

```
In[ ]:= 2 - 2
```

```
Out[ ]:= 0
```

Wieting's Example of Failed Generalized Spectral Decomposition

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```
In[ ]:= a = 3;
```

$$\mathbf{M} = \begin{pmatrix} 1 & a & a \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix};$$

```
In[ ]:= Ω = Eigenvalues[M];
```

$$\omega_1 = \Omega[[1]];$$

$$\omega_2 = \Omega[[2]];$$

$$\omega_3 = \Omega[[3]];$$

```
Table[ωk, {k, 1, 3}]
```

```
Out[ ]:= {1, 1, 1}
```

```
In[ ]:= R = Eigenvectors[M];
```

```
Table[ak = Transpose[{R[[k]]}], {k, 1, 3}];
```

```
Table[MatrixForm[ak], {k, 1, 3}]
```

$$\text{Out[]:= } \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

```
In[ ]:= L = Eigenvectors[Transpose[M]];
```

```
Table[bk = Transpose[{L[[k]]}], {k, 1, 3}];
```

```
Table[MatrixForm[bk], {k, 1, 3}]
```

$$\text{Out[]:= } \left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

```
In[ ]:= Table[Chop[Conjugate[Transpose[ai]].aj][[1]][[1]], {i, 1, 3}, {j, 1, 3}] // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
In[ ]:= Table[Chop[Conjugate[Transpose[bi].bj][[1]][[1]], {i, 1, 3}, {j, 1, 3}] // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
In[ ]:= Table[Chop[Transpose[bi].aj][[1]][[1]], {i, 1, 3}, {j, 1, 3}] // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
In[ ]:= M.  $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ 
```

```
Out[ ]:= {{x + 3 y + 3 z}, {y + 3 z}, {z}}
```

```
In[ ]:= Solve[{x + 3 y + 3 z == x, y + 3 z == y, z == z}, {x, y, z}]
```

 **Solve:** Equations may not give solutions for all "solve" variables.

```
Out[ ]:= {{y -> 0, z -> 0}}
```