Review

Lecture 24

Physics 342
Quantum Mechanics I

Wednesday, April 2nd, 2008

Homework

Reading: Griffiths, pp. 160–170.

Note that the homework is not due until Monday, April 7th, 2008.

Problem 24.1

Define $f_\ell^m(\theta, \phi)$ to be the eigenfunction of L_z and L^2 with eigenvalues given by $\hbar\, m$ and $\hbar^2\, \ell\,(\ell+1)$:

$$L_z\, f_\ell^m = \hbar\, m\, f_\ell^m \qquad L^2\, f_\ell^m = \hbar^2\, \ell\,(\ell+1)\; f_\ell^m. \tag{24.1}$$

Find the "lowest" state $f_\ell^{-\ell}$ using $L_-\, f_\ell^{-\ell} = 0$. Calculate the state $f_\ell^{-\ell+1}$ from this (by using the appropriate raising operation).

Problem 24.2

Griffiths 4.18. Here we work out the constants associated with normalizing the eigenfunctions of L_z.

Problem 24.3

For $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, we have the operator: $\mathbf{L} = \frac{\hbar}{i}\, \mathbf{r} \times \nabla$. Using ∇ expressed in spherical coordinates, find the actual differential operator expressions associated with L_x, L_y and L_z – these should involve $\frac{\partial}{\partial\theta}$ and $\frac{\partial}{\partial\phi}$ (and

associated factors of $\cos\theta$, $\cos\phi$, etc.). Using these, form the explicit, operator expression for L_+ and solve $L_+ f_\ell^\ell(\theta,\phi) = 0$ for $f_\ell^\ell(\theta,\phi)$ to find the "top" state (you could then apply the lowering operator to work down).