Corrections to the Instructor's Solution Manual
Introduction to Quantum Mechanics, 2nd ed.
by David Griffiths
Cumulative errata for the print version—corrected in the current electronic version.
I especially thank Kenny Scott and Alain Thys for catching many of these errors.
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- page 11, Problem 1.14(a), line 1: $|\Psi(x,t)^2 \to |\Psi(x,t)|^2$; line 2: $-\frac{\partial}{\partial t}J(x,t) \to -\frac{\partial}{\partial x}J(x,t)$.
- page 13, Problem 1.18(b), line 2, first inequality: $\dfrac{h^2}{2mk_B} \to \dfrac{h^2}{3mk_B}$.
- page 13, Problem 1.18(b): change lines 5 and 6 to read:
  For atomic hydrogen ($m = m_p = 1.7 \times 10^{-27}$ kg) with $d = 0.01$ m:
  $$T < \frac{(6.6 \times 10^{-34})^2}{3(1.7 \times 10^{-27})(1.4 \times 10^{-23})(10^{-2})^2} = \boxed{6.2 \times 10^{-14}\text{ K.}}$$
- page 14, Problem 2.1(b), lines 1, 2, and 4: $\partial \to d$ (6 times); part (c), lines 1 and 2: $\partial \to d$ (5 times).
- page 15, Problem 2.2, line 6: "requuire" $\to$ "require".
- page 15, Problem 2.3, line 4: $e^{i\kappa a} \to e^{-\kappa a}$.
- page 16, Problem 2.4, second line in the calculation of $\langle x^2 \rangle$: $y^3/4$ should be $y^2/4$.
- page 16, Problem 2.5(a): in the first line, change $\Psi^2\Psi$ to $\Psi^*\Psi$.
- page 18, Problem 2.7(b), last line (in box): $e^{-E_n t/\hbar} \to e^{-iE_n t/\hbar}$.
- page 20, Problem 2.11(a), line five, first integral: $e^{-\xi^2/2} \to e^{-\xi^2}$.
- page 21, Problem 2.11(a), second line of $\langle p^2 \rangle$: $e^{-\xi^2/2} \to e^{-\xi^2}$.
- page 23, Problem 2.13(b), line 2: "E_1 and E_2" $\to$ "E_0 and E_1".
- page 23, Problem 2.13: move "(With ψ_2 in place $\ldots 2\omega$.)" from the middle of part (c) to the end of part (b).
- page 25, Problem 2.17(d): in the first box, change H_0 to H_1; in the second box change H_1 to H_2; in the last line change H_2 to H_3; also, in the first line, $(-2z + \xi) \to (-2z + 2\xi)$.
- page 27, Problem 2.20(d), last term: $e^{ikx} \to e^{-ikx}$.
- page 28, end of last line: $d \to \partial$ (twice).
- page 29, lines 2, 3, and 4: $d \to \partial$ (8 times).
- page 31, Problem 2.27(b), line 2: "$(x < a)$" $\to$ "$(x > a)$".
- page 32, Problem 2.27(b), first line after "Now look for *odd solutions*": "$(x < a)$" $\to$ "$(x > a)$".
- page 33, Problem 2.28, after "(1) Continuity at $-a$": $Ae^{ika} \to Ae^{-ika}$.

- page 33, second line after "Solve these for F ...": $[4-\gamma^2+\gamma] \to [4-\gamma^2+\gamma^2]$.

- page 40, Problem 2.36: at the end of the paragraph starting "If $B=0$", change "$|A|^2/2 \Rightarrow A=\sqrt{2}$" to "$|A|^2 a \Rightarrow A = 1/\sqrt{a}$"; at the end of the paragraph starting "If $A=0$", change "$|a|^2/2 \Rightarrow B=\sqrt{2}$" to "$|B|^2 a \Rightarrow B = 1/\sqrt{a}$".

- page 41, Problem 2.37: add the following, at the end:

  Using Eq. 2.39, $\langle H\rangle = p_1E_1 + p_3E_3 = \left(\frac{9}{10}\right)\frac{\pi^2\hbar^2}{2ma^2} + \left(\frac{1}{10}\right)\frac{9\pi^2\hbar^2}{2ma^2} = \boxed{\frac{9\pi^2\hbar^2}{10ma^2}}.$

- page 42, Problem 2.38(a), end of line 2: remove period.

- page 43, Problem 2.40(b), end of line 2: $z \to a$; line 11: "interms" $\to$ "in terms".

- page 46, Problem 2.43(d), beginning of last line of $\langle p^2\rangle$: $\hbar \to \hbar^2$.

- page 55, line 1: $S_{21}A + S_{22}B \to S_{21}A + S_{22}G$.

- page 56, Problem 2.53(d): in the first line, switch the indices $1 \leftrightarrow 2$ (three times); in all the rest, switch the sign of a.

- page 63, Problem 3.2(d), line 4: "differenting" $\to$ "differentiating"; Problem 3.3, line 6: "particlar" $\to$ "particular".

- page 65, Problem 3.8(b), remove the final sentence ("But notice ... one another.").

- page 65, Problem 3.10: remove the last sentence.

- page 65, Problem 3.11, first line: $e^{-i\omega/2} \to e^{-i\omega t/2}$.

- page 68, Problem 3.17(d), first chain of equations: $\frac{dV}{dx} \to \frac{\partial V}{\partial x}$.

- page 68, Problem 3.18, end of line 1: $E_n \to E_2$.

- page 68, Problem 3.18, line after "Similarly, ...": remove $-E_2^2$ at the very end.

- page 69, Problem 3.18, mid-page, second line after "Meanwhile, ...": in the second expression $a^2 \to a$, and in the last expression $\pi \to \pi^2$ (in the denominators).

- page 70, line 4: "Prob. 2.22(a)" $\to$ "Prob. 2.22(b)".

- page 70, Problem 3.19, line 7 (the one starting with $|\Phi(p,t)|^2$): in the term after the second equals sign, $e^{\frac{1}{2a}\cdots} \to e^{-\frac{1}{2a}\cdots}$.

- page 70, Problem 3.19, first line of σ_H^2: $\hbar^2 \to \hbar^4$.

- page 75, Problem 3.29, 4 lines from the end: "zero" $\to$ "finite $(2\pi\hbar/\lambda)$".

- page 76, Problem 3.31, lines 2 and 3: $\frac{dV}{dx} \to \frac{\partial V}{\partial x}$ (four times).

- page 79, Problem 3.35(e), line 2: "form" $\to$ "from".

- page 83, Problem 3.38(c), first line of B: $e^{-2i\omega t} \to e^{-i\omega t}$.
- page 83, Problem 3.38(c), last line: $(1,2) \to (2,3)$.
- page 87, Problem 4.1(a), penultimate line: $\frac{\partial f}{\partial y} \to \frac{\partial f}{\partial x}$; space after "since".
- page 88, Problem 4.2(a), line 3, second term: $\dfrac{d^2X}{dy^2} \to \dfrac{d^2Y}{dy^2}$.
- page 90, Problem 4.4, attach the following comment at the end: [In truth, this is not as decisive as it appears, since $\int_0^\pi [(\ln[\tan(\theta/2)])^2 \sin\theta\, d\theta = \pi^2/6$, which is perfectly finite; Θ itself blows up at 0 and π, but $\sin\theta$ tames it.]
- page 91, beginning of line 2: $P_3 \to P_3(x)$.
- page 95, Problem 4.11, line 3: "Eq. 4.15" $\to$ "Eq. 4.74".
- page 98, Problem 4.15(b), second line, first term in big parentheses: $a^2 \to a$ in denominator.
- page 100, Problem 4.19(d): insert period after p^2, and replace the rest with the following:

  L_z also commutes with r, as we can show using a test function $f(r)$:

  $$[L_z, r]f = \frac{\hbar}{i}\left[x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}, r\right] f = \frac{\hbar}{i}\left(x\frac{\partial(rf)}{\partial y} - y\frac{\partial(rf)}{\partial x} - rx\frac{\partial f}{\partial y} + ry\frac{\partial f}{\partial x}\right) = -i\hbar\left(x\frac{\partial r}{\partial y} - y\frac{\partial r}{\partial x}\right) f.$$

  But
  $$\frac{\partial r}{\partial x} = \frac{\partial}{\partial x}\sqrt{x^2+y^2+z^2} = \frac{x}{r}, \quad \text{so} \quad \left(x\frac{\partial r}{\partial y} - y\frac{\partial r}{\partial x}\right) = 0,$$
  and hence $[L_z, r] = 0$ (and the same goes for the other two components). So $\mathbf{L}$ commutes with $H = p^2/2m + V(r)$. QED
- page 102, Problem 4.23, first line: $i\cot\theta\frac{\partial}{\partial\theta} \to i\cot\theta\frac{\partial}{\partial\phi}$.
- page 104, Problem 4.27(b), first line: $(12i + 12i) \to (-12i + 12i)$.
- page 104, Problem 4.27(c), second line, after the second equals sign: $\frac{\hbar}{4} \to \frac{\hbar^2}{4}$.
- page 107, Problem 4.32(b), line 2, third term: $\sin\frac{\alpha}{2}e^{i\gamma B_0 t/2} \to \sin\frac{\alpha}{2}e^{-i\gamma B_0 t/2}$.
- page 108, Problem 4.34(a), line 2: "(Eq. 4.143)" $\to$ "(line above Eq. 4.146)".
- page 109, Problem 4.36 (a): after the second comma in the box, it should read "or 0 (probability 6/15)."
- page 111, Problem 4.39, third line from bottom: in the second term, $(l+2) \to (l+1)$.
- page 112, Problem 4.40(a), line 3: "$(-i\delta_{ij})$" $\to$ "$(-i\hbar\delta_{ij})$".
- page 112, Problem 4.40(b), last line: change $\langle T\rangle = \langle V\rangle$ to $\langle T\rangle + \langle V\rangle$.
- page 115, Problem 4.43(c), second line: in the last two terms, $\left(\dfrac{3a}{2}\right)^5$ and $\left(\dfrac{3a}{2}\right)^3 \to \left(\dfrac{3a}{2}\right)^s$.

- page 116, Problem 4.44(a): remove $\cos\theta$ in the middle term; Problem 4.44(c): change 4(5) to 3(4) and 11 to 3.
- page 118, Problem 4.48(b): 4.113 → 4.118.
- page 119, Problem 4.51(a), line 1: "Eqs. 4.136 and 4.144" → "Eq. 4.136 and line after Eq. 4.146"; line 7: in the second square brackets, $S_2 \to s_2$.
- page 120, Problem 4.51(a), line beginning "where $a \equiv \dots$": "Multiply by $(a-b)$" → "Multiply by $(a-m)$".
- page 121, Problem 4.51(a), penultimate line: insert "$= 1$" before the period.
- page 123, Problem 4.54, line 1: begin with "**For positive upper index:**"; $Y_l^{m\pm 1} \to Y_l^{m+1}$.
- page 123-124, Problem 4.54, bottom of page 123 to top of page 124, replace "For $m < 0$" to just before "Now, Problem 4.22" with the following:

For negative upper index: write $Y_l^{-m} = B_l^{-m} e^{-im\phi} P_l^{-m}$ and (Problem 4.18)

$$L_- Y_l^{-m} = A_l^{-m} Y_l^{-m-1} = \hbar\sqrt{(l-m)(l+m+1)}\, Y_l^{-m-1}.$$

Noting (Eq. 4.27) that $P_l^{-m} = P_l^m$, we have (Eq. 4.130)

$$-\hbar e^{-i\phi}\left(\frac{\partial}{\partial\theta} - i\cot\theta\frac{\partial}{\partial\phi}\right) B_l^{-m} e^{-im\phi} P_l^m = \hbar\sqrt{(l-m)(l+m+1)} B_l^{-m-1} e^{-i(m+1)\phi} P_l^{m+1},$$

or

$$\left(\frac{d}{d\theta} - m\cot\theta\right) P_l^m\, B_l^{-m} = -\sqrt{(l-m)(l+m+1)}\, B_l^{-m-1} P_l^{m+1}.$$

As before,

$$\left(\frac{d}{d\theta} - m\cot\theta\right) P_l^m = -P_l^{m+1},$$

so

$$B_l^{-m-1} = \frac{1}{\sqrt{(l-m)(l+m+1)}} B_l^{-m}.$$

Thus (using $m = 0$, $m = 1$, $m = 2$, $\dots$):

$$B_l^{-1} = \frac{1}{\sqrt{l(l+1)}} B_l^0; \quad B_l^{-2} = \frac{1}{\sqrt{(l-1)(l+2)}} B_l^{-1} = \frac{1}{\sqrt{(l+2)(l+1)l(l-1)}} B_l^0, \dots.$$

Evidently

$$B_l^{-m} = (-1)^m B_l^m,$$

and in general,

$$B_l^m = \epsilon\sqrt{\frac{(l-|m|)!}{(l+|m|)!}}\, C(l),$$

where

$$\epsilon \equiv \begin{cases} (-1)^m, & \text{for } m \ge 0, \\ 1, & \text{for } m \le 0, \end{cases}$$

(as in Eq. 4.32).

Begin a new paragraph with "Now, Problem 4.22".

- page 123, Problem 4.54, last box: $(-1)^{l+m} \to (-1)^l\epsilon$; next line: "overall sign, which of course" → "factor of $(-1)^l$, which".
- page 124, Problem 4.55(e), line 3: $s \to j$.
- page 125, Problem 4.55(h): in the integral, $\sin^2\theta \to \sin\theta$.
- page 125, Problem 4.56(e): at the end of the second line the minus sign should be plus.
- page 126, Problem 4.57(b), end of first line: a should be squared.
- page 128, Problem 4.59(b), line 8: $\frac{\partial A_x}{\partial y} \to \frac{\partial A_x}{\partial z}$.
- page 129, Problem 4.60(b), line 4 (the boxed equation): $\nabla\cdot(\mathbf{A}\psi) \to (\nabla\cdot\mathbf{A})\psi$.
- page 129, Problem 4.60(b), line 12 (starting with "Now let ..."): before the colon, insert "(you can also do it in Cartesian coordinates)".
- page 130, Problem 4.60(b), beginning of penultimate line: open parentheses after "$E =$".
- page 133, Problem 5.1(b), end of penultimate line: $+\frac{1}{m_1+m_2} \to = \frac{1}{m_1+m_2}$.
- page 134, Problem 5.2(b), beginning of penultimate line: insert equals sign before $\frac{5}{36}R$.
- page 138, bottom line: $-\frac{ar_1}{a} \to -\frac{ar_1}{4}$.
- page 139, Problem 5.12(b), second line of last box: ${}^2F_{3/2} \to {}^2F_{7/2}$.
- page 142, Problem 5.20, line beginning "In the Figure": "postive" → "positive".
- page 144, Problem 5.23(c), first line: end the boxed answer with a comma, and after the box insert "so the three configurations are all equally likely."
- page 146, Problem 5.25, $N = 4$, second line of "Total": $\frac{1}{24} \to \frac{d}{24}$.
- page 151, Problem 5.35(c), line 2: $\hbar$ should be squared, in the last expression.
- page 155, Problem 6.3(b), end of first line: $\delta(x_2 - x_2) \to \delta(x_1 - x_2)$.
- page 159, Problem 6.7(b), line 4: 6.26 → 6.27.
- page 159, Problem 6.7(c), line 1: $E^1_- \to E^1$.
- page 161, first line: $\sin\left(\frac{3\pi}{4}\right) \to \sin^2\left(\frac{3\pi}{4}\right)$.
- page 161, Problem 6.9(c), line 4: $(0 \quad 1 \quad 0)\begin{pmatrix}0\\0\\1\end{pmatrix} \to (0 \quad 1 \quad 0)\begin{pmatrix}-1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}$.
- page 162, Problem 6.10, first line: "orthonornal" → "orthonormal".
- page 165, Problem 6.14, last line: $3n^2 \to 2n^2$.

- page 168, Problem 6.17, penultimate line: $-3+ \to +3-$.
- page 168, Problem 6.18, under "For $n = 3$", line $j = 1/2$: after the second equals sign, $-9 \to -\frac{9}{2}$.
- page 169, box at bottom, line 2: "$\nu_3 - \nu_3$" $\to$ "$\nu_3 - \nu_2$".
- page 176, 3 lines up from the bottom: remove γ in front of the final parentheses.
- page 178, figure, right end of 4th line up: $2/5 \to 2/3$.
- page 178, bottom line, last term: $[3(\mathbf{S}_p \cdot \hat{r})(\mathbf{S}_e \cdot \hat{r})] \to [3(\mathbf{S}_p \cdot \hat{r})(\mathbf{S}_e \cdot \hat{r}) - \mathbf{S}_p \cdot \mathbf{S}_e]$.
- page 180, Problem 6.29, 3 lines from end: $\left(\frac{10^{-15}}{5\times10^{-11}}\right) \to \left(\frac{10^{-15}}{5\times10^{-11}}\right)^2$.
- page 181, Problem 6.31(c), last line: $\frac{1}{2\hbar\omega_0} \to \frac{\hbar\omega_0}{2}$.
- page 182, at end of Problem 6.31, insert the following: [There is an interesting fraud in this well-known problem. If you expand H' to order $1/R^5$, the extra term has a nonzero expectation value in the ground state of H^0, so there is a non-zero first-order perturbation, and the dominant contribution goes like $1/R^5$, not $1/R^6$ (as desired). The model gets the power "right" in three dimensions (where the expectation value is zero), but not in one. See A. C. Ipsen and K. Splittorff, ArXiv: 1401.8144.]
- page 188, last line: $R_{31} \to R_{32}$.
- page 190, Problem 6.38, line 6: "(Eq. 6.98)" $\to$ "(Eq. 6.93)".
- page 192, line 3 of "Off-diagonal elements": $\langle|y^2\rangle \to \langle|y^2|\rangle$.
- page 194, Problem 6.40(a), penultimate line of (i): "second-order" $\to$ "first order"; 2 lines below: 6.11 $\to$ 6.14.
- page 197, line 3: $\langle T\rangle = -\frac{\hbar^2}{2m}\frac{2b^3}{\pi} \to \langle T\rangle = -\frac{\hbar^2}{2m}\frac{2b^3}{\pi}4$.
- page 197, Problem 7.3, line 2: $\left(\frac{a}{3}\right)^3 \to \left(\frac{a}{2}\right)^3$.
- page 198, Problem 7.4(b), 3 lines from the end: in the expression for $\langle H\rangle$, $\omega \to \omega^2$.
- page 200, line 1: in front of both integrals, $a^3 \to a^2$.
- page 200, line 4, inside first integral: $e^{2R/a} \to e^{2r/a}$.
- page 203, last line, first term inside square brackets: $4n - 1 \to 4n + 1$ (twice).
- page 210, Problem 7.18, line 1: remove spurious extra equals sign.
- page 211, mid-page, line beginning "The term in ...": $(Z_2^2 + Z_1^2) \to (Z_2^2 + Z_1^2)E_1$.
- page 211, last line: second $(Z_1 - 1) \to (Z_2 - 1)$.
- page 212, line 2 of $\langle H\rangle$: $4E_1A^2 \to -4E_1A^2$.
- page 212, mid-page (three lines following $\langle V_{ee}\rangle$): $e \to e^2$ (3 times); in the first of these three lines, $\psi_2(r_1)+\psi_1(r_2)$ $\to \psi_2(r_1)\psi_1(r_2)$ (i.e. remove the plus sign).

- page 213, mid-page (line starting with $\langle V_{ee}\rangle$): $e \to e^2$.
- page 216, line 3: cancel the second 2 in the numerator.
- page 217, line 6: between $]$ and $\}$ insert $\left[\frac{y}{a}\theta(a-x) + \alpha\theta(x-a)\left(1-\frac{y}{a}\right)\right]$.
- page 224, Problem 8.9, bottom part of second line: $|p(x)| \to p(x)$.
- page 225, line 2: "whereα" $\to$ "where α".
- page 225, 2 lines into "Overlap region 1", in the denominator of ψ_{WKB}: $\alpha^{3/2} \to \alpha^{3/4}$.
- page 225, first line of "Overlap region 2": $|p(x')| \to p(x')$.
- page 226-227, Problem 8.10. The statement of the problem has now been corrected (switching the signs in the exponents of the two terms in the first line of Eq. 8.52). Accordingly, the solution should be changed as follows:

...

$$\psi_{\mathrm{WKB}}(x) = \begin{cases} \dfrac{1}{\sqrt{p(x)}}\left[Ae^{-\frac{i}{\hbar}\int_x^0 p(x')\,dx'} + Be^{\frac{i}{\hbar}\int_x^0 p(x')\,dx'}\right] & (x<0) \\ \dfrac{1}{\sqrt{|p(x)|}}\left[Ce^{\frac{1}{\hbar}\int_0^x |p(x')|\,dx'} + De^{-\frac{1}{\hbar}\int_0^x |p(x')|\,dx'}\right] & (x>0) \end{cases}$$

...

In overlap region 1, Eq. 8.43 becomes $\psi_{\mathrm{WKB}} \approx \dfrac{1}{\hbar^{1/2}\alpha^{3/4}(-x)^{1/4}}\left[Ae^{-i\frac{2}{3}(-\alpha x)^{3/2}} + Be^{i\frac{2}{3}(-\alpha x)^{3/2}}\right]$,

...

$A = \sqrt{\dfrac{\hbar\alpha}{\pi}}\left(\dfrac{ia+b}{2}\right)e^{-i\pi/4}$; $\quad B = \sqrt{\dfrac{\hbar\alpha}{\pi}}\left(\dfrac{-ia+b}{2}\right)e^{i\pi/4}$. Putting in the expressions above for a and b :

$$A = \left(\frac{C}{2} + iD\right)e^{-i\pi/4}; \quad B = \left(\frac{C}{2} - iD\right)e^{i\pi/4}.$$

...

$$A = \left(\frac{C}{2} + iD\right)e^{-i\pi/4} = \left(\frac{i}{4}e^{-\gamma}e^{-i\pi/4}F + ie^{\gamma}e^{-i\pi/4}F\right)e^{-i\pi/4} = \left(\frac{e^{-\gamma}}{4} + e^{\gamma}\right)F.$$

$$T = \left|\frac{F}{A}\right|^2 = \frac{1}{(e^{\gamma} + \frac{e^{-\gamma}}{4})^2} = \boxed{\frac{e^{-2\gamma}}{[1 + (e^{-2\gamma}/4)]^2}.}$$

...

- page 228, penultimate line, "Limits", top line: $z \to z_1$; bottom line: $z \to z_2$.
- page 229, Problem 8.13, line 4: $e^{-e} \to e^{-x}$.
- page 230, Problem 8.14, 3 lines from end: $\frac{e^2}{4\pi\epsilon_0} \to -\frac{e^2}{4\pi\epsilon_0}$; same at beginning of the next line.
- page 235, Problem 8.16(d), lines 1 and 3: $2\times 10^{-19} \to 2\times 10^{-16}$.

- page 235, Problem 8.17, insert the following at the end: [Actually, to tunnel *all the way* through the classically forbidden region, the center of mass must not only rise from 0 to x_0, but also drop back to 0. This doubles γ, and makes the final exponent 1×10^{31}.]

- page 236, Problem 9.1, line 3, last expression: $e^{r/2a} \to e^{-r/2a}$.

- page 236, Problem 9.1, last three lines: remove the minus signs in front of all 6 expressions.

- page 237, line 6: $Ae^{i(\omega_0+\omega)/2} + Be^{i(\omega_0-\omega)/2} \to Ae^{i(\omega_0+\omega)t/2} + Be^{i(\omega_0-\omega)t/2}$.

- page 237, end of Problem 9.2, add the following:

  [In light of the **Comment** you might question the initial conditions. If the perturbation includes a factor $\theta(t)$, are we sure this doesn't alter $c_a(0)$ and $c_b(0)$? That is, are we sure $c_a(t)$ and $c_b(t)$ are *continuous* at a step function potential? The answer is "yes", for if we integrate Eq. 9.13 from $-\epsilon$ to ϵ,

  $$c_a(\epsilon) - c_a(-\epsilon) = -\frac{i}{\hbar} H'_{ab} \int_0^{\epsilon} e^{-i\omega_0 t} c_b(t)\, dt.$$

  But $|c_b(t)| \leq 1$, so the integral goes to zero as $\epsilon \to 0$, and hence $c_a(-\epsilon) = c_a(\epsilon)$. The same goes for c_b, of course.]

- page 238, 5 lines up from end: $c_b(t) = -\frac{i\alpha^*}{2\epsilon\hbar\omega} \to c_b(t) = -\frac{\alpha^*}{2\epsilon\hbar\omega}$.

- page 240, Problem 9.4, 3 lines from end: $H_{ba} \to H'_{ba}$.

- page 241, Problem 9.6, penultimate line: $\omega \to \omega_0$.

- page 242, line 6 ("General solution"), second exponent: $+\omega_r \to -\omega_r$.

- page 243, Problem 9.8, line 6: $e^{\hbar\omega/k_B t} \to e^{\hbar\omega/k_B T}$.

- page 244, line 5: $\left[\frac{2a}{3}\right)^5 \to \left(\frac{2a}{3}\right)^5$.

- page 244, line 8: $1\,0\,0\rangle \to |1\,0\,0\rangle$.

- page 245, line 5: insert "$2i\hbar$" right after "{".

- page 250, Problem 9.18, line 4: $\left(\frac{4V_0}{3\pi}\right) \to \left(\frac{4V_0}{3\pi}\right)^2$.

- page 250, Problem 9.20(b), line 2, last term: $\begin{pmatrix} B_0 a & B_{\rm rf} e^{i\omega t} b \\ B_{\rm rf} e^{-i\omega t} a & -B_0 b \end{pmatrix} \to \begin{pmatrix} B_0 a + B_{\rm rf} e^{i\omega t} b \\ B_{\rm rf} e^{-i\omega t} a - B_0 b \end{pmatrix}$.

- page 253, Problem 9.22:
  $\underline{l' = l + 1}$

From Eq. 9.69, $\langle n'l'm'|z|nlm\rangle = 0$ unless $m' = m$, so the only nonzero z term is

$$\begin{aligned}
\langle n'l'm|z|nlm\rangle &= \int R_{n'l'}(Y_{l'}^m)^* r\cos\theta\, R_{nl}Y_l^m\, r^2\, dr\, \sin\theta\, d\theta\, d\phi \\
&= I\sqrt{\frac{(2l'+1)}{4\pi}\frac{(l'-|m|)!}{(l'+|m|)!}}\sqrt{\frac{(2l+1)}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}}\; 2\pi\int_0^\pi P_{l'}^m P_l^m \cos\theta\sin\theta\, d\theta. \qquad (1)
\end{aligned}$$

This is independent of the sign of m, so we might as well assume $m \geq 0$. The integral (changing variables to $x \equiv \cos\theta$) is (using Eq. 9.99)

$$\text{Int}_\theta = \int_{-1}^1 x\, P_{l+1}^m(x) P_l^m(x)\, dx = \frac{1}{(2l+1)}\left[(l+m)\int_{-1}^1 P_{l+1}^m P_{l-1}^m\, dx + (l-m+1)\int_{-1}^1 P_{l+1}^m P_{l+1}^m\, dx\right].$$

Now, it follows from Eq. 4.33 that the associated Legendre functions satisfy the orthogonality relation

$$\int_{-1}^1 P_{l'}^m(x) P_l^m(x)\, dx = \frac{2}{(2l+1)}\frac{(l+|m|)!}{(l-|m|)!}\,\delta_{ll'}, \qquad (2)$$

so

$$\text{Int}_\theta = \frac{(l-m+1)}{(2l+1)}\frac{2}{(2l+3)}\frac{(l+1+m)!}{(l+1-m)!} = \frac{2}{(2l+1)(2l+3)}\frac{(l+1+m)!}{(l-m)!}.$$

Putting this into Eq.(1):

$$\begin{aligned}
\langle n'(l+1)m|z|nlm\rangle &= \frac{I}{2}\sqrt{(2l+3)\frac{(l+1-m)!}{(l+1+m)!}}\sqrt{(2l+1)\frac{(l-m)!}{(l+m)!}}\,\frac{2}{(2l+1)(2l+3)}\frac{(l+1+m)!}{(l-m)!} \\
&= I\sqrt{\frac{(l+1)^2-m^2}{(2l+1)(2l+3)}}. \qquad (3)
\end{aligned}$$

From Eq. 9.72, $\langle n'l'm'|x|nlm\rangle = 0$ unless $m' = m \pm 1$; let's start with $m+1$:

$$\langle n'l'(m+1)|x|nlm\rangle = \int R_{n'l'}(Y_{l'}^{m+1})^* r\sin\theta\cos\phi\, R_{nl}Y_l^m\, r^2\, dr\, \sin\theta\, d\theta\, d\phi$$

$$= I\sqrt{\frac{(2l'+1)}{4\pi}\frac{(l'-m-1)!}{(l'+m+1)!}}\sqrt{\frac{(2l+1)}{4\pi}\frac{(l-m)!}{(l+m)!}}\int_0^\pi P_{l'}^{m+1}P_l^m \sin^2\theta\, d\theta\int_0^{2\pi}\cos\phi\, e^{-i(m+1)\phi}e^{im\phi}\, d\phi. \qquad (4)$$

$$\text{Int}_\phi = \frac{1}{2}\int_0^{2\pi}(e^{i\phi}+e^{-i\phi})e^{-i\phi}\, d\phi = \frac{1}{2}\int_0^{2\pi}(1+e^{-2i\phi})\, d\phi = \pi. \qquad (5)$$

Changing variables ($x \equiv \cos\theta$), and using Eqs. 9.100 and (2):

$$\begin{aligned}
\text{Int}_\theta &= \int_{-1}^1 \sqrt{1-x^2}\, P_{l+1}^{m+1}(x) P_l^m(x)\, dx = \frac{1}{(2l+1)}\left[\int_{-1}^1 P_{l+1}^{m+1}P_{l+1}^{m+1}\, dx - \int_{-1}^1 P_{l+1}^{m+1}P_{l-1}^{m+1}\, dx\right] \\
&= \frac{2}{(2l+1)(2l+3)}\frac{(l+m+2)!}{(l-m)!}.
\end{aligned}$$

Thus (4) becomes

$$\begin{aligned}\langle n'(l+1)(m+1)|x|nlm\rangle &= \frac{I}{2}\sqrt{(2l+3)\frac{(l-m)!}{(l+m+2)!}}\sqrt{(2l+1)\frac{(l-m)!}{(l+m)!}}\,\frac{1}{(2l+1)(2l+3)}\,\frac{(l+m+2)!}{(l-m)!} \\ &= \frac{I}{2}\sqrt{\frac{(l+m+2)(l+m+1)}{(2l+1)(2l+3)}}. \qquad (6)\end{aligned}$$

Now we do the same for $m' = m-1$:

$$\langle n'l'(m-1)|x|nlm\rangle = \int R_{n'l'}(Y_{l'}^{m-1})^* r\sin\theta\cos\phi\, R_{nl}Y_l^m\, r^2\,dr\,\sin\theta\,d\theta\,d\phi$$

$$= I\sqrt{\frac{(2l'+1)}{4\pi}\frac{(l'-m+1)!}{(l'+m-1)!}}\sqrt{\frac{(2l+1)}{4\pi}\frac{(l-m)!}{(l+m)!}}\int_0^\pi P_{l'}^{m-1}P_l^m\sin^2\theta\,d\theta\int_0^{2\pi}\cos\phi e^{-i(m-1)\phi}e^{im\phi}\,d\phi. \qquad (7)$$

$$\text{Int}_\phi = \frac{1}{2}\int_0^{2\pi}(e^{i\phi}+e^{-i\phi})e^{i\phi}\,d\phi = \frac{1}{2}\int_0^{2\pi}(e^{2i\phi}+1)\,d\phi = \pi. \qquad (8)$$

Changing variables ($x \equiv \cos\theta$), and using Eqs. 9.100 and (2):

$$\begin{aligned}\text{Int}_\theta &= \int_{-1}^{1}\sqrt{1-x^2}\,P_{l+1}^{m-1}(x)P_l^m(x)\,dx = \frac{1}{(2l+3)}\left[\int_{-1}^{1}P_{l+2}^m P_l^m\,dx - \int_{-1}^{1}P_l^m P_l^m\,dx\right] \\ &= -\frac{2}{(2l+1)(2l+3)}\frac{(l+m)!}{(l-m)!},\end{aligned}$$

and (7) becomes

$$\begin{aligned}\langle n'(l+1)(m-1)|x|nlm\rangle &= -\frac{I}{4}\sqrt{(2l+3)\frac{(l-m+2)!}{(l+m)!}}\sqrt{(2l+1)\frac{(l-m)!}{(l+m)!}}\,\frac{2}{(2l+1)(2l+3)}\frac{(l+m)!}{(l-m)!} \\ &= -\frac{I}{2}\sqrt{\frac{(l-m+2)(l-m+1)}{(2l+1)(2l+3)}}. \qquad (9)\end{aligned}$$

Meanwhile, Eq. 9.70 says $|\langle n'l'm'|y|nlm\rangle|^2 = |\langle n'l'm'|y|nlm\rangle|^2$, so

$$\begin{aligned}&|\langle n'(l+1)(m+1)|\mathbf{r}|nlm\rangle|^2 + |\langle n'(l+1)m|\mathbf{r}|nlm\rangle|^2 + |\langle n'(l+1)(m-1)|\mathbf{r}|nlm\rangle|^2 \\ &= 2\left[\frac{I}{2}\sqrt{\frac{(l+m+2)(l+m+1)}{(2l+1)(2l+3)}}\right]^2 + \left[I\sqrt{\frac{(l+1)^2-m^2}{(2l+1)(2l+3)}}\right]^2 + 2\left[-\frac{I}{2}\sqrt{\frac{(l-m+2)(l-m+1)}{(2l+1)(2l+3)}}\right]^2 \\ &= \frac{I^2}{2}\left\{\frac{(l+m+2)(l+m+1)+2[(l+1)^2-m^2]+(l-m+2)(l-m+1)}{(2l+1)(2l+3)}\right\} \\ &= I^2\frac{(2l^2+5l+3)}{(2l+1)(2l+3)} = I^2\frac{(l+1)}{(2l+1)}. \qquad (10)\end{aligned}$$

Therefore, $|\wp|^2$ (summed over the three allowed transitions) is $e^2I^2(l+1)/(2l+1)$, and the spontaneous emission rate (Eq. 9.56) is

$$\boxed{A_{l\to l+1} = \frac{e^2\omega^3 I^2}{3\pi\epsilon_0\hbar c^3}\frac{(l+1)}{(2l+1)}.} \tag{11}$$

$\underline{l' = l-1}$

Return to Eq. (1). This time the integral is

$$\begin{aligned}
\text{Int}_\theta &= \int_{-1}^{1} x\, P_{l-1}^m(x)P_l^m(x)\,dx = \frac{1}{(2l+1)}\left[(l+m)\int_{-1}^{1} P_{l-1}^m P_{l-1}^m\,dx + (l-m+1)\int_{-1}^{1} P_{l-1}^m P_{l+1}^m\,dx\right] \\
&= \frac{(l+m)}{(2l+1)}\frac{2}{(2l-1)}\frac{(l-1+m)!}{(l-1-m)!} = \frac{2}{(2l-1)(2l+1)}\frac{(l+m)!}{(l-m-1)!}.
\end{aligned}$$

Therefore

$$\begin{aligned}
\langle n'(l-1)m|z|nlm\rangle &= \frac{I}{2}\sqrt{(2l-1)\frac{(l-1-m)!}{(l-1+m)!}}\sqrt{(2l+1)\frac{(l-m)!}{(l+m)!}}\,\frac{2}{(2l-1)(2l+1)}\frac{(l+m)!}{(l-m-1)!} \\
&= I\sqrt{\frac{l^2-m^2}{(2l-1)(2l+1)}}.
\end{aligned} \tag{12}$$

From $\langle n'l'm'|x|nlm\rangle$ with $m' = m+1$, Eqs. (4) and (5) are unchanged; this time

$$\begin{aligned}
\text{Int}_\theta &= \int_{-1}^{1} \sqrt{1-x^2}\, P_{l-1}^{m+1}(x)P_l^m(x)\,dx = \frac{1}{(2l+1)}\left[\int_{-1}^{1} P_{l-1}^{m+1}P_{l+1}^{m+1}\,dx - \int_{-1}^{1} P_{l-1}^{m+1}P_{l-1}^{m+1}\,dx\right] \\
&= -\frac{2}{(2l-1)(2l+1)}\frac{(l+m)!}{(l-m-2)!},
\end{aligned}$$

and (4) becomes

$$\begin{aligned}
\langle n'(l-1)(m+1)|x|nlm\rangle &= -\frac{I}{2}\sqrt{(2l-1)\frac{(l-m-2)!}{(l+m)!}}\sqrt{(2l+1)\frac{(l-m)!}{(l+m)!}}\,\frac{1}{(2l-1)(2l+1)}\frac{(l+m)!}{(l-m-2)!} \\
&= -\frac{I}{2}\sqrt{\frac{(l-m)(l-m-1)}{(2l-1)(2l+1)}}.
\end{aligned} \tag{13}$$

Now we do the same for $m' = m-1$. Eqs. (7) and (8) are unchanged, the θ integral is

$$\begin{aligned}
\text{Int}_\theta &= \int_{-1}^{1} \sqrt{1-x^2}\, P_{l-1}^{m-1}(x)P_l^m(x)\,dx = \frac{1}{(2l-1)}\left[\int_{-1}^{1} P_l^m P_l^m\,dx - \int_{-1}^{1} P_{l-2}^m P_l^m\,dx\right] \\
&= \frac{2}{(2l-1)(2l+1)}\frac{(l+m)!}{(l-m)!},
\end{aligned}$$

and (7) becomes

$$\begin{aligned}\langle n'(l-1)(m-1)|x|nlm\rangle &= \frac{I}{2}\sqrt{(2l-1)\frac{(l-m)!}{(l+m-2)!}}\sqrt{(2l+1)\frac{(l-m)!}{(l+m)!}}\frac{1}{(2l-1)(2l+1)}\frac{(l+m)!}{(l-m)!} \\ &= \frac{I}{2}\sqrt{\frac{(l+m)(l+m-1)}{(2l-1)(2l+1)}}. \end{aligned} \tag{14}$$

Thus

$$\begin{aligned} & |\langle n'(l-1)(m+1)|\mathbf{r}|nlm\rangle|^2 + |\langle n'(l-1)m|\mathbf{r}|nlm\rangle|^2 + |\langle n'(l-1)(m-1)|\mathbf{r}|nlm\rangle|^2 \\ &= 2\left[-\frac{I}{2}\sqrt{\frac{(l-m)(l-m-1)}{(2l-1)(2l+1)}}\right]^2 + \left[I\sqrt{\frac{l^2-m^2}{(2l-1)(2l+1)}}\right]^2 + 2\left[\frac{I}{2}\sqrt{\frac{(l+m)(l+m-1)}{(2l-1)(2l+1)}}.\right]^2 \\ &= \frac{I^2}{2}\left\{\frac{(l-m)(l-m-1)+2(l^2-m^2)+(l+m)(l+m-1)}{(2l-1)(2l+1)}\right\} \\ &= I^2\frac{(2l^2-l)}{(2l-1)(2l+1)} = I^2\frac{l}{(2l+1)}, \end{aligned} \tag{15}$$

and the emission rate is

$$\boxed{A_{l\to l-1} = \frac{e^2\omega^3 I^2}{3\pi\epsilon_0\hbar c^3}\frac{l}{(2l+1)}.} \tag{16}$$

(Of course, I is different for the two cases $l \to l \pm 1$.)

- page 254, 4th line from bottom: $\frac{imb}{\hbar w} \to \frac{imv}{\hbar w}$.
- page 255, Problem 10.1(b), line 3, upper limit of second integral: $\pi \to a$; end of that line: $c_n' \to c_{n'}$.
- page 255, penultimate line, $\sin\left(\frac{n\pi}{a}\right) \to \sin\left(\frac{n\pi}{a}x\right)$.
- page 256, in the first box: "$T_e a/v$" $\to$ "$T_e = a/v$".
- page 256, Problem 10.1(c), penultimate line: $\frac{mva^2}{2\hbar a} = \frac{mva}{2\hbar} \to \frac{mv(2a)^2}{2\hbar(2a)} = \frac{mva}{\hbar}$.
- page 257, line 2, lower row: first cos $\to$ sin.
- page 257, line 8: $\frac{\omega_1(\omega_1-\omega)}{\lambda} \to \frac{i\omega_1(\omega_1-\omega)}{\lambda}$.
- page 259, Problem 10.4, final box: $\hbar^2 \to \hbar^3$.
- page 265, Problem 10.9(b), line 4: remove $\hbar$ after $(n+\frac{1}{2})$.
- page 265, 4 lines from end: $\frac{d\Psi_n}{dz} \to \frac{d\psi_n}{dz}$.
- page 266, Problem 10.9(e), line 4: remove $\hbar$ after $(n+\frac{1}{2})$.
- page 267, Problem 10.10(b), part (1), line 3: ", and (Eq. 10.39)" $\to$ "; Eqs. 10.39 and 10.93 yield"; line 4, insert after $\hbar\omega t$: $-\frac{m\omega^2}{2\hbar}\int_0^t [f(t')]^2\,dt'$.

- page 268, Problem 11.1(a), first line: square $\dot{r}$.
- page 270, Problem 11.1(c), in the expression for σ: $8 \to 16$.
- page 271, line beginning "**(1)** ψ continuous": $sinka \to \sin ka$".
- page 272, Problem 11.5(a), line 2: $B^{-ikx} \to Be^{-ikx}$; line 10: $B^{ika} \to Be^{ika}$.
- page 272, Problem 11.5(a), line 4: $-k'\psi \to -(k')^2\psi$.
- page 273, Problem 11.6, line 3, in the denominator: $j_l(x) + in_l(x) \to j_l(ka) + in_l(ka)$.
- page 275, line 3: $dr \to dr_0$.
- page 275, Problem 11.10, line 4: $\cos(\kappa a) = \to \cos(\kappa a) \approx$.
- page 276, Problem 11.12, final box: $k \to \hbar$.
- page 278, line 3: insert plus sign at beginning.
- page 279, line 6: before G insert $\int$.
- page 280, Problem 11.18, end of line 4: $\alpha \to \alpha^2$.
- page 280, Problem 11.18, 4 lines from end: $R = \left[\frac{m}{\hbar^2 k}\right)^2 \left(\frac{V_0}{k}\sin(2ka)\right]^2 \to R = \left[\left(\frac{m}{\hbar^2 k}\right)^2 \left(\frac{V_0}{k}\sin(2ka)\right)\right]^2$.
- page 288, Problem A.13, end of line 2: $\det \mathsf{U} = 1 \to |\det \mathsf{U}| = 1$.
- page 289-290, Problem A.15: starting with the last line on page 289, we should have

$$\hat{i} = -\hat{j}'; \quad \hat{j} = \hat{i}'; \quad \hat{k} = \hat{k}', \quad \text{so (Eq. A.61)} \quad \boxed{\mathsf{S} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.} \quad \mathsf{S}^{-1} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

$$\begin{aligned} \mathsf{ST}_x\mathsf{S}^{-1} &= \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ \cos\theta & 0 & -\sin\theta \\ \sin\theta & 0 & \cos\theta \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} = \mathsf{T}_y(-\theta). \end{aligned}$$

$$\begin{aligned} \mathsf{ST}_y\mathsf{S}^{-1} &= \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -\cos\theta & \sin\theta \\ 1 & 0 & 0 \\ 0 & \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} = \mathsf{T}_x(\theta). \end{aligned}$$

  Is this what we would expect? Yes, for rotation about the x axis now means rotation about the $-y'$ axis, and rotation about the y axis has become rotation about the x axis.
- page 297, Problem A.28(a), first line of part (ii): $\mathsf{M}^3 = -\theta^3\mathsf{M} \to \mathsf{M}^3 = -\theta^2\mathsf{M}$.