EXERCISES

Mathematica 6 ~ Lab Number 0

Problem 1. Ask *Mathematica* about **`?Binomial`** and use that information to evaluate

$$\text{the binomial coefficient } \binom{81}{14}, \text{ pronounced “81 choose 14”}$$

Compare your result with the result obtained by evaluation of

$$\frac{81!}{14!(81-14)!}$$

Evaluate the sum

$$\sum_{k=0}^{n} \binom{n}{k}$$

Problem 2. The infinite sum

$$G \equiv 1 - \tfrac{1}{9} + \tfrac{1}{25} - \tfrac{1}{49} + \cdots = \sum_{k=0}^{\infty} (-1)^k (2k+1)^{-2}$$

turns up frequently in combinatorial contexts. What does *Mathematica* have to say about that sum? Use **`?`** to make sense of its answer, and **`N[ , ]`** to obtain an evaluation to 50 decimal places. Creat a link—named “CatalanBiography”—to the appropriate Wikipedia website.

Problem 3. Ask *Mathematica* about the closely related sum

$$1 - \tfrac{1}{3} + \tfrac{1}{5} - \tfrac{1}{7} + \cdots = \sum_{k=0}^{\infty} (-1)^k (2k+1)^{-1}$$

and obtain an evaluation accurate to 50 decimal places.

Problem 4. Obtain 20-place evaluations of π^π, e^π , π^e and e^e and see what *Mathematica* has to say about the assertion that

$$\pi^\pi > e^\pi > \pi^e > e^e$$

Problem 5. Evaluate the following:

$$\sum_{k=0}^{\infty} x^k$$

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)^4}$$

$$\sum_{k=1}^{\infty} \frac{1}{2^k k^2}$$

$$\prod_{k=1}^{\infty} \left\{1 + \frac{(-1)^{k+1}}{2k-1}\right\}$$

$$x \prod_{k=1}^{\infty} \left\{1 - \frac{x^2}{k^2\pi^2}\right\}$$

To the final result, bring the command **Simplify[%, $x > 0$]**.

Problem 6. Evaluate the indefinite integral

$$\int \frac{1}{1-x^3}\,dx$$

and observe how *Mathematica* responds to the post-command

TraditionalForm[%]

Also evaluate

$$\int \frac{1}{1-x^5}\,dx$$

$$\int \frac{1}{1-x^7}\,dx$$

and render the last result both as standard output and in **TraditionalForm**.

Problem 7. Ask what *Mathematica* has to say in response to the query **?Table**. Use that information to construct a table of the values assumed by

$$\sum_{k=1}^{n} k \quad : \quad n = 1, 2, \ldots, 10$$

and give that table (list) the name **triangularnumbers**. Do the same for

$$\sum_{k=1}^{n} k^3 \quad : \quad n = 1, 2, \ldots, 10$$

Command **triangularnumbers^2**. What do you conclude? Does *Mathematica* support your conjecture? Ask for a respond to the assertion

$$\sum_{k=1}^{n} k^3 = \left[\sum_{k=1}^{n} k\right]^2$$