Demonstrating a Negative index of Refraction

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__________________________
Lucas Illing
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This thesis involved bringing together a lot of different parts from a lot of different places, and it would have never gotten very far at all if not for the help of a few people.

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# Table of Contents

**Introduction** .................................................. 1

**Chapter 1: Theory** ........................................ 5
  1.1 Maxwell’s Equations in Matter .......................... 5
  1.2 Linear Media ............................................... 6
  1.3 Waves in Matter ........................................... 6
  1.4 A Complex Index of Refraction .......................... 7
  1.5 Snell’s Law at the Boundary of a NIM ................. 10

**Chapter 2: Building a Negative Index Material** .......... 13
  2.1 The Drude Model .......................................... 13
  2.2 Parallel Plates ........................................... 14
  2.3 Split Ring Resonators ................................... 15
  2.4 Combining the Effects: A Metamaterial ................. 16

**Chapter 3: Experimental Methods** .......................... 19
  3.1 Equipment ................................................ 19
    3.1.1 Klystron oscillator ................................ 19
    3.1.2 Voltage controlled oscillator ..................... 19
    3.1.3 Detector ............................................. 21
  3.2 Metamaterial Fabrication ................................ 22
  3.3 Experimental Setup ...................................... 23
  3.4 Null Test ................................................ 24
  3.5 Characterization of the Metamaterial ................. 26

**Conclusion** ................................................... 33

**References** ................................................... 35
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Refraction of light at an interface between materials.</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Refraction of light at an interface with a NIM.</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>Split ring resonator and NIM construction.</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>NIM planar lens compared to conventional lens.</td>
<td>3</td>
</tr>
<tr>
<td>1.1</td>
<td>Square root mapping for $\tilde{n} = \sqrt{\tilde{\varepsilon}_r \tilde{\mu}_r}$.</td>
<td>8</td>
</tr>
<tr>
<td>1.2</td>
<td>Positive $Re(\tilde{\varepsilon}_r)$ and $Re(\tilde{\mu}_r)$.</td>
<td>9</td>
</tr>
<tr>
<td>1.3</td>
<td>Negative $Re(\tilde{\varepsilon}_r)$ and Positive $Re(\tilde{\mu}_r)$.</td>
<td>9</td>
</tr>
<tr>
<td>1.4</td>
<td>Negative $Re(\tilde{\varepsilon}_r)$ and $Re(\tilde{\mu}_r)$.</td>
<td>10</td>
</tr>
<tr>
<td>1.5</td>
<td>Direction of fields at interface between NIM and conventional material.</td>
<td>11</td>
</tr>
<tr>
<td>2.1</td>
<td>Direction of fields in a non-magnetic material with a negative permittivity.</td>
<td>15</td>
</tr>
<tr>
<td>2.2</td>
<td>Direction of fields through a split ring resonator.</td>
<td>16</td>
</tr>
<tr>
<td>2.3</td>
<td>Polarization of fields incident on metamaterial.</td>
<td>17</td>
</tr>
<tr>
<td>3.1</td>
<td>Power spectrum of klystron oscillator output.</td>
<td>20</td>
</tr>
<tr>
<td>3.2</td>
<td>Power spectrum of voltage controlled oscillator output.</td>
<td>21</td>
</tr>
<tr>
<td>3.3</td>
<td>Oscillator Circuit Ordering.</td>
<td>21</td>
</tr>
<tr>
<td>3.4</td>
<td>Power spectrum of voltage controlled oscillator output with amplifier and multiplier.</td>
<td>22</td>
</tr>
<tr>
<td>3.5</td>
<td>Image of printed SRR design.</td>
<td>23</td>
</tr>
<tr>
<td>3.6</td>
<td>Experimental setup.</td>
<td>24</td>
</tr>
<tr>
<td>3.7</td>
<td>No prism data.</td>
<td>25</td>
</tr>
<tr>
<td>3.8</td>
<td>PVC Prsim data.</td>
<td>27</td>
</tr>
<tr>
<td>3.9</td>
<td>Frequency dependence of power.</td>
<td>28</td>
</tr>
<tr>
<td>3.10</td>
<td>Metamaterial Prism data at 9.45 GHz.</td>
<td>29</td>
</tr>
<tr>
<td>3.11</td>
<td>Metamaterial Prism data at 9.25 GHz.</td>
<td>30</td>
</tr>
<tr>
<td>4.1</td>
<td>Resonator detail.</td>
<td>34</td>
</tr>
</tbody>
</table>
Abstract

A metamaterial intended to have a negative index of refraction is fabricated using aluminum plates and split ring resonators printed onto circuit boards. The angle of refraction is used to characterize the index of refraction of the material. A prism constructed out of the metamaterial is found to result in several refracted beams, one of which is consistent with the predicted angle of refraction. Due to the presence of other beams, the material is not the homogeneous linear negative index of refraction metamaterial that was the intended goal. It behaves in a more complex manner.
Introduction

Shine light through a prism and it will cast a rainbow. A specially shaped piece of glass will reveal a full spectrum of colors in the light. A diffracting prism has a frequency dependent index of refraction. That is, light of different wavelengths (colors) are slowed by different amounts as they pass through it. At the surface of the glass, the light changes direction as it moves from one substance to another. We can predict this change of direction using Snell’s law.

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2. \] (1)

![Figure 1: Refraction of light at an interface between materials.](image)

In Fig. 1, \( \theta_1 \) and \( \theta_2 \) are the angles of a chosen beam of light, measured from the normal to the interface and \( n \) is the value for the corresponding material’s index of refraction. In the case of the diffracting prism, this value also depends on the wavelength of the light making the transition. The colors appear when the many frequencies that make up white light are separated by the prism. It bends each one in a slightly different direction.

As light passes through any matter, it interacts with the charged particles that are the medium. On a macroscopic scale, we observe a change in the phase velocity and wavelength of the light as it passes through the material. The index of refraction for this medium is the ratio of the phase velocity of light in a vacuum to the phase velocity of light within the material,
\[ n = \frac{c}{v_{\text{phase}}}. \quad (2) \]

The speed of light in a vacuum, \( c = 3.00 \times 10^8 \text{ m/s} \), and the phase velocities of light in typical materials are large enough that measuring the speed of light directly requires very precise equipment. It is much more practical to measure a value of \( n \) using Snell’s law. The index of refraction for air is very close to 1. The gaseous medium does comparatively little to slow down light. Thus, if we consider air to be our first medium, the index of refraction for the second medium can be found by measuring the angle of the incident light, \( \theta_i \), and the angle of the refracted light, \( \theta_r \):

\[ n(\text{unknown}) = \frac{\sin \theta_i}{\sin \theta_r}. \quad (3) \]

This thesis involves the fabrication of a prism with a negative index of refraction. This is something of a novel concept. No object found in nature has a negative value for \( n \); negative index materials (NIMs) have only recently become a topic of academic research [1]. Consider a beam of light incident on a NIM. Equation (1), Snell’s Law, requires that the refracted angle have sign opposite that of the incident angle as shown in Fig. 2.

![Figure 2: Refraction of light at an interface with a NIM.](image)

Negative index materials are not found in nature. Instead, they are part of a group of artificial materials called metamaterials. Unlike conventional materials, which interact with light based on their chemical composition, the properties of metamaterials come from their geometric structure. The metamaterial used in this thesis utilizes a combination of waveguides and etched circuits called split ring resonators, as shown in Fig. 3. Together they produce a negative index of refraction at a specific microwave frequency.

Metamaterials are a very active current area of research. In optics, metamaterials are used to create planar lenses, as can be seen in Fig. 4. Conventional lenses use curved surfaces of glass or plastic to refract light in the desired direction [2]. The
curved nature of these lenses, however, leads to aberrations and distortion. Using
metamaterials, it is possible to construct a lens which refracts light in the desired
direction with only planar interfaces, thereby avoiding any aberrations [3].

Another area of research involves focusing high intensities of microwaves in or-
der to perform a wireless transfer of electric energy [4]. Furthermore, metamaterial
antennae are already in common use. They have structures that resonate with in-
coming light in order to produce a higher gain received signal. Most metamaterials,
including the one associated with this thesis, are designed to work in the microwave
spectrum. Microwaves have wavelengths on the order of a few centimeters, meaning
that metamaterials designed to interact with them share that convenient scale. Be-
yond microwaves, some research groups are attempting the much more difficult task
of producing a metamaterial that interacts with visible light [5]. Since visible light has
a wavelength only a few hundred nanometers long, visible spectrum metamaterials
are very difficult to fabricate.

Metamaterials also have some more exotic uses. Optical cloaking involves bending
light around an object such that no detectable scattering occurs far from the object [3].
Metamaterials can also be designed to work with sound or other forms of propagating
waves. It has been suggested, for example, that seismic metamaterials can be designed
to protect buildings from earthquakes [1].
This thesis seeks to fabricate a metamaterial prism that demonstrates a negative index of refraction via Snell’s law. The intention is to create a process for the simple fabrication and characterization of these NIMs in an undergraduate laboratory.
Chapter 1

Theory

1.1 Maxwell’s Equations in Matter

In classical physics, light is characterized by propagating transverse electric and magnetic fields. These fields satisfy Maxwell’s Equations. In matter, and without free charges or currents, Maxwell’s equations are

\[ \nabla \cdot \mathbf{D} = 0 \quad (1.1) \]

\[ \nabla \cdot \mathbf{B} = 0 \quad (1.2) \]

\[ \nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} \quad (1.3) \]

\[ \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \quad (1.4) \]

where \( \mathbf{D} \) and \( \mathbf{H} \) are the electric and magnetic auxiliary fields. We also need to consider the polarization and magnetization fields \( \mathbf{P} \) and \( \mathbf{M} \). These fields are the distributions of electric and magnetic dipoles in the material. Together with the electric and magnetic fields, these fields define the auxiliary fields

\[ \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \quad (1.5) \]

\[ \mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M} \quad (1.6) \]

where \( \varepsilon_0 \) and \( \mu_0 \) are the permittivity and permeability of free space. These equations are completely general on a non-relativistic macroscopic scale [6]. However, alone they are insufficient to find any useful results. We need to specialize to a case where we have some information about the relationship between the auxiliary fields and the electric and magnetic fields.
1.2 Linear Media

Linear media are a set of materials where the auxiliary fields are linearly related to the others by the constitutive relationships

\[ \mathbf{P} = \epsilon_0 \chi_e \mathbf{E} \]  
\[ \mathbf{M} = \chi_m \mathbf{H}. \]  

Here, \( \chi_e \) and \( \chi_m \) are the electric and magnetic susceptibilities of the medium. They represent the degree to which the dipoles in the material are affected by the total fields in the medium. Using Eqs. (1.5) and (1.6), we can define the relative permittivity \( \epsilon_r \) and the relative permeability \( \mu_r \), and write the relationships

\[ \mathbf{D} = \epsilon_0 (\chi_e + 1) \mathbf{E} = \epsilon_0 \epsilon_r \mathbf{E} \]  
\[ \mathbf{H} = \frac{1}{\mu_0} \frac{1}{\chi_m + 1} \mathbf{B} = \frac{1}{\mu_0} \frac{1}{\mu_r} \mathbf{B}. \]  

Note that in vacuum the polarization and magnetization fields are both zero. Here, \( \epsilon_r \) and \( \mu_r \) are both one, so the auxiliary fields are entirely determined by the electric and magnetic fields [6].

1.3 Waves in Matter

Using Eqs. (1.9) and (1.10), we can write Eqs. (1.1) and (1.4) entirely in terms of the electric and magnetic fields.

\[ \nabla \cdot \mathbf{E} = 0 \]  
\[ \nabla \times \mathbf{B} = \epsilon_0 \epsilon_r \mu_0 \mu_r \frac{\partial \mathbf{E}}{\partial t}. \]  

Together with Eqs. (1.2) and (1.3), we can derive a form of the wave equation for waves in matter, as we do for fields in a vacuum:

\[ (\nabla^2 - \epsilon_0 \epsilon_r \mu_0 \mu_r \frac{\partial^2}{\partial t^2}) \mathbf{E} = 0 \]  
\[ (\nabla^2 - \epsilon_0 \epsilon_r \mu_0 \mu_r \frac{\partial^2}{\partial t^2}) \mathbf{B} = 0 \]  

Note that Eqs. (1.12) and (1.3) let us determine the magnetic field from the electric field, to a phase shift. Thus, as in a vacuum, we can discuss only the electric field without loss of generality. The form of the wave equation gives us the relationship

\[ \epsilon_0 \epsilon_r \mu_0 \mu_r = \frac{1}{v_{\text{phase}}^2}. \]
From the wave equation for fields in a vacuum, we know that the speed of light $c$ can be found from the permittivity and permeability of free space,

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}. \quad (1.16)$$

Using Eq. (2), we can derive the relationship between Maxwell’s equations and the index of refraction,

$$n = \sqrt{\varepsilon_r \mu_r}. \quad (1.17)$$

While not very realistic, it is useful to consider the plane wave set of solutions to Eq. (1.13). With no envelope, these solutions involve planes formed by uniform magnitude electric field vectors that move at the wave’s phase velocity and are oriented perpendicular to the axis of propagation. For an electric field polarized in the $x$ direction and propagating along the $z$-axis, a plane wave has the form

$$E(r, t) = E_0 \text{Re}[e^{-i(kz - \omega t)}] \hat{x}, \quad (1.18)$$

where $E_0$ is the amplitude of the wave and $\omega$ is the oscillatory frequency of the wave in radians. $k$ is the wave vector, defined as

$$k = \frac{2\pi n}{\lambda_0}; \quad (1.19)$$

where $\lambda_0$ is the vacuum wavelength of the light.

### 1.4 A Complex Index of Refraction

For waves in a material, we also must consider a complex valued index of refraction,

$$\tilde{n} = n + i\kappa. \quad (1.20)$$

Here, the real part, $n$, is the ratio described in Eq. (2). The imaginary part, denoted $\kappa$, is called the extinction coefficient [3]. It corresponds to the rate at which light is absorbed by the material. This can be seen by rewriting Eq. (1.18) in terms of $n$ and $\kappa$;

$$E(r, t) = E_0 \text{Re}[\exp(-i(2\pi \frac{\tilde{n}z}{\lambda_0} - \omega t))] \hat{x}, \quad (1.21)$$

which reduces to

$$E(z, t) = E_0 \exp(-2\pi \frac{\kappa}{\lambda_0} z) \cos(2\pi \frac{n}{\lambda_0} z - \omega t) \hat{x}. \quad (1.22)$$

Note that the oscillating cosine term is now modulated by an exponentially decaying envelope dependent on $\kappa$. This constant describes the rate at which the wave is absorbed by the material as it passes through it. Like the index of refraction, the
relative permittivity and permeability also are complex. It is useful to consider these values in terms of a complex phase,

\[ \tilde{\epsilon}_r = |\epsilon_r|e^{i\theta} \]  

(1.23)

\[ \tilde{\mu}_r = |\mu_r|e^{i\phi}. \]  

(1.24)

Here, \( \phi \) and \( \theta \) are complex phases on the interval \([-\pi, \pi]\) [3]. When taking a square root function of a complex number, we must consider both a positive and negative valued square root. Thus, the square root of a complex number has two possible values for its square root, only one of which corresponds to our physical system. When applying this logic to our index of refraction, Eq. (1.17) becomes

\[ \tilde{n} = \pm \sqrt{\epsilon_r\mu_r} e^{i\frac{1}{2}(\theta+\phi)}. \]  

(1.25)

Here, the physical result is the one in which the imaginary part, \( \kappa \) is positive. Just as a positive extinction coefficient corresponds to the exponential decay of a wave in a medium, a negative value for \( \kappa \) corresponds to exponential growth. Here, we are only considering passive materials, where there is no external energy source. In order to satisfy conservation of energy, then, we must select the value of the square root in Eq. (1.25) such that \( \kappa \) is positive. Thus, our square root mapping becomes the one shown in Fig. 1.1.

![Figure 1.1: Square root mapping the complex number \( \tilde{\epsilon}_r\tilde{\mu}_r \) to the complex number \( \tilde{n} = \sqrt{\epsilon_r\mu_r} \) and selecting the sign of the square root such that Im(\( \tilde{n} \)) = \( \kappa \) is positive.](image)

The vast majority of materials are non-magnetic [2]. For these media, the magnetic dipoles of the atoms within the material are small or misaligned. As a result, the magnetization field is close to zero, and Eqs. (1.5) and (1.10) show us that the relative permeability is very nearly positive real valued with a magnitude of one.

For non-magnetic materials that are transparent to the wave, such as glass, there are only very small losses in the electric field as the wave propagates through the medium. This case is similar to our real valued case where the complex phases \( \theta \)
and $\phi$ are both very close to zero, and $\tilde{\mu}_r$ and $\tilde{\varepsilon}_r$ are dominated by their real parts. As a result, the wave easily propagates through the material. Figure 1.2 shows the mapping for this case.

For reflective materials such as metal, $\tilde{\varepsilon}_r$ is primarily real and negative. Using, Eq. (1.9), we can interpret this case as the electric field, $\mathbf{E}$, having opposite direction as its corresponding auxiliary field, $\mathbf{D}$.

Equation (1.25) shows us that the physical result of this interaction is an index of refraction dominated by its imaginary part, $\kappa$, as in Fig. 1.3. This produces a very over-damped oscillating field that quickly decays to zero as the wave enters the medium. This is called an evanescent wave [6]. Such materials are opaque (and highly reflective).

In order to achieve a negative valued index of refraction, we must find a material with magnetic and electric response such that the real parts of both $\tilde{\varepsilon}_r$ and $\tilde{\mu}_r$ are negative. In this case, the square root mapping requires that the real part of the value
Chapter 1. Theory

1.5 Snell’s Law at the Boundary of a NIM

At an interface between materials, such as the one considered in Eq. (1), Snell’s law, components of the electromagnetic fields are continuous. These continuity equations demand that components of the $E$ and $H$ fields parallel to the interface remain continuous, as well as the components of the $B$ and $D$ fields perpendicular to the interface [6].

Consider a plane wave incident on a medium with a negative index of refraction. All the interface conditions hold, but Eqs. (1.9) and (1.10) demand that the auxiliary fields have direction opposite to their corresponding electric and magnetic fields. Figure 1.5 shows the direction of fields as a polarized wave crosses into a medium with either a positive or negative index of refraction.

Note that the continuity conditions are met in both cases. Also pictured is the Poynting vector, $S = E \times H$, which describes the direction of energy transfer [6]. Note that within the negative index material, the unusual phenomenon occurs where the wave vector points in the opposite direction of the direction of energy transfer.
1.5. *Snell’s Law at the Boundary of a NIM*

Figure 1.5: Direction of fields at interface between NIM and conventional material.
Chapter 2

Building a Negative Index Material

2.1 The Drude Model

In a conductor, electrons are free to move about the material. We can treat conducting metals as a plasma of electrons scattering off of stationary protons. The Drude model of electrical conduction assumes that electrons can be treated classically as they move throughout the solid. For a non-magnetic material, these particles have a negligible interaction with the magnetic field. So, starting with the Lorentz force law, we can write Newton’s 2nd law for an electron as

$$\frac{d}{dt} \langle p \rangle = qE - \frac{\langle p \rangle}{\tau}, \quad (2.1)$$

where $\langle p \rangle$ is the average momentum of the electrons in the material, $q$ is the charge of one electron, and $\tau$ is the mean free time between electron collisions with protons. The $\frac{\langle p \rangle}{\tau}$ term corresponds to friction due to the inability of electrons to pass through the conductor unimpeded. We can find the average displacement of these electrons using

$$\langle p \rangle = m_e \frac{d}{dt} \langle r \rangle \quad (2.2)$$

by taking an anti derivative of Eq. (2.1). This gives us

$$\frac{d}{dt} \langle r \rangle = -\frac{e}{m_e} \int E(r, t) \, dt - \langle r \rangle \frac{1}{\tau} \quad (2.3)$$

Now, we can specialize to the case of an electromagnetic wave moving though the metal. Using the complex polarized plane wave solution, Eq. (1.18), as the form of the electric field, we can write the above equation in terms of the angular frequency of the wave and the time dependent component of the electric field,

$$\frac{d}{dt} \langle x \rangle = -\frac{e}{m_e} \omega \bar{E}(z) \, e^{-i\omega t} - \frac{\langle x \rangle}{\tau}. \quad (2.4)$$

We can solve this differential equation to find an expression for the average displacement of each electron over time, in the asymptotic limit (large $t$):
\[ \langle x \rangle = - \frac{e}{m_e \omega^2} \left( \frac{1}{1 + i \frac{1}{\omega \tau}} \right) \hat{E}(z) \, e^{-i\omega t}. \] (2.5)

The displaced electrons act as electric dipoles. By averaging over all of these dipole moments, we can find an expression for the polarization field of the material in terms of the frequency of the wave and the electric field. The polarization field is defined as

\[ \mathbf{P} = n_e \, \mathbf{e} \, \langle \mathbf{r} \rangle. \] (2.6)

where, \( n_e \) is the number of electrons per unit volume. Equation (2.6) gives us a relationship between the electric and polarization fields,

\[ \mathbf{P} = -\frac{n_e e^2}{m_e} \frac{1}{\omega^2} \left( \frac{1}{1 + i \frac{1}{\omega \tau}} \right) \mathbf{E}. \] (2.7)

From equation (1.7), we find \( \chi_e \) in terms of the frequency,

\[ \chi_e(\omega) = -\frac{\omega_p^2}{\omega^2} \left( \frac{1}{1 + i \frac{1}{\omega \tau}} \right), \] (2.8)

where \( \omega_p \) is the conductor’s plasma frequency, defined as

\[ \omega_p = \sqrt{\frac{n_e q^2}{m_e \varepsilon_0}}. \] (2.9)

For most materials, we can choose a frequency much greater than the characteristic frequency \( \frac{1}{\tau} \). Thus, the \( \frac{1}{\omega \tau} \) term will be much less than one. In this case, we can simplify Eq. (2.7) to write a real expression for the relative permittivity,

\[ \varepsilon_r = 1 + \chi_e = 1 - \frac{\omega_p^2}{\omega^2}. \] (2.10)

The charge carrier density for aluminum is about \( 6.02 \times 10^{28} \text{ electrons/m}^3 \). From this, we can find that the plasma frequency for electrons in aluminum is \( 1.38 \times 10^{16} \text{ rad/s} \). This equation, then, shows that for frequencies below the x-ray range, aluminum has a relative permittivity that is very negative, with a comparatively small positive imaginary part. From Eq. (1.25), we can see that this results in a complex index of refraction dominated by a large \( \kappa \). Physically, this is consistent with what we know about aluminum; incident waves on the metal are reflected, and any wave that propagates inside such a conductive material decays to zero amplitude almost instantly (Fig. 2.1).

### 2.2 Parallel Plates

While a slab of aluminum is an effective way to obtain a negative permittivity, the magnitude of this value is much larger than is practical to work with. Additionally,
there is no way to produce a magnetic response from a homogeneous conductor made of aluminum, much less a negative response.

The electric field of a linearly polarized beam of light only oscillates in one transverse direction. To obtain the desired response from this beam, our material need only have a negative permittivity in the direction the beam is polarized. This can be achieved by, for example, a parallel plate waveguide consisting of conducting metal sheets in the plane spanned by the direction of the propagation vector and the direction of the polarization vector. This waveguide has an effective plasma frequency that depends on the spacing $d$ of the metal sheets,

$$\omega_p = \frac{\pi c}{d}. \quad (2.11)$$

This relationship is based on the fraction of the allowed transverse electric modes propagating through a parallel plate waveguide [7]. We can use Eq. (2.11) to rewrite Eq. (2.10) as

$$\epsilon_r = 1 - \left(\frac{\pi c}{d \omega}\right)^2. \quad (2.12)$$

Waveguides have the additional benefit of only occupying a portion of the space in our material. There is room left for a material with a magnetic response. A final use for waveguides in this project is that they can also be used to polarize light. Light polarized in the direction transverse to the planes of the plates will encounter very large impedance as it passes through the material [2]. Thus, waveguides require us to use polarized light, but also give us the means to easily polarize light.

### 2.3 Split Ring Resonators

While a negative permittivity is a common occurrence, a negative permeability is not found in nature. Equation (1.10) shows that a negative relative permeability requires the magnetic field, $B$ and its auxiliary field, $H$ to point in opposite directions. Using Eq. (1.6), we note that this can be accomplished with a sufficiently large magnetization field.
A split ring resonator is a geometric design of conductors on dielectric materials that resonates with the magnetic portion of the wave, producing a large magnetic dipole [8]. As a magnetic field oscillates back and forth through a split ring resonator, it induces a current that flows through the ring. The two rings that make up the resonator are coupled, they act as a circuit with a capacitance and an inductance. As a result, the rings have a resonant frequency at which the maximum oscillation amplitude of the current induced in the rings is much greater than at other frequencies. In some aspects, these rings act as cavity resonators. The main difference to typical resonators is scale, the split ring resonators are much smaller than the wavelength of the wave they resonate with.

When at resonance, the current oscillating in the split ring resonators induces a magnetic dipole. By averaging over many magnetic dipoles, we can find the magnetization field, as shown in Fig. 2.2. If the spacing between resonators is small enough, the material can have a magnitude of $\mathbf{M}$ at resonance large enough to have the $\mathbf{B}$ and $\mathbf{H}$ fields point opposite directions. This, as seen in Eq. (1.10), implies a negative value for the relative permeability [8].

Calculating the permeability due to the resonators is a numerical analysis beyond the scope of this thesis. It involves considering the inductance, capacitance and resistance of small areas along the perimeter of the ring [9]. By summing over all these areas, the current through the ring and the induced fields can be found. This method is based on the methods used to engineer antennae.

### 2.4 Combining the Effects: A Metamaterial

Like the permittivity of our material, we only need the permeability to be negative in the direction of the magnetic field. We can accomplish this by stacking sheets of split ring resonator arrays together with the desired spacing of metal plates of the waveguide (Fig. 2.3). The horizontally polarized electric field interacts with the metal plates while the vertically polarized magnetic field interacts with the split ring resonators. This metamaterial is referred to as a one dimensional negative index material. The material only works for a single polarization of light traveling in a
2.4. Combining the Effects: A Metamaterial

Figure 2.3: Polarization of fields incident on metamaterial.

plane parallel to that of the sheets in the metamaterial.

Either a negative relative permittivity due to waveguides or a negative permeability due to resonators would alone cause a wave to decay inside the material, but together they allow microwaves to propagate through. One way to consider the interaction between these effects is to think of the magnetic fields produced by the resonators as propagating forward the evanescent electric wave between the metal plates. The result is a material that acts as a linear material with a negative index of refraction on a scale larger than its smallest structure (the split ring resonator).
Chapter 3

Experimental Methods

3.1 Equipment

3.1.1 Klystron oscillator

In this experiment, a klystron oscillator was used to test the viability and precision of the experimental setup for determining the index of refraction of a prism. It was eventually replaced by voltage controlled oscillator circuits for the characterization of the metamaterial.

The klystron oscillator I used is part of a demonstration kit for a 1970s stock course in physics called Project Physics. It uses a type 6116 reflex klystron oscillator tube to produce linearly polarized radiation at 9.8 GHz \[10\]. It has an attached horn that matches the impedance of the tube to the impedance of air. The microwave signal produced by the klystron is a very clean sine wave, the power spectrum of the klystron’s output (Fig. 3.1) shows that the klystron frequency is close to 30 decibels greater in strength than any other frequency. This data was collected on an Agilent D5SO 81204A Oscilloscope with a bandwidth of 12 GHz. The power spectrum was calculated using a discrete Fourier transform.

3.1.2 Voltage controlled oscillator

In order to test the metamaterial’s response over a range of frequencies, I used a tunable oscillator circuit as a source of microwaves. The oscillator was a Mini-Circuits ZX95-5400+ voltage controlled oscillator (VCO). It has a control voltage that sets the frequency at which the circuit oscillates. The VCO could produce frequencies between 4,300 and 5,400 MHz at -1.3 dBm. At 4,600 MHz, the VCO produced the power spectrum seen in Fig. 3.2. The main peak in the power spectrum is notably wider than in the klystron oscillator power spectrum. It is still about 20 decibels above the next highest peak, which occurs at the first harmonic frequency.

In order to produce a signal at roughly the same frequency as the klystron, an amplifier and \(\times 2\) frequency multiplier were connected to the VCO. The amplifier was a Mini-Circuits ZX60-V82+, with a frequency range of 20 to 6,000 MHz. At the frequencies I was interested in, the amplifier produced about a 10 dB gain.
This amplified signal was passed through one of two $\times2$ frequency multipliers. Mini-Circuits ZX90-2-50+ input a signal between 3,500 and 5,000 MHZ and output a signal between 7,000 and 10,000 MHz. Mini-Circuits ZX90-2-24+ requires input signals between 5,000 and 10,000 MHZ and produces output signals between 10 and 20 GHz. Both amplifiers had similar losses of about -11 dB. Together, these circuits could output a signal between 8.6 and 10.8 GHz at about -2 dBm.

With the amplifier and frequency multiplier, the circuit produced the power spectrum seen in Fig 3.4. The data for this spectrum was taken at the same VCO frequency as the spectrum in Fig. 3.2. Note that the frequency that was before the first harmonic is now the dominant frequency. This peak is about 15 dB above the VCO frequency.

The microwave from the frequency multiplier was passed through a JDS Uniphase Model H301 wide-band amplifier. This amplifier increased the power of the signal by a factor of 15. Then, to polarize the signal, it was passed through a Narda Microwave Model 640 rectangular waveguide. The ordering of these circuits together is shown in Fig. 3.3. The waveguide only allows a linearly polarized component of the electromagnetic wave to pass. This waveguide is able to guide microwaves with frequencies between 8.2 and 12.4 GHz. As a result, other frequencies, such as the VCO frequency, should be further suppressed in the waveguide. Finally, a gain horn was used to match the impedance of the waveguide to the impedance of the air, and emit the microwave as a wave propagating in free space (air).
3.1. Equipment

3.1.3 Detector

I used a silicon diode in a resonator cavity waveguide to detect the intensity of microwaves at a given angle. The diode produces a DC voltage when aligned with an oscillating electric field. This diode was designed to work at the same range of microwave frequencies as the waveguide. To match the signal from the air into the cavity, I used a gain horn identical to the one used with the microwave source.

The wavelength of radiation at the relevant range of frequencies is about 3 cm. The maximum length of the resonator cavity is about 9 cm, or 3 wavelengths. This length was adjustable using a screw, in order to align the diode in the cavity with an anti-node in the oscillating electric field. Before taking any measurement, it is necessary to adjust the cavity to maximize the diode’s output voltage.
3.2 Metamaterial Fabrication

The split ring resonators used in this thesis are based on a design in an Optics Express paper by E. Ekmekci and co-authors [11]. They are printed on 0.5mm thick FR4 laminate, which has a relative permittivity of 4.6 at microwave frequencies, and is, by itself, non-magnetic. The resonators are pictured in Fig. 3.5. They are predicted to resonate at a frequency of 9.48 GHz. FR4 is very available and inexpensive, but it is a relatively absorbent material for microwaves. Water molecules, which absorb microwaves, are trapped within the laminate [10]. This makes FR4 less than ideal for experimenting with microwaves. It is used in this experiment since it is the most available and least expensive laminate available for commercial printing.

The metal plates are aluminum sheet metal also about 0.5 mm thick. Cardboard was used as a spacer between alternating metal plates and resonator boards, since it is transparent to microwaves. The spacing between plates is about 11mm. Equation 2.11 gives the plasma frequency for this spacing as 13.6 GHz. For a 9.5 GHz wave, Eq. 2.12 gives a relative permittivity of -1.06.

The stack of aluminum plates, cardboard spacers, and resonator laminates is the metamaterial fabricated in this thesis. To characterize it, each part is cut and re-stacked into a 20 degree prism. In addition, a 20 degree polyvinyl chloride (PVC) prism is cut. It is used as a check to verify that the characterization procedure can accurately identify a traditional index of refraction. For microwaves, the index of refraction of PVC is 1.7 [7].
3.3 Experimental Setup

The experimental setup for determining the index of refraction of a prism is shown in Fig. 3.6.

The diameter of the circle would be the total travel distance of the wave from the microwave source horn to the detector horn. I chose this distance to be 30 cm, or about 10 wavelengths. The shield was positioned so that only the portion of the wave passing directly through the center of the circle would be able to pass. A shelf was attached behind the opening in the center of the shield so that the radiation passing through could then pass through a prism. If I placed a prism there, the wave would enter the prism normal to the first surface, then refract through the slanted surface on the other side. The central point of this far surface would be centered directly above the center of the circle.

I placed the detector so that it could be moved around the perimeter of the circle. I marked 5 degree increments around the circle on the appropriate semicircle. This spacing was based on the width of the receiving gain horn, which was about 6 cm. The horn covered about 10 degrees of space total. The voltage output by the detector diode would be proportional to the total power radiated within that arc. This set the resolution of the angular data at about half the detector’s angular width: 5 degrees.

For the shielding around the prism, I initially used bags of water as an absorbent medium. They were able to reduce the signal passing through them by about 2 dB. These were later replaced by a reflective aluminum foil shield, which was opaque to the microwave signal.
3.4 Null Test

The first experiments with the experimental setup were done using the klystron tube as a microwave source and bags of water as shielding. With no prism, the detector diode output voltage was recorded at five degree integrals about the perimeter. This procedure was considered a test of the experimental setup. The relationship between angle and measured power was a distribution about the shield normal and peaked in the center, as expected for a case with no refraction. By taking a weighted average, I found that the center of the distribution was -1.2 degrees relative to the shield normal. I used this calculation as a starting point to determine the uncertainty in my angular measurements.

In each experiment, I also recorded the diode voltage due to background microwave waves. I was also careful to keep myself out of the microwave beam whenever possible, especially my eyes.

Later, I replaced the klystron tube with the oscillator circuits in Fig. 3.3. I also replaced the absorptive water shield with a reflective aluminum foil one. I repeated the aforementioned experiment using the new equipment. The results of this experiment are shown in Fig. 3.7.

This data is also a distribution centered about the shield normal. The weighted
3.4. Null Test

Figure 3.7: No prism: microwave power as a function of angle relative to shield normal.
average angle is well within the experimental uncertainty of zero degrees. Also note that the microwave signal does not exhibit diffraction or any other periodic behavior as it passes through the opening in the shield.

Next, I placed the 20 degree PVC prism in the shield opening. I then performed a similar experiment, where instead of measuring the angle relative to the shield normal, I measured the angle relative to the prism normal. Using 1.7 as the index of refraction of PVC for microwaves, I used Snell’s law (Eq. (1)) to calculate the predicted angle of refraction as 36 degrees relative to the prism normal. The data collected from this procedure is shown in Fig. 3.8.

Taking a weighted average of an equal number of points about the peak of the distribution, I found that the central angle of refraction was 36±1 degrees relative to the prism normal, which is consistent with the predicted angle. I also noted that this distribution had roughly the same width and total power as the power distribution for the no refraction experiment.

From these measurements, I concluded that this experimental setup was sufficient for determining the angle of refraction, and thus the index of refraction, for prisms made of homogeneous materials.

For all these experiments, I found that the background reading of the detector diode was at least 4 decibels smaller than any single measurement. This source of systemic error has a negligible contribution to the uncertainty in these measurements. This does not, however, account for background radiation due to dispersion of the microwaves as they pass through the prism.

3.5 Characterization of the Metamaterial

Using the voltage controlled oscillator as a microwave source, I was able to determine the frequency dependence of the signal propagating through the metamaterial. I began by placing the metamaterial prism I had constructed into the shield opening. I then moved the detector closer to the prism, so that the horn profile covered a larger arc. I placed the detector horn so that it covered the range of angles over which I expected a negative angle of refraction to occur. In this setup, I took power measurements over a range of frequencies near the expected resonance frequency of 9.48. This data is depicted in Fig. 3.9.

The function I found shows a maximum transmittance at a source frequency of 9.25 GHz. In addition, there was a local maximum in the transmittance at a source frequency of 9.45 GHz. After moving the detector back to its place on the edge of the circle, I repeated the procedure of measuring diode voltage as a function of angle around the circle. At first, I performed this experiment at a source frequency of 9.45 GHz. The data collected in this experiment are shown in Fig. 3.10.

This intensity distribution shows at least four separate peaks. The largest of these peaks is at a position where we would expect to see a wave refracted due to a prism with a positive index of refraction. The second largest peak is where we would expect to see a wave refracted due to a prism with a negative index of refraction. Near the prism’s normal there are two smaller peaks.
Figure 3.8: PVC Prism: microwave power as a function of angle relative to prism normal.
Chapter 3. Experimental Methods

A first conclusion is that the prism does not approximate a homogeneous material. Instead of the single angle of refraction we would expect from a homogeneous prism made of a linear medium, we find four peaks. All peaks show significant losses compared to both the no-prism and PVC prism signals. The leftmost peak in Fig. 3.10 likely corresponds to a portion of the wave refracting through the prism conventionally, though with significant losses. The other peaks, however, are not at an angle at which we would expect to see refraction through a conventional prism.

The metamaterial does direct a portion of the wave at a reasonable negative angle of refraction. It also does so at a frequency (9.45 GHz) roughly consistent with the expected resonance frequency of the resonators (9.48 GHz).

I repeated this experiment using the frequency at which I measured maximum transmittance (9.25 GHz) as my source frequency. The data collected are shown in Fig. 3.11.

Here, the signal has three major peaks. The largest peak is at roughly the same angle and at the same power as in the previous experiment. I expect that the portion of the microwave signal that refracted normally in the previous case would refract similarly in this experiment. The other peaks, however, are very different. There is only one central peak instead of two, and it carries much more power. This peak is at an angle less than the prism angle of 20 degrees relative to the prism normal. Angles of refraction between 0 and 20 degrees correspond to an index of refraction less than one.

I measured the rightmost peak of the distribution in Fig. 3.11 at the same angle as I measured it in the data is Fig. 3.10. Here, the peak is somewhat narrower, and reaches a higher peak power. These peaks are consistent with each other and both occur in a band of higher transmission through the metamaterial. As a result, I believe that these peaks do correspond to some portion of the incoming wave being refracted due to a negative index of refraction.

As an additional check, I measured the losses due to each of the effects that make...
Figure 3.10: Metamaterial Prism: 9.45 GHz microwave power as a function of angle relative to prism normal.
Figure 3.11: Metamaterial Prism: 9.25 GHz microwave power as a function of angle relative to prism normal.
up the metamaterial separately. I started with the spaced out aluminum plates, without the resonator circuits. When aligned with the polarization vector of the electric field, the waveguide should produce an index of refraction dominated by its imaginary part, resulting in high losses. When rotated so that the plates are perpendicular to the electric field polarization vector of the wave, the waveguide should have much less of an effect on the propagation of the waves. Compared to an uninterrupted signal, the parallel plate waveguide reduced the signal by 79 percent when aligned with the electric field polarization. When anti-aligned with the electric field polarization, they reduced the signal by 17 percent.

I performed a similar check using only the split ring resonators. The printed circuit boards reduced the signal by 88 percent when anti-aligned with the magnetic polarization vector, and reduced the signal by 17 percent when aligned with the magnetic polarization vector. This check shows that the resonators interact with the magnetic field when correctly aligned with it, but has much less of an effect on the microwaves when incorrectly aligned. This check also demonstrates that both of these effects are orientation dependent independent of one another.
Conclusion

In conclusion, the metamaterial I fabricated did not exhibit the approximately homogeneous negative index of refraction that was my intention. It did, however, demonstrate an effect that is consistent with a negative index of refraction, if only for a portion of the total transmitted signal.

The printed resonator circuits are the likely source of disparity between expectation and measurement in this experiment. The material on which the resonators are printed, FR4, interacts with the wave much too significantly. I hypothesize that the dominant refraction peaks in both metamaterial power distributions are due to a portion of the incident wave interacting with the laminate.

This problem is easily overcome by using laminates designed to have low losses when interacting with microwaves. Unfortunately, even though I had procured several such materials, etching turned out to be too time intensive a procedure to attempt. The split ring resonator design for this process used a calculator developed by the Department of Electronics and Electromagnetism at the University of Seville [7] to determine the resonant frequency of the split ring resonators.

Another problem with the factory-printed circuits is the resonator design. In order to have these patterns printed, I had to substitute the circular resonators for square edged ones. While there should be no loss in their effectiveness as magnetic resonators, the altered geometry interacts much more strongly with the electric field [12]. In addition, the inner ring of the factory-printed resonators was distorted due to the tooling process (see Fig. [4.1]).

The enormous dependence of the electric field on the orientation of the individual components of the metamaterial is consistent with what I expected. Given more time, I would have liked to address these effects separately across a variety of frequencies. With this information it would be much easier to isolate individual refractive effects within the material.

On the more positive side, the testing equipment all worked very well. The voltage controlled oscillator circuits and the detector diode produced consistent, reproducible data throughout the experiment. The aluminum foil shielding and experimental setup worked as intended in every null test.

The next step in this experiment is to develop a procedure for etching the metamaterial onto microwave laminates. From there, there are possible topics in planar lensing, superluminal propagation, wireless energy transfer, or even cloaking.
Figure 4.1: Resonator detail.
References


