Avalanches in Granular Material

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______________________
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Abstract

In this thesis the physics of granular avalanches are investigated. A rough inclined plane is constructed and the angles of inclination at which a layer of known uniform thickness is either stable, meta-stable or unstable are measured. We find that the stable, meta-stable and unstable behavior form well defined regions in the inclination-layer-thickness parameter space. The location of these boundaries are compared to theoretical predictions and we find a good match to the data if the characteristic length scale parameter from the theory is set equal to 2.5 times the grain diameter. The significance and physical explanation for this characteristic length scale remains an open question. Avalanches are triggered on the rough inclined plane and their velocities are measured using an edge detection algorithm. We find that avalanche velocity is negatively correlated with the angle of inclination. A possible explanation for this relationship is discussed.
Chapter 1

Introduction

In January of 2016 I saw an interesting demonstration during a three day avalanche safety course. The demonstration was intended to show how the variations in the density of the snowpack can drastically effect the destructive potential of an avalanche. In the first half of the demonstration a tray was filled with a layer of sugar and then a layer of flour. When the tray was inclined and then jolted, a relatively small amount of flour avalanched down. When the layering was reversed, so that the denser sugar sat on top of the flour, the resulting avalanche was much larger. This demonstration sowed the seeds of what later became this thesis.

1.1 The Challenge of Granular Material

Granular materials (also referred to as granular media) are defined as a collection of discrete, solid particles that are sufficiently large for Brownian motion to be irrelevant [2]. These granular materials appear practically everywhere. They are ubiquitous in everyday life and are used in a wide range of industrial applications. Sand, snow, soil, coffee, rice, ball bearings, cereals and pills are just a tiny subset of common granular materials. In fact, after water, granular media is the material that is said to be most often manipulated by man [5].

At first glance, it seems that the physics behind granular materials should be well understood. After all, granular materials are everywhere and the properties that govern them should be dictated by basic classical mechanics. However, and despite significant study, granular materials are not particularly well understood. The theory behind granular materials is comprised of a large number of often contradictory theoretical approaches that only explain a limited number of granular phenomena. Granular materials have been so difficult to understand for three major reasons: First, interactions between granular particles dissipate energy due to friction and inelastic collisions. Second, granular material often exists in a state far from thermodynamic equilibrium. Avalanches are a prime example of this non-equilibrium. The snow (or other granular material) sitting on a slope is initially static due to friction, but a small perturbation can trigger a large slide. Finally, granular materials can behave as either a solid, fluid or gas under the right circumstances. This requires physicists to blend
elements of fluid mechanics, solid mechanics and statistical physics in order to create an accurate theory of granular materials. Unsurprisingly this blending is not an easy task. These three factors eliminate many of the common modeling tools, theories and assumptions that physicists use, making the behavior of granular materials a particularly vexing problem.

1.2 A Brief Overview of Snow Avalanches

Snow avalanches present a significant danger to skiers, mountaineers, buildings and infrastructure in mountainous regions. In countries with large swathes of alpine terrain significant resources are invested in avalanche forecasting and prevention. Switzerland, unquestionably the world leader in avalanche related science and engineering, invests in excess of $50 million each year in order to prevent and predict avalanches [6]. However, little of this money goes towards monitoring the snowpack, mitigating avalanche risk or building avalanche defense structures.

The physics of snow avalanches is poorly understood not due, mainly, to a lack of funding but rather because of the nature of snow itself. Snow is an incredibly complex granular material with properties that can change depending on temperature, precipitation, wind patterns, solar radiation, age and slope aspect. Anyone who has ever packed a snowball knows that the properties of snow can vary significantly. Furthermore, the snowpack is often made up of layers of different snow that has been subsequently transformed by a number of environmental factors. The interactions of these layers within the snowpack strongly affect avalanche conditions. In some cases it can be essentially impossible to trigger an avalanche, in other cases avalanches are practically guaranteed. The likelihood of triggering an avalanche is largely determined by recent weather, terrain and how well the snowpack is bonded to itself and its underlying surface. Unsurprisingly, it is practically impossible to deduce the likelihood of triggering an avalanche just by looking at the snow surface. Extensive monitoring of the snowpack and weather are necessary for avalanche forecasting. In many mountainous regions of the United States the Forest Service partners with non-profit avalanche centers to employ teams of avalanche professionals who monitor the snowpack from December to April.

The behavior of an avalanche, once it is triggered, is also strongly dependant on the structure of the snowpack. Under the right weather conditions snow is strongly cohesive and slides in large slabs when an avalanche is triggered. This is called a slab avalanche and the slab breaks up as it slides down hill. Slab avalanches are generally considered more dangerous and are responsible for the majority of avalanche fatalities. In other cases, called a loose snow avalanche, the avalanche begins as unconsolidated snow falling from a single point and entrains more snow as it travels downhill. Loose snow avalanches leave behind the fan shaped path shown in Fig. 1.1a, which closely resembles the avalanche path for the granular avalanches I trigger in this thesis. In contrast, slab avalanches do not have a point origin, instead a slab of some area breaks off and begins sliding, the resulting avalanche path, shown in Fig. 1.1b, looks very
A complete overview of avalanche dynamics is far beyond the scope of this thesis. However, the important point is that snow is an incredibly complex substance that evolves in time and has widely varying properties. For avalanches, the interactions between various layers in the snowpack make the problem far more complex. Any laboratory experiment using a different granular materials is a significant simplification and researchers must be careful when extrapolating their conclusions to actual snow avalanches.

1.3 Granular Avalanches

For obvious reasons it is quite difficult to take experimental measurements of snow avalanches. However, it is relatively simple to create gravity driven granular flows in a controlled laboratory environment. Most of these experiments involve an entirely mobilized mass of granular material moving under the influence of gravity. While these experiments give important insight into the dynamics of granular materials, they fail to simulate a key aspect of real avalanches. In real world avalanches the mass of moving granular material changes as the static material sitting on the slope is entrained by the flow. A small subset of experiments, pioneered by Daerr [4], mimic this setup by triggering avalanches on a rough inclined plane that is already covered with a layer of static granular material.

These experiments have several major findings. First, granular material on a rough inclined plane exist in one of three states. In the stable state the material is static and any perturbed grains will quickly come to rest. Alternatively, the granular
material can be in an unstable state where it flows downhill. In between the stable and unstable state there exists a third, meta-stable state in which the granular material is initially static but a small perturbation will trigger an avalanche.

A second major finding pertains to the shape of the avalanches. Avalanches can either travel downhill from their starting point forming long, fan shapes or travel both uphill and downhill forming more balloon like shapes. Examples of these two avalanche shapes can be seen in Fig. 1.2. In uphill avalanches the grains above the avalanche fall when the supporting material below them is removed. Whether the avalanche travels uphill or downhill is determined by the systems location in the meta-stable region. The 3-dimensional shape of granular avalanches has also been measured. Essentially, the front of the avalanche forms a wave crest that protrudes above the surrounding static granular material as seen in Fig. 1.3 [3, 4]. As one moves uphill the wave crest tapers off to eventually form a trough in the static granular material.
1.3. Granular Avalanches

Figure 1.3: Experimental height measurements of an avalanche. From [3]

material. Finally, velocity measurements of the avalanches are also taken. These measurements show that avalanches undergo a short acceleration phase after being triggered and then reach some terminal velocity.

In this thesis I construct a rough inclined plane setup, quantify the location of the stable, unstable and meta-stable states and compare my results to a theory developed by Aranson and Tsimring [7]. I also measure the velocities of downhill avalanches as a function of various system parameters.
Chapter 2

Experimental Setup

2.1 Granular Material

For my granular material I used a sand blasting medium made up of the small, roughly spherical, glass beads seen in Fig. 2.1. Manufacturer specifications claim that these beads range in diameter from 212-300 micron.

2.2 The Rough Inclined Plane

The experimental setup I used is almost identical to that described by Daerr [4]. The setup consists of a plane with a rough bed surface and a variable angle of inclination. The plane itself is essentially a plywood tray with walls on three sides. The fourth and bottom side of the tray has no walls, allowing moving granular material to flow off the apparatus. The rough bed is provided by a piece of short-nap velvet cloth affixed to the bottom of the tray using a spray adhesive. Finally, the tray is attached to the counter-top by a hinge that allows for variable inclination that can be controlled using a motor and pulley system. A camera, line laser and an angle sensor are attached to the rough inclined plane in order to collect data. Additionally, an LED strip mounted to the top wall of the tray and a table lamp sitting on the counter top and shining onto the tray are used to achieve proper illumination. A photo of the experimental

Figure 2.1: Brightfield image of the granular material.
Figure 2.2: A picture of the rough inclined plane

setup can be seen in Figure 2.2. The camera is a a Nikon J5, shooting video at 1080p resolution and 60 frames per second.

### 2.3 Measuring Layer Height

It is often necessary to measure the height of the layer of granular material that sits on the inclined plane. Under the correct conditions this layer forms with a uniform thickness allowing depth measurement at a single point to be sufficient. Measurements are achieved by shining a line laser onto the inclined plane at a small angle of incidence, $\theta_i$. The line laser is affixed to the edge of the inclined plane such that $\theta_i$ is measured with respect to the inclined plane and is independent to the angle of inclination of the plane itself. In my setup $\theta_i$ has been measured as $18^\circ$ with an angle sensor and is fixed at that angle. The laser line has been aligned such that it is parallel to the edge of the inclined plane and also parallel to the edge of the camera frame. Since the top of the plane is always kept free of granular material, it is possible to use the displacement of the line laser in order to calculate the absolute height of the granular layer. The geometry of this measurement scheme is shown in Figure 2.3. Following a measurement of $d$, the displacement, Eqn. 2.1 can be used to calculate the layer height:

$$h = d \tan \theta.$$  \hspace{1cm} (2.1)
The camera is used in order to measure $d$. First an image of the displaced line laser is taken and converted to gray scale to produce an image like that in Figure 2.4a.

The image in Fig 2.4a is then converted to the average intensity distributions seen in Fig. 2.4b by averaging the intensity values of pixels with the same x coordinate. This averaging is done over 2 boxes, one box which encompasses only the granular material and one box which covers only the bare velvet, in order to produce two intensity distributions. Next a Gaussian is fitted to these intensity distributions in order to find the location of the intensity peaks which are then used to calculate $d$. The Mathematica script for this process can be found in App. A.

2.4 Avalanche Edge Detection

It is useful to find the edge of the avalanche in order to quantify how it evolves over time. In order to accomplish this, I implemented an edge finding algorithm that processes individual video frames in order to take measurements of the avalanche path (the area that has participated in the avalanche) as the avalanche propagates downhill. There are essentially two factors which need to be optimized in order to have accurate edge detection. First, contrast between the avalanche path and the static granular material must be maximized in order to make the avalanche edge visually prominent. Second, a number of image processing steps are necessary in order to detect the avalanche edge.

2.4.1 Maximizing Contrast

My final experimental setup used a black bed in order to maximize contrast with the off-white granular material. Various prototype setups that used an off-white bed failed to produce accurate edge detection due to a lack of contrast. Proper lighting is also necessary in order to maximize contrast between the bed and the granular
Chapter 2. Experimental Setup

2.4. Experimental Setup

Figure 2.4: An image of (a) the line laser displacement and (b) the corresponding intensity distribution and their fitted Gaussians. The distance between the centers of the two Gaussians is $d$.

material. Additionally, a directional light source is in order to ensure that the raised avalanche front casts a shadow and thus contrasts with the static granular material around it. In the case of my experimental setup I affixed an LED strip to the top wall of the plywood tray in order to achieve this directional lighting. However, if the directional light source causes part of the camera frame to be significantly brighter than the rest of the camera frame, it is possible to blind the camera. This occurred while prototyping my experimental setup so it was necessary to add a second, non-directional light source at the bottom of the tray. This non-directional light source does not cause the raised avalanche front to cast deep shadows and serves to even out the overall brightness of the camera frame. My non-directional light source consisted of the table lamp in Fig. 2.2 located at the bottom of the inclined plane. With these two light sources in place I was able to achieve sufficient contrast and image quality.

2.4.2 Edge Detection Algorithm

The Edge Detection Algorithm is implemented on individual frames of a video that shows an avalanche propagating downhill. Fig. 2.5 shows an example of a raw avalanche image, which is a single raw video frame. It is fairly difficult for the human eye to discern the avalanche path in Fig. 2.5 and computer edge detection is unable to locate the path. In order to make the avalanche path more obvious, image subtraction is used. An image of the undisturbed bed of granular material plus some offset is subtracted from the image in Fig 2.5 in order to obtain Fig. 2.6. Without the offset the subtraction image would simply be too dark to easily discern the avalanche path. The offset yields the medium gray color that comprises the static material in Fig. 2.6. Next, derivative filtering is preformed on the subtracted image in order to obtain the image shown in Fig 2.7. A number of different filters were tested at this step and the derivative filter offered the best compromise between computational speed and edge detection accuracy. After derivative filtering has been completed the built-in Mathe-
2.4. Avalanche Edge Detection

Figure 2.5: Raw Avalanche Image

Figure 2.6: Subtraction Image

Figure 2.7: Derivative filtered subtraction image
matematica edge detection algorithm can be run in order to produce the image shown in Fig. 2.8. Infilling, smoothing and small feature deletion is performed on this initial edge detection in order to produce the detected avalanche path shown in Fig. 2.9. These paths are then overlaid on the subtraction images, as seen in Fig. 2.10, in order to compare the detected edge with the actual avalanche path.

Length measurements are taken by fitting a bounding box to the detected edge and measuring the boxes length.
Figure 2.10: The subtraction image with the detected avalanche path outlined in red.
Chapter 3

Results and Analysis

3.1 Characterizing Regions of Stability

There are three possible regions of stability for granular material sitting on a rough inclined plane. The granular material can be in a stable state in which grains dislodged by a small perturbation will quickly come to rest without dislodging an ever increasing number of grains. The granular material can also be in an unstable state where all grains flow due to the influence of gravity. Between the stable and unstable state, a meta-stable state exists in which the granular material is initially static but a small perturbation will trigger an avalanche of grains. I began by quantifying the location of these states for my experimental setup. In order to do this I measured the angle of inclination and layer height at which the granular material began moving or stopped flowing.

The data used to characterize the regions of stability was obtained in batches using the sequential process illustrated in Fig. 3.1. First the rough inclined plane was set at a low inclination angle and loaded with granular material. The inclination angle is then increased until the granular material began to flow. Once the granular material begins to flow, the pulley is immediately turned off in order to set a constant angle of inclination, thus the granular material will eventually come to rest on the inclined plane, forming a layer of uniform thickness. The height of this layer and the angle of inclination are then measured in order give point 1. The angle of inclination is then increased again until the static layer of granular material begins to flow at which point the pulley stopped. This new angle of inclination along with the layer height that existed before the grains mobilized are used to determine the location of point 2. Point 3 is created when the flowing granular material comes to stop, forming a new layer height at the same angle of inclination as point 2. This process is repeated until the layer height becomes too thin to measure. The entire sequence is repeated multiple times in order to gather more data.

The regions of stability can be quantified by using the data shown in Fig. 3.2 to partition the phase space into three regions. The empirically derived curves in
Figure 3.1: Data collection process. Numbers and arrows indicate the order in which the data is collected.
3.2 Triggering an Avalanche

Avalanches can only be triggered in the meta-stable regions shown in Fig. 3.2. However, when flowing granular material comes to a rest, the system exists on the boundary between the stable and meta-stable regions. Avalanches are difficult to trigger at this boundary. In order for a small perturbation to trigger an avalanche, the system must be firmly in the meta-stable region. To ensure that the system is well within the meta-stable region the inclination of the plane is raised slightly by $\delta \theta$, causing the system to translate to the right in the inclination-layer thickness phase space. The granular material is then poked with the end of a wire in order to trigger the avalanche. All avalanches were created using $\delta \theta = 0.9^\circ$.

3.3 Measuring Avalanche Velocities

The edge detection algorithm described in chapter 1 outputs a list of length measurements that describe the avalanche path. In order to determine the velocity of the avalanches I fit linear models to the position-time data of each avalanche to get plots like the one shown in Fig. 3.3. An examination of the blue model in Fig. 3.3 and its residuals which are shown in Fig. 3.4 clearly indicate that the avalanche is not immediately traveling at a constant velocity. The non random residuals and the slight positive curvature of the data shown in Fig. 3.3 indicates that the avalanche
Chapter 3. Results and Analysis

Figure 3.2: Stability regions in the inclination-layer thickness phase space.
3.3. Measuring Avalanche Velocities

Figure 3.3: Position-Time data for an avalanche with two different fitted linear models. The model indicated by the blue line is fitted to all the data points and has a slope of 1.54. The model indicated by the orange line is fitted to only the orange data points and has a slope of 1.74.

...undergoes some small acceleration, something which the blue linear model fails to account for. Furthermore, Daerr [4] finds that the avalanches quickly reach a terminal velocity. In order to obtain these terminal velocities, I simply fit linear models to the last 13 data points (equivalent to 2 seconds) of each avalanche. The orange model in Fig. 3.3 is an example of one of these 2 second models. The residual plots for these new linear models show a stochastic pattern indicating that linearity is a reasonable assumption. With these velocities, the relationship between the angle of inclination and avalanche velocity can be plotted as shown in Fig. 3.5. As Fig. 3.5 shows, the angle of inclination is inversely related to the velocity of the avalanches. In other words, the counter intuitive result is that steeper slopes lead to slower avalanches.

The leading explanation for the inverse relationship between velocity and inclination centers on a layer-thickness dependent drag force and is described in Daerr [4]. Essentially, the rough bed of the inclined plane exerts some type of drag force on the avalanche front as it propagates downhill. This drag force is dependent on the layer thickness. In other words, it is greatest at the bed and decreases as one moves up through the granular material. As a result, layers of greater thickness, which occur at lower inclinations, exert less average drag on an avalanche as compared to layers of smaller thickness, which occur at greater inclinations. Fig. 3.6 shows the relationship between layer thickness and velocity. The best fit line does a poor job of prediction but it does indicate the positive relationship between layer thickness and velocity that the drag force explanation prescribes. In theory, combining the equation for the best fit line in Fig. 3.6 with equation for the orange curve in Fig. 3.2 should give an
Figure 3.4: A residual plot for a linear model fit of avalanche position-time data.

Figure 3.5: Terminal velocity of an avalanche vs angle of inclination. The curve is described by Eqn. 3.2
3.3. Measuring Avalanche Velocities

Figure 3.6: Terminal velocity vs layer thickness. A best fit line with the equation $y = 1.40 + 0.14x$ is included.
equation for velocity as a function of inclination angle. Simple algebra gives

\[ v = 0.645 - 0.758 \log \left( \frac{\tan \theta - 0.429}{0.251} \right) \]

(3.2)

where \( v \) is velocity. The curve defined by Eqn. 3.2 is included in Fig. 3.5. This curve does not match the data particularly well and clearly tends to overestimate the velocity of the avalanches. Nevertheless, Eqn. 3.2 correctly predicts the counter intuitive relationship in which greater angles of inclination lead to slower avalanches.

### 3.4 Summary and Comparison

In summary, I measured the location of the various states of stability for the rough inclined plane and find a counter intuitive relationship between avalanche velocity and angle of inclination. My stability measurements closely match previously published results by Daerr [4], Pouliquen [8]. As expected, the exact location of the regions of stability differ from published results due to the different bed surface material and granular material but the shapes and relative sizes of the stability regions match similar measurements closely. Following Pouliquen [8] and Daerr [4], I fit curves to the boundaries of the stability regions using Eqn. 3.1. These curves do a reasonable job of fitting the data but they aren’t strongly connected to underlying fundamental physics. Furthermore, these curves result from a three parameter fit so they could reasonably approximate data with a wide range of different curvatures.

I also find that avalanche velocity is inversely related to the angle of inclination which is a surprising and counter intuitive result. Daerr [4] finds a similar relationship between velocity and inclination when granular flows are triggered using the “bulldozing method”. In the bulldozing method a layer of granular material is prepared using the same procedure for triggering an avalanche except, the angle of inclination is not increased by \( \delta \theta \). A straight front of moving material is then created by pushing the granular material at the top of the plane with a bar on the entire width of the plane. Daerr [4] finds that the velocity of these bulldozed fronts is inversely related to angle of inclination and that velocity and layer thickness are positively correlated, as shown in Fig. 3.7. These findings suggest that a similar, layer-thickness dependent drag force is at play in here.
Figure 3.7: Main Graph: Terminal Velocity vs Angle of inclination for bulldozed fronts. Inset: Terminal velocity vs layer thickness. From Daerr [4]
Chapter 4

Theory

The transition from static equilibrium to granular flow is one of the most interesting and complex properties of granular materials. In the natural world snow avalanches are an obvious and spectacular example of this transition. Despite significant work, the physics that describe the transition from static to flowing material are not particularly well understood. The same can also be said for many other granular phenomena. In general “the theoretical description of granular systems remains largely a plethora of different, often contradictory concepts and approaches” [2].

In this chapter I compare the predictions of one theory of granular flows to my results. Aranson and Tsimring [7] postulate a continuum theory for partially fluidized granular flows. This theory attempts to describe the transition between the static equilibrium and granular flow for several scenarios including avalanches in shallow inclined layers.

4.1 A brief overview of the theory

The theory developed by Aranson and Tsimring [7] is a based on the Navier-Stokes equation and incorporates an order parameter $\rho$ that describes the fluidity of the system. Central to the theory is the idea that, in partially fluidized flows, some grains maintain prolonged static contact with their neighbors while other grains are in a more fluid state. This multiphase behavior is quantified by an order parameter which can be thought of as the proportion of static contacts in a small volume of granular material. This definition of the order parameter implies that $\rho = 1$ for the static state and $\rho = 0$ for the fully fluid state. The order parameter $\rho$, which varies depending on its spatial coordinates, is also a dynamic variable. Following Landau’s theory of phase transitions, Aranson and Tsimring [7] have developed a partial differential equation that determines the evolution of $\rho$ in space and time. Both the static state in which $\rho = 1$ and fluid state in which $\rho = 0$ are possible solutions to this equation and there exists a “flow” that connects these two solutions, corresponding to transitions between static behavior and fluid behavior at a given location in the granular material.
Aranson and Tsimring [7] analysis gives predictions of the boundaries of the stable, meta-stable and unstable regions shown in Fig. 3.2. The left-hand curve, which describes the height at which moving grains freeze, is given by

\[ h(\theta)_{\text{stop}} = \frac{\pi}{2\sqrt{\delta - 1}}, \]  

(4.1)

where

\[ \delta(\theta) = \frac{\tan(\theta)^2 - \tan(\theta_0)^2}{\tan(\theta_1)^2 - \tan(\theta_0)^2}. \]  

(4.2)

Here, \( \theta \) is the inclination angle of the plane and \( \theta_1 \) is the minimum angle at which granular material of a very large layer thickness will flow, in other words the vertical asymptote of the unstable region. \( \theta_0 \) is the dynamic angle of repose, which is the slope angle formed by grains moving continuously downhill. The most common way to measure the dynamic angle of repose is to partially fill a transparent, rotating cylinder with granular material and measure the angle slope angle that forms as the material cascades down.

Similarly, the left-hand curve at which static grains begin to flow is given by the minimum of the following integral with respect to \( \rho_0 \) where \( \rho_0 \in [0, 1] \)

\[ h(\theta)_{\text{start}} = \min \int_{\rho_0}^{1} \frac{d\rho}{\sqrt{\frac{\rho^4}{2} - \frac{2(\delta + 1)\rho^3}{3} + \delta\rho^2 - c(\rho_0)}}. \]  

(4.3)

In Eqn. 4.3, \( c(\rho_0) \) is given by

\[ c(\rho_0) = \frac{\rho_0^4}{2} - \frac{2(\delta + 1)\rho_0^3}{3} + \delta\rho_0^2. \]  

(4.4)

All values of the height, \( h \), in this theory are expressed in non-dimensionalized form, where the characteristic length scale \( l \) is expected to be on the order of the average grain diameter \( d \).

### 4.2 Comparison with experiment

#### 4.2.1 Choosing \( \theta_0 \) and \( \theta_1 \)

The values of \( \theta_0 \) and \( \theta_1 \) in Eqn. 4.2 need to be chosen in order to plot the curves given by Eqn. 4.1 and Eqn. 4.3. \( \theta_0 \) cannot be read directly off the graph shown in Fig. 4.1, however \( \tilde{\theta}_0 \) corresponds to the value of the orange data points vertical asymptote as indicated in Fig. 4.1. With \( \tilde{\theta}_0 \), \( \theta_0 \) can be easily calculated once I have also selected a value for \( \theta_1 \), as \( \delta(\tilde{\theta}_0) = 1/2 \). I select \( \tilde{\theta}_0 = 23.19^\circ \), which corresponds to the vertical asymptote formed by the orange curve in Fig. 4.1. The orange curve in Fig. 4.1 is obtained by fitting Eqn. 3.1. Eqn. 3.1 (and Fig. 3.2) is chosen because of the relationship introduced in Pouliquen [8]. Pouliquen [8] finds an empirical relationship where

\[ \tan(\theta_{\text{stop}}) = \mu(\infty) + (\mu(0) - \mu(\infty)) \exp\left(-\frac{h}{c \cdot d}\right) \]  

(4.5)
4.2. Comparison with experiment

Figure 4.1: The vertical asymptotes that correspond to the values of $\theta_0$ and $\tilde{\theta}_1$. Data and fit lines are the same as shown in Fig. 3.2.

describes the curve at which moving grains freeze for a number of different rough inclined planes. Eqn. 4.5 is identical to Eqn. 3.1 up to a choice of coefficient names. In Eqn. 4.5, $d$ is the grain diameter, $c$ is a rescaling constant, and $\mu(h)$ gives the coefficient of friction at some height $h$ within the layer of granular material sitting on the plane. While Pouliquen’s relationship is largely empirical, it is a robust enough relationship to be exhibited across a number of different systems and attempts to describe the system in terms of physical attributes.

Choosing the value for $\theta_1$ is a more subjective process. Following [4], I began by fitting the equation

$$
\tan(\theta) = a + b \exp(-h/c)
$$

(4.6)

for the blue data points in Fig. 4.1, where $a$, $b$, and $c$ are all free parameters. Eqn. 4.6 has the same form as Eqn. 4.5. However, the values of $a$ and $b$ do not have the same physical significance as they do in Eqn. 4.5. The asymptote from the fit, which occurs at $25.738^\circ$, is treated as an initial estimate for $\theta_1$. I then select a $\theta_1$ in the neighborhood of $25.738^\circ$, which gives best fit. In the end I use $\theta_1 = 26^\circ$.

4.2.2 Numerical Integration of Eqn. 3.3

Unsurprisingly Eqn. 4.3 cannot be solved analytically for most values of $\theta$. The following Mathematica code is used to solve the integral numerically.

```mathematica
Plot[FindMinimum[
  NIntegrate[
    1/(Sqrt[p^4/2 - (2 (((Tan[x Degree]^2 - t0sq)/(Tan[t1]^2 - t0sq)) + 1) p^3)/
      3 + ((Tan[x Degree]^2 - t0sq)/(Tan[t1]^2 - t0sq)))*
    p^2 - (p0^4/2 -
```
28  

Chapter 4. Theory

Figure 4.2: The theoretical stability curves given by Eqn. 3.1 and Eqn. 3.3 compared with the empirical data.

\[
2 \left(\frac{(\tan(x \text{ Degree})^2 - t0^sq)/(\tan(t1)^2 - t0^sq)) + 1}{3 + ((\tan(x \text{ Degree})^2 - t0^sq)/(\tan(t1)^2 - t0^sq))}\right)p0^3/3 + ((\tan(x \text{ Degree})^2 - t0^sq)/(\tan(t1)^2 - t0^sq))p0^2\]  

4.2.3 Comparison With Experiment

Plotting the curves given by Eqn. 4.1 and Eqn. 4.3 gives the results shown in Fig. 4.2. Clearly these two curves are not a good fit for the data, their curvature is far too steep. However, by introducing a new scaling parameter, the fit of these curves can be significantly improved. The new scaling parameter simply multiplies Eqn. 3.1 and Eqn. 3.3 by some constant \( C \). Setting \( C = 2.5 \) gives the graph shown in Fig. 4.3. The curves shown in Fig. 4.3 fit the data much better than those shown in Fig. 4.2.

It is important to note that the rescaling parameter \( C \) is not a part of the theory introduced by Aranson and Tsimring [7]. However the original theory does call for rescaling of height measurements by introducing \( l \), the characteristic length scale which is set equal to \( d \), the average grain diameter. My introduction of \( C \) simply alters this characteristic length scale such that \( l = C \cdot d \). In other words I’ve redefined the characteristic length scale as being equal to 2.5 times the average grain diameter. Intuitively, setting \( l = d \) makes much more sense. However, as seen in Fig. 4.2, leaving \( l = d \) does not produce curves that match the data. My rescaling seems to
4.2. Comparison with experiment

Figure 4.3: The rescaled theoretical stability curves given by Eqn. 3.1 and Eqn. 3.3 compared with the empirical data.

indicate that that characteristic length scale should be on the order of $l \approx 2-3d$. While this rescaling of $l$ does not seem to have any obvious physical significance, it should be noted that Pouliquen [8] and Daerr [4] find similar rescalings ($c \approx 2$) when fitting Eqn. 4.5 to grain flow on a number of different rough inclined planes. Unlike Pouliquen [8] and Daerr [4], fitting Eqn. 4.5 to my data does not yield $c \approx 2$. Instead I find $c = 5.09591$, as shown in Table 3.1. However, the value of $c$ I find implies that the $c/C \approx 2$ for the rough inclined plane I construct. The ratio of $c/C$ is similar for the experimental setup in [4]. As previously mentioned, Daerr [4] finds $c \approx 2$. Aranson and Tsimring [7] set $C = 1$ and achieve reasonable fits to the data from [4] using the curves given by Eqn. 4.1 and Eqn. 4.3. Once again, there is no obvious physical reason for why $c/C$, which is simply a ratio of scaling parameters from two different theories, should remain constant from one rough inclined plane to another.
Chapter 5

Conclusion

I constructed a rough inclined plane with a variable angle of inclination in order to create granular avalanches. I measured the plane inclination angles at which a layer of known uniform thickness is stable, meta-stable and unstable. I find my measurements to be in qualitative agreement with measurements of similar setups. The experimental measurements of these stability regions are compared to the theory developed by Aranson and Tsimring [7]. The predictions made by the theory do not match up with experimental measurements unless the characteristic length scale used in the theory is set equal to 2.5 times the average grain diameter. There is no obvious physical explanation for this rescaling.

I also optimized the lighting setup of the plane and developed an edge detection algorithm that tracks avalanches as they move downhill. This allows for easy velocity measurements of an avalanche. I find, counterintuitively, that steeper angles of inclination lead to slower avalanches. These measurements agree qualitatively with the bulldozing measurements done by Daerr [4]. There is a positive correlation between layer thickness, which is inversely related to inclination angle, and velocity, indicating that drag from the surface of the rough inclined plane is inversely correlated to the number of layers. In other words, fewer layers lead to a greater average drag force. These two relationships together indicate that the gain in gravitational force from increasing the angle of inclination is outweighed by the increase in drag force that results from a smaller layer thickness. This results in the counter intuitive relationship in which avalanche velocity decreases with greater angles of inclination.

An unexplained phenomena of snow avalanches is that very large avalanches travel much further than would be expected when extrapolating from smaller avalanches [4]. Future work could investigate the relationship between velocity and layer thickness for large (> 10) layer thicknesses in order to gain insight into this phenomenon. A related extension could involve measuring the duration of an avalanche’s acceleration phase for these larger layer thicknesses.
Appendix A

Layer Height Code

The Mathematica code for layer height measurements is shown below. The code takes a photo and asks the user to specify the approximate intersection of the line laser and the upper edge of the granular material. Then the process described in chapter 1.2 is used to find layer height.

```mathematica
depthmeas[vidpath_, convfac_] := (files = FileNames["*", vidpath, Infinity];
test1 = ImageRotate[Import[files[[1]]], 88 Degree];
Input[DynamicModule[{position = None},
  Column[{EventHandler[
    Image[test1,
    ImageSize -> Full],{"MouseClicked" :> (middle = 
    MousePosition["Graphics"]}), 
    Dynamic[position]]}]];
tempimg = 
  ImageTrim[
    ColorSeparate[test1,
    "G"], {{middle[[1]]} + 200, 
    middle[[2]] + 350}, {middle[[1]] - 200, middle[[2]] - 350}]];
topdata = Mean[ImageData[tempimg][[450 ;; 650]]];
bottomdata = Mean[ImageData[tempimg][[50 ;; 250]]];
btop = b /. 
  FindFit[topdata,
    a*E^(-(x - b)^2/(2*c^2)) + 
    d, {{a, 0}, {b, 
      Flatten[Position[topdata, Max[topdata]][[1]]]}, {c, 23}, {d, 
      0}}, x, PrecisionGoal -> Infinity];
bbottom = 
  b /. FindFit[bottomdata,
    a*E^(-(x - b)^2/(2*c^2)) + 
    d, {{a, 0}, {b, 
      Flatten[Position[bottomdata, Max[bottomdata]][[1]]]}, {c, 
      23}, {d, 0}}, x, PrecisionGoal -> Infinity];
delta = btop - bbottom;
```
Appendix A. Layer Height Code

\[
depth = \text{Abs}[((\text{delta} \times \text{Tan}[18\ \text{Degree}]) \times \text{convfac})/0.0256)]
\]
Appendix B

Edge Detection Code

The Mathematica code for the edge detection algorithm is shown below. The code executes the edge detection algorithm on every sixth video frame and outputs a list of length and width measurements of the avalanche path for every frame. The code also outputs images that consist of the outline of the detected avalanche path overlaid on the appropriate subtraction image. Width measurements are taken by fitting an ellipse to the boundary of the detected avalanche path. Length measurements are taken using a convex hull which is defined as the smallest possible region that includes all points inside the avalanche path and that ensures that a straight line between any two points inside the path falls inside this region.

```mathematica
avmeas[background_, vidpath_, wrpath_, starti_, endi_, topx_] := (
files = FileNames["*", vidpath, Infinity];
numframes = Import[files[[2]], "FrameCount"];
lengthlist = {};
widthlist = {};
For[ i = starti, i < endi, i = i + 10,
tempimg = ImageTrim[
  ColorConvert[Import[files[[2]], {"ImageList", i}],
  "Grayscale"], {{50, 283}, {topx, 958}}];
tempsubimg = ImageSubtract[ImageSubtract[tempimg, -.3], background];
tempedge =
  Dilation[
    DeleteSmallComponents[
      Erosion[DeleteSmallComponents[
        FillingTransform[
          Dilation[
            EdgeDetect[
              DerivativeFilter[tempsubimg, {1, 0}, 3] // ImageAdjust, DiskMatrix[2]]], DiskMatrix[8]]], DiskMatrix[3]]];
  AppendTo[lengthlist, ComponentMeasurements[MorphologicalComponents[tempedge], "CaliperLength"]];
  AppendTo[widthlist,
```
ComponentMeasurements[MorphologicalComponents[tempedge], "Width"];
highlightimg = HighlightImage[tempsubimg, EdgeDetect[tempedge]];
Export[
StringJoin[
"C:\Users\Giovanni\Documents\Wolfram Mathematica\", wrpath,
"\f", ToString[i], ".png"], highlightimg]]}
References


