections between number and other magnitudes. For example, even symbolic numbers, for which there are no visual confounds, affect representations of continuous magnitudes: Subliminally primed Arabic numerals bias adults’ cumulative area judgments (Lourenco et al. 2016). Crucially, Lourenco et al. ruled out the possibility of priming effects at a decision stage, arguing instead for representations of number and area that partially overlap. Indeed, recent functional magnetic resonance imaging (fMRI) evidence is consistent with overlapping representations in parietal cortex. Harvey et al. (2015) showed not only that number and area share topographic organization, but also that there was a correlation across these maps, with voxels displaying greater activation for larger numerosity also displaying greater activation for items larger in area – a finding that cannot reflect inhibitory processing, as in other neuroimaging studies discussed by Leibovich et al., because the correlation corresponded to a monotonic increase between magnitudes from different displays.

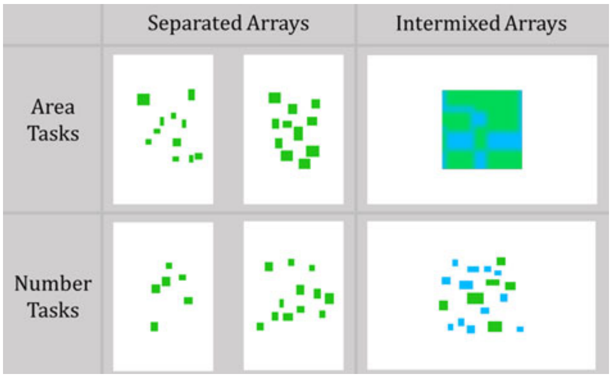

Figure 2. (Lourenco et al.). Examples of stimuli used by Bonny and Lourenco (in preparation) to compare the precision of children’s number and area representations. In number and area tasks, children were asked to designate which array contained the greater magnitude. The left column shows displays that were spatially separated, with children selecting either a left or right array. The right column shows intermixed arrays, with children selecting either blue or green as having greater “paint” (area task) or “boxes” (number task). Cumulative area in this study was tested with both discrete (left column) and amorphous (right column) stimuli. Regardless of stimulus type, children never showed an advantage in making area judgments over number, as would be predicted by Leibovich et al.’s theory. Some studies report greater accuracy for area than for number judgments, but “area” in these studies involved single elements (DeWind & Brannon 2012; Piazza et al. 2013) or uniform displays akin to single elements (Odic et al. 2013). When comparing precision, cumulative area is the more appropriate counterpart to number because, like number, it applies to the set of items. Moreover, although infants and children might show greater sensitivity to other continuous magnitudes such as contour length (Cantrell & Smith 2013), the relevant contrast to number, based on Leibovich et al.’s logic, is cumulative area (or perhaps convex hull) because contour length requires object individuation akin to number.

In summary, although we appreciate the prominence given to continuous magnitudes within Leibovich et al.’s sense of magnitude theory, we have argued that disregarding number representations early in development is a weakness (see also Lourenco & Bonny 2016) and that the emphasis on stimulus confounds misses the deeper underlying connections between numerical and non-numerical representations (see also Holmes & Lourenco 2011; Lourenco 2015; 2016). The theory, though a well-intentioned alternative to extant number sense models, suffers from its own limitations.