Is Current a Vector?

So many people have written to me complaining about my treatment of current as a vector that I have decided to write out my explanation once and for all. Some of what follows may not be relevant to your personal concern, but I hope I have addressed the most common objections. Let me say at the outset that if it bothers you to think of current as a vector, by all means stick with the scalar notation! This is purely a matter of convenience, and most people think it is simpler to do it that way. But for consistency with more advanced work it is (in my opinion) preferable to switch over at some stage to the more general notation.

Let me begin with an analogy: A locomotive on a curving track. Suppose I want to tell someone how fast it is going. Should I specify its *velocity* (magnitude and direction), or just its *speed* (the magnitude)? Obviously it would be simplest to quote the *speed*—after all, the direction of the motion is dictated by the shape of the track, and it seems awkward and unnecessary to say "60 mph due east," and a moment later "60 mph at 37° west of north," and so on—why not just say the train is going 60 mph? In this context it is simpler to use the scalar quantity. But if you are a physics teacher, you know that in the next chapter you will be discussing projectile motion (say), in which the object in question moves in two dimensions, and its direction is *not* constrained by the shape of any track. In this context you might prefer to talk about the *velocity.* Is the rate of change of position a scalar, or a vector? I guess it depends on the context which is the more natural terminology.

In the case of current exactly the same issue arises: If you are talking about current constrained to flow along a wire (the analog to the track), then no doubt it is simpler to treat it as a scalar quantity. But when it comes to surface currents (charge flowing over a 2-dimensional surface) or volume currents (charge flowing through 3-dimensional space), then you *have* to keep track of the directions, and so we adopt the vector notation ($\mathbf{K}$ and $\mathbf{J}$, respectively). Looking back, one asks how come we didn't use a vector for the 1-dimensional case ($\mathbf{I}$). Perhaps, for consistency, we *should* have!

It is a peculiar fact that in almost all formulas (Biot-Savart, Lorentz, etc.) current appears multiplied by an element of displacement along the wire: $I\,d\mathbf{l}$. Because the current (I) and the displacement ($d\mathbf{l}$) are in the same direction, it doesn't matter whether you associate the vector sign with the one or the other—in elementary work everyone attaches it to $d\mathbf{l}$, keeping I a scalar. But again, when you come to surface and volume currents, where the analogous expression is $\mathbf{K}\,da$ and $\mathbf{J}\,d\tau$, you have no choice: here the direction is that of the current ($d\tau$ doesn't even *have* a direction). So if you write $\mathbf{I}\,dl$ for the one-dimensional current, everything is consistent, even though in that case you can do it either way.

Here's another consideration: If you have charges of one sign, with charge density ρ, flowing with velocity $\mathbf{v}$ (in 3-dimensions), this constitutes a volume current density

$$\mathbf{J} = \rho\mathbf{v}.$$

If you have a surface charge σ flowing over a surface with velocity $\mathbf{v}$, the surface current is

$$\mathbf{K} = \sigma\mathbf{v}.$$

What about a line charge λ, flowing down a wire at velocity $\mathbf{v}$? I would like to write the analogous expression

$$\mathbf{I} = \lambda\mathbf{v}.$$

But if you forbid me to treat current as a vector I have to adopt a different notation for the one-dimensional case. Doesn't that seem awkward?

OK: I have told you why I prefer (in appropriate cases) to represent current as a vector. Let me now address the standard objections to this:

- **In high school I was told that current is a scalar.** Sure—so was I. And that is entirely appropriate at that level, where surface and volume currents do not occur. It's not *wrong*, exactly, just limiting. At the college level it is time to liberate yourself from the more constraining notation.

- **Charge is a scalar, and time is a scalar, so $I = dq/dt$ must be a scalar.** To put it another way, the current in a wire is the amount of charge per unit time passing a particular point—there is no reference here to direction. Well, actually there *is*: if positive charge is passing to the right we say the current is positive (say), and if it is going to the left we say the current is negative. Of course, in a wire those are the only two possibilities, so a simple sign suffices to keep track of it. But in general we are talking about charge per unit time flowing in a particular direction—it's just that the direction (up to a sign) is dictated by the shape of the wire. Again, one could run the same argument with the locomotive on the track: distance (traveled) is a scalar, and time is a scalar, so velocity must be a scalar(?)! No: it is not distance we meant, but distance *in a particular direction*, which is to say *displacement*.

- **Currents do not add vectorially.** This is the most subtle objection, because it misconstrues the meaning of vector addition. Kirchhoff's law says that the sum of the currents *into* a junction is zero (counting currents flowing *out* as negative). They do *not* add vectorially—it doesn't matter what direction the wires meet at. Correct, but this is a *law of physics*, not a mathematical case of addition of vectors/scalars. If I say the sum of the momenta of some colliding particles is the same after the collision as before, that is (again) a *law of physics*, not a question of vector addition (though in this case the law says you should add them vectorially). Kirchhoff's law has nothing to do with the vectorial nature of current, since it refers to the *magnitudes*. The number of locomotives entering a junction is equal to the number leaving (in any given time interval)—scalar addition—but this tells me nothing about the vectorial nature of velocity, only about the law of conservation of locomotives.

  In point of fact, currents *do* add vectorially. Here's an example. Suppose I have a fat pipe, and inside that pipe there are some wires, each carrying a current. The wires do not run parallel to the axis of the pipe, but make various angles. When they reach the edge of the pipe, they bend back so as to make the same angle (like light rays reflecting off the surface of a light pipe). If you ask "What is the total current flowing down the pipe?" you would of course have to take into account the direction of each wire. If it happens to run parallel to the pipe, it counts at full value, but if it's going at 45° you'll need to throw in a factor of $1/\sqrt{2}$ (and if it is perpendicular to the pipe it contributes nothing). In other words, you must add the currents *vectorially*—the total current is the *vector* sum of the individual currents.

I hope this helps. But again: if you prefer to think of current as a scalar, that's fine. It won't hurt you; it just isn't very elegant.

David Griffiths