Mass of the Muon

Introduction

Earth is continually bombarded by cosmic rays, high-speed subatomic particles produced by astrophysical processes such as supernovae. Many of the first experiments in elementary particle physics used cosmic rays as their particle source, and even today, they are used in experiments that probe the very highest particle energies. While this source of high-speed particles is free and omnipresent, the flux of cosmic rays is relatively low. Thus experiments done with cosmic rays can best probe processes with high intrinsic rates.

For over forty years, the presence of high-speed muons at Earth's surface has allowed educators to design instructional laboratory experiments that illustrate the concerns and experimental methods of elementary particle physics. These muon are the byproduct of collisions between cosmic rays and gas molecules near the top of Earth's atmosphere. By far, the most widely adopted such experiments measures the muon's lifetime [1-5], but others include observation of the time dilation effect [5, 6] and determination of the muon's magnetic moment [7]. In 1964, researchers at the University of Michigan suggested that the mass of the muon could be measured through an experiment that analyzed tracks produced by muon decay within a spark chamber [8]. These researchers built a prototype system and demonstrated the feasibility of this approach. However, at that time, the necessary setup required too much specialized technical expertise for most educators to copy and the experiment was too cumbersome to perform (because track images were captured on photographic film). Thus, this experiment has never been embraced by instructional lab developers. In this paper, we describe the muon mass experiment and how it can be implemented today with relative ease through the use of modern instrumentation, most notably CCD-based image acquisition. We analyze the data gleaned from our experimental setup and go on to show how limitation in the experimental apparatus can be account for through an available software simulation package called GEANT.

Theory

A muon is a negatively charged elementary particle from the lepton family with a rest mass that is about 200 times that of an electron. With effectively 100% probability, a free muon decays into an electron, an electron-antineutrino, and a muon-neutrino with a mean lifetime of $2.2\ \mu\text{s}$. That is, the muon's principal decay mode is $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$. Similarly, the principal decay mode of the muon's antiparticle is $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$. As a result of the interaction of cosmic rays with air molecules at the top of Earth's atmosphere, Earth's surface is bombarded by a stream of high-speed "secondary cosmic ray" particles, 75% of which are a roughly equal mixture of muons and antimuons. In the vertical direction $\left(\theta = 0^\text{o}\right)$, the total flux per unit solid angle of these secondary particles is about $0.66/\text{cm}^2 \cdot \text{sr} \cdot \text{min}$. Using this value, along with the empirically determined

$\cos^2\theta$ variation of particle flux with angle θ from vertical, it can be shown that the flux of secondary cosmic-ray particles impinging on a horizontally aligned detector is on the order of $1/\text{cm}^2 \cdot \text{min}$.

The goal of our experiment is to stop many secondary cosmic-ray muons within a spark chamber and observe each decay into an electron (we will simply describe processes involving the muon, but our discussion likewise applies to the analogous processes involving the antimuon). The spark chamber is a charged-particle detector which makes visible the muon's path prior to being stopped as well as the path of the product electron after the decay and allows the electron's initial kinetic energy to be determined. Two product neutrinos are also produced by the muon's decay, but because they are neutral their paths are not recorded by the spark chamber. Our chamber consists of a vertical stack of about 20 aluminum disks. Most cosmic-ray muons that enter the chamber have energies greater than 100 MeV and so will pass completely through the chamber. However, a small population of low-energy muons will be slowed as they pass through the aluminum to speeds on the order of the aluminum's valence electrons, suppressing the time-dilation effect that allowed these short-lived particles to traverse the height of the atmosphere. All of the "stopped" antimuon then decay freely via $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$. However, roughly hlalf of the "stopped" muons are captured by atoms and undergo the process inverse beta $\mu^- + \text{Al} \to \nu_\mu + \text{Mg}^*$ (which is undetected by the spark chamber), while the other half decay freely via $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$.

Consider the decay of a stationary muon of rest energy $m_\mu c^2$ into three product particles (electron, anti-electron neutrino, muon neutrino). Applying conservation of energy to this process, we find $m_\mu c^2 = E_e + E_{\bar{\nu}_e} + E_{\nu_\mu}$, where the energies E_e, $E_{\bar{\nu}_e}$, and E_{ν_μ} of the product particles can be taken (to a good approximation) to be entirely kinetic energy because each of these product particles has a rest energy much less than $m_\mu c^2$. Additionally, applying conservation of momentum, we get $0 = \vec{p}_e + \vec{p}_{\bar{\nu}_e} + \vec{p}_{\nu_\mu}$, where $\vec{p}_e$, $\vec{p}_{\bar{\nu}_e}$, and $\vec{p}_{\nu_\mu}$ are the relativistic momenta of the product particles. With three-product particles, the kinetic energy of the electron is not uniquely determined by the conservation laws, but instead a range of E_e-values is possible. The minimum value of this range is $E_e = 0$ (neglecting the electron's rest energy) in which case the electron is at rest and the two neutrinos move oppositely directed with equal momentum magnitude. At the other extreme, the maximum electron energy occurs when the two neutrinos both move in the opposite direction to the electron. Then, assuming the product particles are all massless, it is easy to show from the conservation laws that $E_e = \frac{1}{2} m_\mu c^2$. Thus, the allowed range of electron energies is $0 \le E_e \le \frac{1}{2} m_\mu c^2$. The quantum electrodynamics analysis of muon decay [6], in which the decay (in lowest order) is mediated by a W boson of mass M_W, yields the following relation for the probability per unit time $d\Gamma$ that a product electron will be produced with energy in the interval E_e to $E_e + dE_e$

$$\frac{d\Gamma}{dE_e} = \left(\frac{g_w}{M_W c}\right)^4 \frac{m_\mu{}^2}{2\hbar(4\pi)^3} E_e{}^2 \left(1 - \frac{4}{3}\frac{E_e}{m_\mu c^2}\right) \qquad 0 \le E_e \le \frac{1}{2} m_\mu c^2 \qquad [1]$$

where g_w is the weak coupling constant. Higher order electromagnetic effects modify this expression slightly (by approximately 1%). From [1] it follows that if many muons are observed to decay during an experiment of fixed duration, the mean daughter electron energy is $\bar{E}_e = \frac{7}{20} m_\mu c^2$.

System Overview

Our system consists of four main components: spark chamber, particle-decay detection, high-voltage pulsing electronics, and CCD-based image acquisition. A short description of each of these components follows.

The spark chamber consists of a series of conducting plates, where each plate is separated from its neighboring plates by gaps filled with a noble gas (in our case, neon). The odd-numbered plates are electrically grounded, while the even numbered plates are connected to a high-voltage, high-current pulser. When a charged particle moves within the chamber, it ionizes the noble gas along its path, leaving behind a trail of ionized atoms and liberated electrons. If the pulser can be triggered to apply sufficiently high voltage (typically 4000 V) to the even numbered plates more quickly than the mena ion-electrons recombination time (on order of $5\ \mu\text{s}$), sparks will form in the gaps between the plates along the low-resistance ionized track, making visible the trajectory of the moving charged particle.

The decay of a cosmic ray muon within the spark chamber is detected through the use of three scintillation detectors stacked vertically about the chamber. Each detector consists of a horizontally aligned light-tight $20\ \text{cm} \times 20\ \text{cm} \times 2\ \text{cm}$ plastic scintillator coupled to a blue-light sensitive photomultiplier tube (PMT). When a high-speed charged muon passes through such a detector, it causes the scintillator material to fluoresce and the fluorescent photons are detected and converted to a short $(\sim 5\ \text{ns})$, negative-going electrical signal by the attached PMT. This signal is then passed through a discriminator which converts the PMT signal into a digital pulse with duration of about 50 ns. As shown in Fig xx, two of these detectors (call them X and Y) are placed above the chamber, separated by a vertical distance of approximately 1 meter, while the third detector (Z) is placed slightly below the chamber. When a muon with (close to) normal incidence enters the chamber from above, and then subsequently decays within the chamber, digital pulses will be produced by X and Y, while a digital pulse will not be produced by Z, assuming the decay-product electron is stopped within the chamber). Thus, if the digital output of the three discriminators are input to a logic unit that performs the coincidence/anticoincidence operation $X \cdot Y \cdot \bar{Z}$ (X AND Y AND not Z), the output of this logic unit will signals the muon's decay within the spark chamber.

The output of the coincidence/anticoincidence logic unit is connected to the input of a high-speed pulser ("driver"). This driver unit produces the proper voltage pulse to trigger a high-voltage, high-current switch ("thyratron"), which then causes a bank of capacitors to discharge, applying a (negative) pulse of several thousand volts to the even-numbered plates of the spark chamber.

Finally, a video camera focused on the spark chamber is configured to send its analog video signal to the video input of a frame grabber board within the expansion slot of a PC, while the output of the coincidence/anticoincidence logic unit is connected to the board's trigger input. The frame grabber is controlled via a LabVIEW program, which acquires and stores a single image of the chamber whenever the logic unit detects a muon decay event.

Experimental Procedure

With high voltage applied to all three scintillation detector PMTs, each PMT output (after passing through a preamplifier) is viewed on a fast oscilloscope. By noting the typical amplitude of a signal pulse (relative to the noise level), an appropriate value for the threshold is set for each of the three detector channels on a multi-channel (at least three) constant fraction discriminator (CFD). Using the oscilloscope, it is then verified that each PMT signal pulse input to a CFD channel results in a logic pulse (typical width 50 ns) at the CFD output for that channel. In our setup, each detector's cross-sectional area $A = 20 \text{ cm} \times 20 \text{ cm} = 400 \text{ cm}^2$, so the rate of logic pulses produced by secondary cosmic ray particles on each channel should be about
$\left(1/\text{cm}^2 \cdot \text{min}\right)\left(400 \text{ cm}^2\right)\left(1 \text{ min}/60 \text{ s}\right) \approx 7/\text{s}$.

After properly configuring the CFD as above, the logic-pulse outputs of the three channels X, Y, and Z are input into the logic unit. The output of this logic unit is connected to the fast voltage pulser, which triggers the spark chamber operation. Proper functioning of the setup can be verified by programming the logic unit to perform the logic operation $X \cdot Y$, so that the spark chamber is triggered for any charged particle (i.e., not just those that decay) that passes into the chamber through the two top detectors. The expected rate of such $X \cdot Y$ coincidence is found as follows: X and Y each have area $A = 400 \text{ cm}^2$, and are separated by distance $d = 100 \text{ cm}$. Thus, at each point on Y, the solid angle subtended by X is $\Delta\Omega \approx \frac{A}{d^2} = \frac{400 \text{ cm}^2}{(100 \text{ cm})^2} = 0.04 \text{ sr}$, and so the expected rate at which vertically downward secondary cosmic ray particles (75 % of which are muons and antimuons) will pass through both detectors producing coincidence signals is
$\left(0.66/\text{cm}^2 \cdot \text{sr} \cdot \text{min}\right)\left(400 \text{ cm}^2\right)\left(0.04 \text{ sr}\right) = 10/\text{min}$.

Finally, the logic unit is configured to perform the logic operation $X \cdot Y \cdot \bar{Z}$, the video camera is focused on the spark chamber along with its perpendicular reflection, and a LabVIEW-based image acquisition program is started. Each time the logic unit triggers the spark chamber, it also sends a TTL trigger pulse that triggers the frame grabber board to acquire the video camera's current frame and save it as a .jpeg file on the computer's hard drive with a unique file name. The experiment is run under computer-control in a darkened room overnight, typically resulting in about 100 saved images per hour.

After a data run, one must inspect all the images manually, eliminating those obviously not due to muon decay (e.g., two charged product particles) or those in which the product electron from muon decay appears to have left the chamber. After this selection process, one is typically left with about 50 images that clearly record muon decay (see Fig xx for a sample image).

Analysis of Data

For each decay image, we construct two lines, one delineating the path of the incoming muon, and the other marking the path of the product electron. The intersection of these lines defines the location at which the muon decayed. Once point of decay is established, one then counts the number n of aluminum plates, each of thickness $t = 3/8'' = 0.953\ \text{cm}$, traversed by the electron as it is brought to rest, and also estimates the angle θ of the electron's path relative to the normal to the faces of the aluminum plates. When traveling in aluminum, the stopping distance $d\ (\text{cm})$ of a beta particle with initial kinetic energy E (MeV) has been show empirically to be

$$d = 0.196E - 0.039 \qquad [2]$$

,for $2.5\ \text{MeV} < E < 100\ \text{MeV}$ [10]. Thus, the electron's energy at the location of the decay is found from

$$E(\text{MeV}) = \frac{1}{0.196}\left[\frac{n(0.953\ \text{cm})}{\cos\theta} + 0.039\right] \qquad [3]$$

For small angles, [3] reduces to $E(\text{MeV}) \approx 5n$, i.e., the electron loses about 5 MeV of kinetic energy in each plate it traverses.

Repeating this process for all decay images, and choosing an energy bin size of 5 MeV, a histogram of the number N of occurrences of electron energy E can be generated as shown in Fig. XX. Based on this histogram, a value for the muon mass can be determined in several ways. First, and most simply, the maximum electron energy equal $E_e = \frac{1}{2} m_\mu c^2$. From our plot we see that this maximum value is 45 MeV, thus $m_\mu c^2 = 2(45\ \text{MeV}) = 90\ \text{MeV}$. Second, from the histogram values, it is found that $\bar{E}_e = 29\ \text{MeV}$. Theoretically, we expect $\bar{E}_e = \frac{7}{20} m_\mu c^2$, thus

$m_\mu c^2 = \frac{20}{7}(29\ \text{MeV}) = 83\ \text{MeV}$. Finally, from Eq. [1], we expect that for an experimental run of finite time, the values for N vs. E should follow the functional form $N = \alpha E^2 \left(1 - \frac{4}{3}\frac{E}{\beta}\right)$, where α and $\beta\left(= m_\mu c^2\right)$ are constants. A least-squares fit of this function form to the data in Fig. Xxx yields the solid curve shown with $\beta = m_\mu c^2 = 80^{+15}_{-23}\ \text{MeV}$.

Appendix: Construction Details

Spark Chamber: The design of the spark camber is an adaptation of a spark chamber constructed by A.M. Sachs in the mid-1960s [7]. The chamber is constructed of $3/8''$ thick, round aluminum plates. The chamber consists of 20 gaps and 21 plates, and is illustrated in Fig xx. The 19 inner plates are six inches in diameter; the top and bottom plates are $7''$ in diameter. The conductive aluminum plates of the chamber are separated by $6''$ outer diameter, $5\,1/4''$ inner diameter plexiglass spacer rings, $1/4''$ in thickness. It is essential that the chamber be completely leak tight, and O-rings provide a simple cost effective means of sealing the chamber. Therefore, the gas seals between the aluminum plates and plexiglass spacers are made with O-rings. The plates of the chamber have grooves to accommodate $1/8''$ thick, $5\,1/2''$ inner diameter O-rings. The 19 inner plates have $1/4''$ holes to allow gas to flow freely in the chamber. The chamber is held together by four threaded rods; these rods are electrically shielded from the pulsed plates with $1/2''$ O.D., $1/4''$ I.D. plexiglass sleeves.

The chamber is filled with pure neon at slightly over atmospheric pressure (approximately 1.1 atm) to ensure that any leakage in the chamber will be outward, not inward. This prevents any atmospheric gas impurities from leaking into the chamber. Electronegative gas impurities such as oxygen have detrimental effects on the chamber's operation.

Scintillation Detectors: Each scintillation detector consists of a light-tight $20\ \text{cm} \times 20\ \text{cm} \times 2\ \text{cm}$ plastic scintillator (Saint-Gobain BC-408) coupled to a blue-light sensitive photomultiplier tube (Hamamatsu R7400). The detectors were purchased as assembled units (Saint-Gobain 8X8.8BC408/.5L-X). The PMTs of all three detectors are connected in parallel to a high-voltage supply (Stanford Research Systems PS310) and power operated at 1000 V. The signal output of each PMT is passed through a fast (140 MHz) noninverting preamplifier (LeCroy 612AM) with adjustable (2.5 to 40) gain.[15]

Coincidence/Anti-Coincidence Electronics: Each of the amplified PMT signals from X, Y, and Z is sent to an input of a multi-channel constant fraction discriminator

(CFD). We use the ECL-logic 16-channel LeCroy 3420, a CAMAC module whose input channel thresholds and output positive-logic pulse widths are computer-controllable. Typical threshold and pulse-widths values in our experiment are 200 mV and 100 ns, respectively. The three ECL logic output signals are input to a homemade logic unit, which functions as shown in Fig. Xxx. In the first stage of this unit, the input ECL signals are regenerated by a receiver chip (MC10H116), then converted from ECL to TTL logic levels(MC10H125). These TTL representations of the input signals are available for monitoring at the X, Y, and Z outputs and are also passed to the heart of the circuit, which is a four-input TTL-based AND gate (7421) that can be configured to perform the logical operation $X \cdot Y$ or else $X \cdot Y \cdot \bar{Z}$ via a switch. Finally, the output of the AND is available for monitoring at LOGIC OUT and is also used to trigger two "one-shot" monostable multivibrators (74123). The one-shot TTL outputs each have a width of 800 ns, which is of sufficient duration to trigger the high-voltage pulser as well as the frame grabber board.

High-Voltage Pulsing Electronics: Triggerable high-voltage pulses are applied to the spark chamber plates using the circuit shown in Fig. XXX. Inside a homemade pulser box, several thousand (typically 4000) Volts from a high-voltage supply (Stanford Research Systems PS350) is applied to the anode of a hydrogen thyratron (Perkin Elmer HY-6) as well as the "high-voltage" side of each $C = 500$ pF capacitors in a parallel bank of capacitors. The "low-voltage" side of each capacitor is connected to a unique plate of the spark chamber as well as a current limiting $R = 3\ \mathrm{k}\Omega$ resistor, whose opposite end is grounded. This circuit is triggered by a commercially available Thyratron Driver (Perkin Elmer TM-27, modified to trigger on 1.5 V). When the Driver receives a TTL pulse from one of the logic unit's one-shots, it outputs a $2\text{-}\mu\mathrm{s}$ wide pulse of amplitude 800 V with a rise time of less than 150 ns. This trigger pulse is passed into the homemade pulser box, where it is applied to the grid of a hydrogen thyratron (Perkin Elmer HY-6), driving the thyratron into conduction for $2\ \mu\mathrm{s}$ and causing the "low-voltage" sides of the capacitors to drop to large negative voltage for a time period of approximately $\tau = RC = (3\ \mathrm{k}\Omega)(500\ \mathrm{pF}) = 1.5\ \mu\mathrm{s}$.

Image Acquisition System: A monochrome progressive scan CCD camera head (Sony XC-55) with zoom lens (Navitar Zoom 7010) is focused on the spark chamber directly as well as its perpendicular reflection is a mirror angled at 45°. With the camera's shutter speed at $1/30$ s, the analog video signal is sent to a frame grabber board (National Instruments PCI-1408) plugged into a PCI expansion slot of a PC. The output from one of the logic unit's one-shots is connected the frame grabber's TRIGGER input. Under the control of a LabVIEW based program, when the one-shot sends a TTL pulse to the TRIGGER input, the current frame acquired by the camera is stored under a unique filename as a jpeg file on the PC's hard drive.

1. See, e.g., Adrian C. Melissinos and Jim Napolitano, Experiments in Modern Physics, Second Edition (Academic Press, San Diego, 2003) pp. 399-409; http://web.mit.edu/8.13/www/14.shtml; http://phys-advlab.physics.lsa.umich.edu/Phys441%5F442/Muon%20Lifetime/Muon%20Lifetime.htm

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10. L. Katz and A.S. Penfold, Rev. Mod. Phys., **24**, 28-44 (1952)

15. Alternately, ORTEC VT120C noninverting 350 MHz preamplifier with voltage gain of 20 could be used.

15. Alternately, North Star Research TT-G2 Thyratron Driver could be used.

Because of their relatively fast update rate and good spatial resolution, such spark chambers found many applications in the 1960s and early 1970s in high energy and comic ray particle physics.