Laboratory II: Transistors

The common-emitter amplifier with bypassed emitter resistor

1 Disclaimer

I will discuss silicon based NPN-type bipolar transistors such as the ones used in the lab. For other transistors, such as PNP-type transistors and field-effect transistors these considerations have to be modified, although the basic approach to the analysis remains unchanged.

2 Preliminaries

2.1 Transistors need biasing

Transistors have to be biased, meaning that the base pin and the collector pin have to be connected to a DC-voltage source such that for NPN transistors the following holds for the collector voltage V_C, base voltage V_B, and emitter voltage V_E :

$$\boxed{V_C > V_B > V_E \qquad V_B > 0.6\text{V}}$$

2.2 DC and AC voltages are analyzed separately and independently

This principle, that DC and AC voltages can be treated separately and independently, is derived from the fact that Maxwell's Equations are linear. It also means that we always have to add DC voltage and AC voltage at any instant to give the total voltage. Often we denote DC voltages and currents using upper-case letters and the AC voltages and currents using lower-case letters, e.g. $V_{base}(t) = V_B + v_B(t)$. (Note that in this notation lower-case i refers to AC currents. It should be clear from the context when i denotes the imaginary unit, $i = \sqrt{-1}$, instead.)

Kirchoff's laws represent the basis of the analysis. These laws state that (1) the sum of all the voltage changes as you follow around a loop in a circuit is always exactly zero, and (2) the current going into any point in a circuit is equal to the current going out of it. But before we can apply these laws, equivalent circuits MUST be drawn. There is no way around that fact. Furthermore, the equivalent diagrams for AC and DC voltages are different and must be drawn separately.

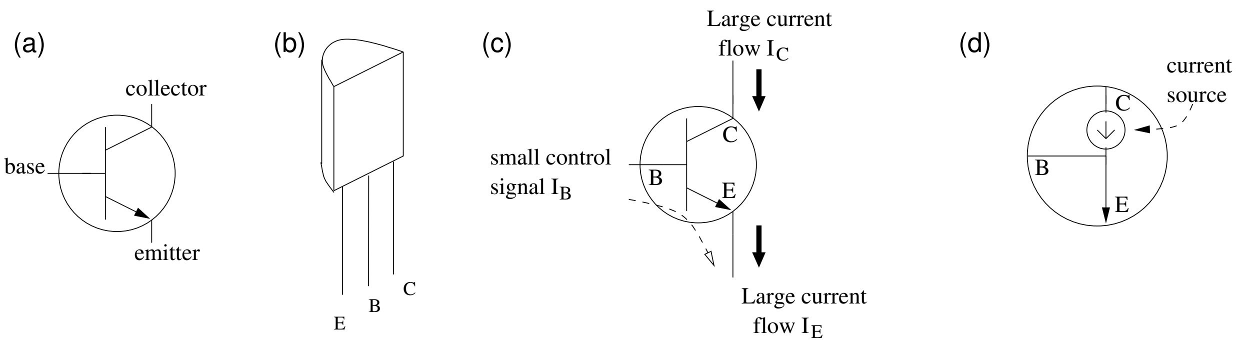

Figure 1: (a) transistor schematic (b) physical (c) the transistor as a current-controlled "valve" (or amplifier). (d) the equivalent diagram

2.3 Know and use at least two transistor models

In understanding transistors circuits it is useful to analyze them at different levels of complexity, starting with the simplest level first.

Simple Model - Current Amplifier : $\boxed{\text{(I) } I_C = \beta \cdot I_B \quad \text{(II) } V_{BE} = 0.6\text{V} = \text{const.}}$

Here I_C and I_B represent the total (AC+DC) collector current and base current, respectively. In reality, β is slightly frequency dependent so that the β factor for AC and DC voltages is not identical, however this difference can be safely neglected. V_{BE} is the potential difference between the base pin and the emitter pin.

One way to understand how a transistor works is to think of the transistor as a valve, where the base current I_B is the small control signal that determines the potentially large amount of current that flows from the collector (I_C) to the emitter ($I_E \approx I_C$). Since β is large, typically ranging from 20 to 200, you may often simply take $I_E = I_C$. This model is depicted graphically in Fig. 1c and the corresponding equivalent circuit diagram is shown in Fig. 1d. This model is very useful in analyzing DC-voltages. It can also provide some insights into the AC-voltage behavior, as shown in the next example.

Example: This simple model can provide us with a basic understanding of why the common-emitter amplifier shows a voltage gain. The reasoning is a follows (see Fig. 2):

1. My incoming AC voltage will wiggle the total voltage on the base-terminal of the transistor, *i.e.* introduce a change ΔV_B, which we will simply write as v_B. The lower case letter for voltage indicates that we are talking about AC-voltages.
2. A wiggle in the base voltage (v_B) will introduce a wiggle in the emitter voltage (v_E) because the voltage difference of bias and emitter, V_{BE}, is constant. This will lead to a wiggle in the emitter current i_E due to the presence of R_E. The current will be big if R_E is small ($i_E = v_E/R_E$).

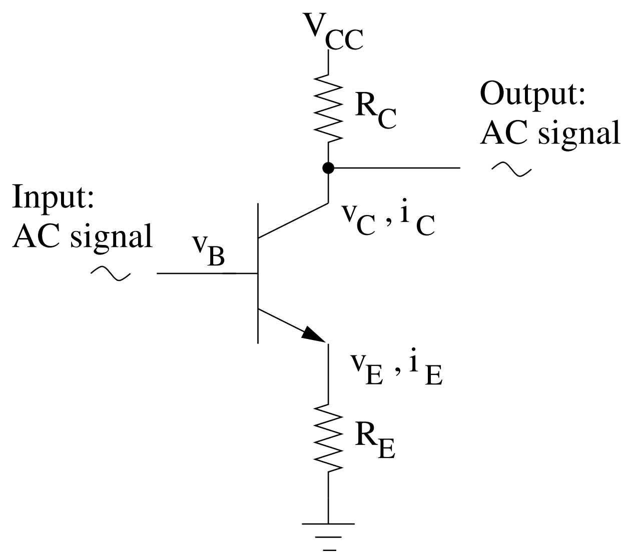

Figure 2: The common-emitter amplifier with emitter resistor.

3. Since i_C is essentially equal to i_E, the collector current i_C will wiggle, resulting in a wiggle of the collector voltage due to the presence of R_C. In detail, because

$$V_{CC} - V_C = I_C^{total} R_C,$$

a change of I_C^{total}, denoted by $i_C = \Delta I_C^{total}$, will result in a change of the collector voltage

$$i_C R_C = \Delta I_C^{total} R_C = -\Delta V_C = -v_C.$$

The collector AC-voltage v_C is also the output voltage, $v_{out} = v_C$. Thus, we obtain for the AC-part of the output

$$v_{out} = -i_C R_C = -i_E R_C = -\frac{R_C}{R_E} v_E = -\frac{R_C}{R_E} v_B.$$

Voilà, a voltage gain! The minus sign simply means the signal is inverted. The gain $g_V = |-R_C/R_E|$ will be large in magnitude if R_C is big and R_E is small.

However, for temperature stability of the bias voltages we want R_E to be big. This means we have a design problem. Additionally, clearly this model breaks down if we consider $R_E \to 0$. Then $v_{out} = \infty$, meaning that our gain is predicted to be infinite. This, of course, is wrong.

Improved Model - Transconductance Amplifier:

$$\boxed{\text{(I) } I_C = I_S(T)\left[e^{V_{BE}/V_T} - 1\right] \qquad \text{(II) } I_B = I_C/\beta}$$

Here, $V_T = kT/q = 25.3$ mV at room temperature, q is the electron charge (1.6×10^{-19} C), k is the Boltzman constant (1.38×10^{-23} J/K), and T is the temperature in Kelvin. I_S is the saturation current, which is strongly temperature dependent.

Condition (I) is called the "Ebers-Moll" equation. The interpretation of (I) is that V_{BE} is not constant as in the previously discussed simple model but, instead, small changes in the base-emitter voltage V_{BE} around its nominal value of 0.6 V will lead to exponentially large changes in the collector current I_C. The transistor acts as a voltage controlled current source, V_{BE} controls I_C! Therefore, we call the model the transconductance amplifier model, because the transconductance gain g_m is defined as the ratio of the current at the output port and the voltage at the input port: $g_m = i_{out}/v_{in}$.

The correct interpretation of condition (II) is that the base current also depends on V_{BE}, in a fashion similar to the collector current I_C. That is why I_C and I_B are approximately proportional to each other. Furthermore, $I_B \ll I_C$ because $\beta \sim 100$. As was the case for the simple model, in practice we can often take the condition $I_B \ll I_C$ to mean $I_B = 0$ (equivalent to $\beta = \infty$). Indeed this is preferably all we should use condition (II) for because if we actually use is for calculating I_B for a known I_C we run into the trouble that the value of β for the same transistor type can vary by more than 50%. Unfortunately in some computations we have to use β explicitly, such as when evaluating the transistor's input impedances (see Appendix).

One important consequence of the Ebers-Moll equation is the prediction of an *intrinsic* emitter resistance r_e. To see this, let us rewrite the Ebers-Moll equation in terms of V_{BE}

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S} + 1\right). \tag{1}$$

Then the dynamic emitter resistance is by definition,

$$r_e = \frac{dV_{BE}}{dI_E}. \tag{2}$$

With $I_E \approx I_C$ we therefore obtain

$$r_e = \frac{d}{dI_C}\left[V_T \ln\left(\frac{I_C}{I_S} + 1\right)\right] = V_T \frac{\frac{1}{I_S}}{\frac{I_C}{I_S} + 1} = \frac{V_T}{I_C + I_S} \tag{3}$$

Since $I_C \gg I_S$ one can safely neglect the I_S term, yielding

$$r_e = \frac{V_T}{I_C} = \frac{25\text{mV}}{I_C} = \frac{25\,\Omega}{I_C(\text{in mA})} \tag{4}$$

Note that I_C in this context denotes the total collector current (AC and DC).

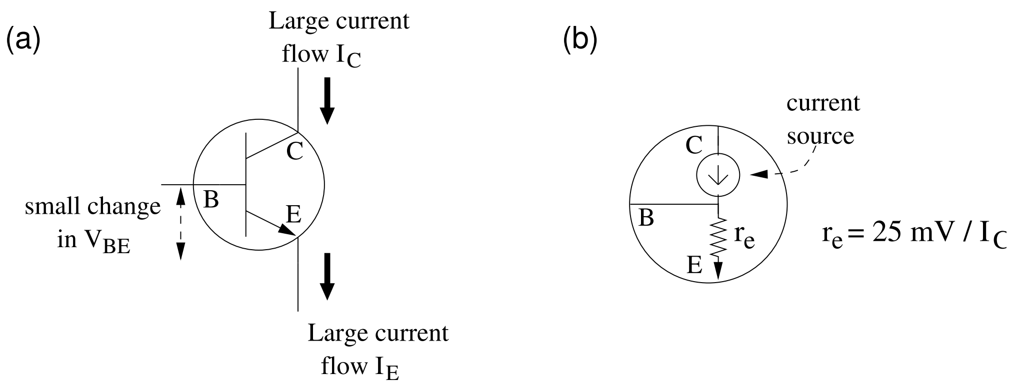

Figure 3: (a) The transistor as transconductance amplifier. (b) equivalent diagram

Another important consequence is the temperature dependence. Consider the Ebers-Moll model

$$I_C = I_S(T)\left[e^{\frac{q}{kT}V_{BE}} - 1\right] \approx I_S(T)e^{\frac{q}{kT}V_{BE}} \tag{5}$$

where we can always drop the "–1" term because in the active region $V_{BE} \gg V_T = kT/q$. Clearly, Eq. (5) says that I_C is temperature dependent, but will I_C increase or decrease with rising T? For fixed V_{BE} an increase of T would decrease the exponential factor and so one might think decrease I_C. However, due to the thermal generation of minority carriers, I_S, the saturation current, does not stay constant but increases very fast with temperature. As a result, the collector current *increases* for *fixed* V_{BE}. Entirely equivalent is the statement that the base-emitter voltage required for a given collector current decreases with increasing temperature. V_{BE} falls by 2.5 mV/°C, if one holds I_C *fixed*. [Simpson: -2.5 mV/°C, Horowitz & Hill: -2.1 mV/°C].

Summary: For our analysis, the improved transistor model has two main consequences

- Intrinsic dynamic emitter resistance: $r_e = 25\text{mV}/I_C^{total}$
- Temperature dependence: As T increases; V_{BE} decreases for fixed I_C, I_C increases for fixed V_{BE}.

3 Objectives for the circuit design

1. Forward bias the transistor using a voltage divider (R_1 and R_2). This means that we have to pull up the potential of the base such that it is at the very least 0.6 V (for Si) above ground.

2. Achieve the condition that the output impedance of the voltage divider ($R_1 + R_2$) is much smaller than the input impedance of the transistor.

3. Compliance (compliance is short for *output voltage compliance* and is defined as the limit of the output voltage that can be supplied by a circuit) must be such that maximum swing of the AC voltage can occur without clipping. Typically this means $V_C = 0.5V_{CC}$.

4. The biasing should be stable against temperature variations.

In addition, the circuit design should not depend on the β of the transistor because for a given transistor type β varies greatly.

4 DC voltages: Is the transistor correctly biased?

The DC analysis can be done entirely with just the simple transistor model (current amplifier model). The circuit diagram of the common-emitter amplifier with bypassed emitter resistor is shown in Fig. 4a. Any question concerning the absence of closed loops to write Kirchoff's equations for a circuit is resolved by understanding the conventions for drawing circuits. The loops are immediately apparent if we redraw the circuit as shown in Fig. 4b.

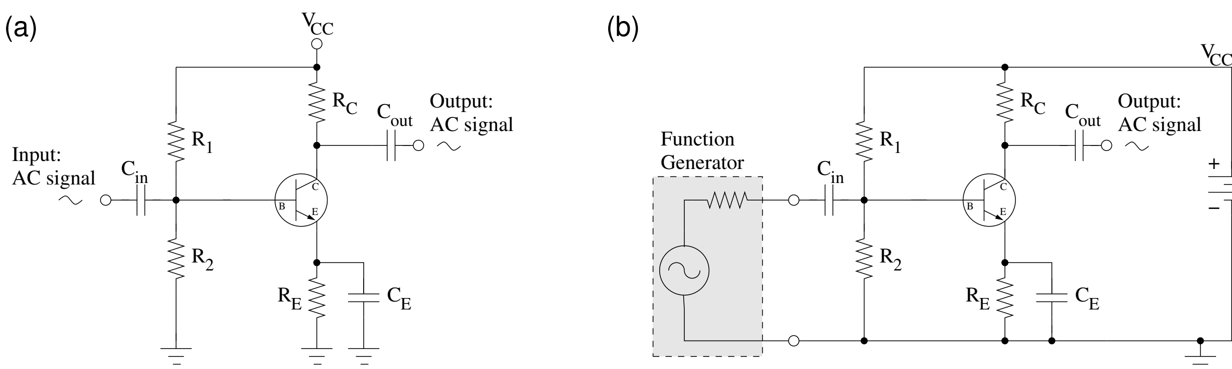

Figure 4: (a) Bypassed emitter resistor circuit. (b) same circuit with voltage loops.

4.1 Equivalent circuit

Before we start the DC-voltage analysis we have to arrive at the corresponding equivalent diagram.

1. Capacitors block DC-currents. Their DC-impedance is infinite:

$$\lim_{\omega \to 0} |Z_{cap}| = \lim_{\omega \to 0} \left| \frac{-i}{\omega C} \right| = \infty.$$

Therefore capacitors represent OPEN circuits and we typically just remove the corresponding "non-functional" part of the circuit from the diagram.

2. An AC-voltage source becomes a short in a DC equivalent circuit. Then symbolically we have

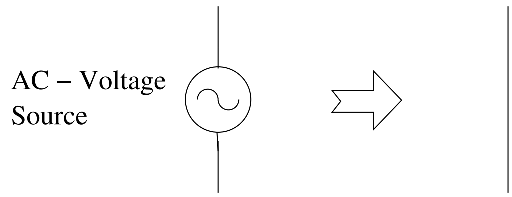

Figure 5: DC equivalent of an AC-voltage input or AC source

3. For the DC analysis we utilize the simple transistor model, *i.e.* the current amplifier model for the transistor. Therefore, we replace the transistor by the equivalent model shown in Fig.1d.

Utilizing these three rules, we arrive at the equivalent diagram of Fig. 4, namely Fig. 6a.

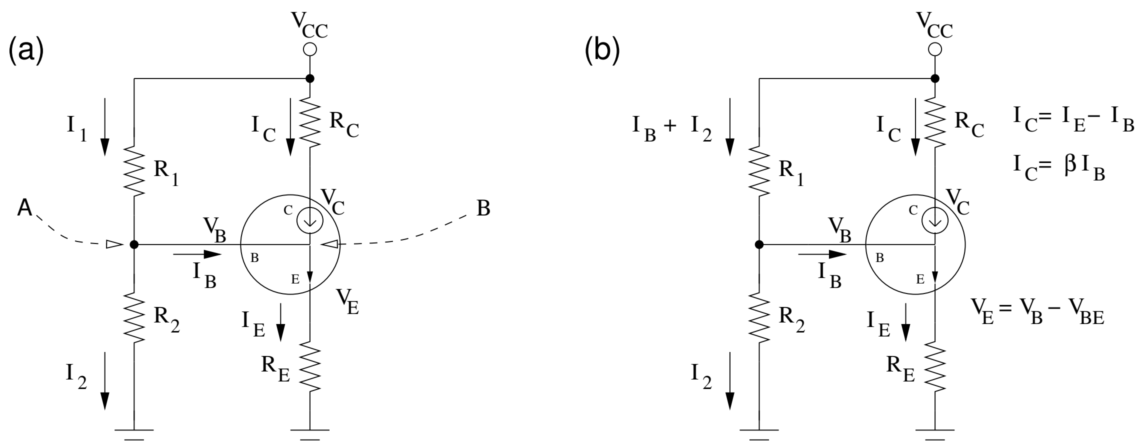

Figure 6: (a) DC equivalent of Fig. 4 (b) Using Kirchoff's law for currents

4.2 Calculating DC-voltages

To begin the DC analysis we first draw current arrows, as shown in Fig. 6a, and then reduce the number of currents that we have to deal with by using Kirchoff's law for currents at the two points labeled A and B.

$$\begin{aligned} &\text{At point } A: \qquad I_1 = I_2 + I_B \\ &\text{At point } B: \qquad I_C = I_E - I_B \end{aligned} \tag{6}$$

As a result we obtain Fig. 6b. The voltages are easily expressed in terms of the unknown currents

$$\begin{aligned} V_B &= I_2 R_2 &(7)\\ V_C &= V_{CC} - I_C R_C = V_{CC} - I_E R_C + I_B R_C &(8)\\ V_E &= V_B - V_{BE} = I_E R_E &(9) \end{aligned}$$

Kirchoff's laws can now be used to arrive at a system of three coupled equations for the three unknown currents I_2, I_B and I_E. One possibility is

$$\begin{aligned} V_{CC} &= I_2 R_2 + (I_2 + I_B) R_1, &(10)\\ V_{CC} &= I_E R_E + V_{BE} + (I_2 + I_B) R_1, &(11)\\ I_B &= I_E/(\beta + 1). &(12) \end{aligned}$$

Using what you've learned in linear algebra you can solve this directly. However, we may significantly simplify our lives if we take advantage of the fact that β is large. Taking formally $\beta \to \infty$, Eq. (12) implies $I_B = 0$, which, in turn, implies $I_E = I_C$. With this approximation Eqs. (10-12) become

$$\begin{aligned} V_{CC} &= I_2 (R_2 + R_1),\\ V_{CC} &= I_E R_E + V_{BE} + I_2 R_1. \end{aligned}$$

The solution is

$$\begin{aligned} I_2 &= \frac{V_{CC}}{R_2 + R_1}, &(13)\\ I_E &= \frac{1}{R_E}\left(V_{CC} - \left(\frac{V_{CC}}{R_2 + R_1}\right) R_1 - V_{BE}\right) = \frac{1}{R_E}\left(\frac{R_2}{R_2 + R_1} V_{CC} - V_{BE}\right). &(14) \end{aligned}$$

The voltages can now be calculated. Using Eq. (7) we get

$$V_B = I_2 R_2 = \frac{R_2}{R_2 + R_1} V_{CC} \tag{15}$$

and using Eq. (8) together with our approximation $I_B = 0$ yields

$$V_C = V_{CC} - \frac{R_C}{R_E}\left(\frac{R_2}{R_2 + R_1} V_{CC} - V_{BE}\right) = \left(1 - \frac{R_C}{R_E}\frac{R_2}{R_2 + R_1}\right) V_{CC} - \frac{R_C}{R_E} V_{BE}. \tag{16}$$

You may wonder whether our approximation is reasonable. That is, what error do we make by setting $\beta = \infty$? It turns out that the error is typically well below 10% and therefore acceptable in view of the fact that all resistors have a 5% tolerance. To convince you of this, I will evaluate the voltages and currents for a particular circuit based on our $\beta = \infty$ approximation below and you may compare these results to the values that one obtains using $\beta = 100$. I put this second calculation in the appendix.

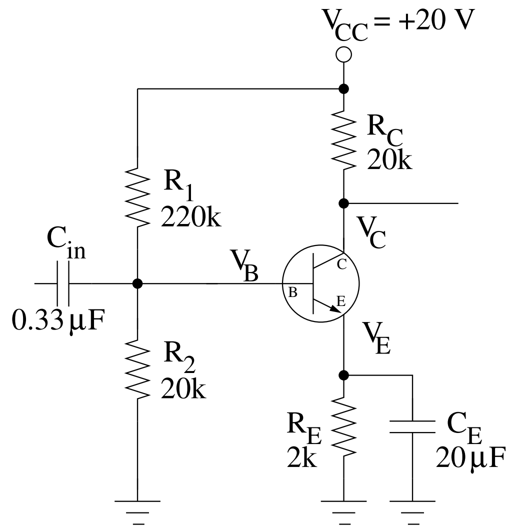

Figure 7: Common emitter amplifier

4.3 Example

We want to obtain values for I_2, I_E, I_C, V_C, and V_B, given the values for resistances and voltages as in Fig. 7.

From Eq. (13), which is reproduced here;

$$I_2 = \frac{V_{CC}}{R_2 + R_1} = \frac{20\,V}{20 \times 10^3\,\Omega + 220 \times 10^3\,\Omega} = 83\,\mu\text{A}.$$

Next obtain the value for I_E from Eq. (14) :

$$\begin{aligned} I_E &= = \frac{1}{R_E}\left(\frac{R_2}{R_2 + R_1} V_{CC} - V_{BE}\right) \\ &= \frac{\left(\frac{20\times10^3\,\Omega}{20\times10^3\,\Omega + 220\times10^3\,\Omega}\right)(20\,V) - 0.6V}{2 \times 10^3\,\Omega} \\ &= 0.53\,\text{mA}. \end{aligned} \tag{17}$$

Proceeding with the program, the value of V_C is determined by using either Eq. (8) or Eq. (16) and making the approximation that $I_C = I_E$. From Eq. (8) follows

$$V_C = V_{CC} - I_E R_C = (20\,V) - (0.53 \times 10^{-3}\,A) \times (20 \times 10^3\,\Omega) = 9.3\,\text{V}. \tag{18}$$

Finally, V_B is obtained from Eq. (15):

$$V_B = V_{CC}\frac{R_2}{R_1 + R_2} = (20\,V)\,\frac{(20 \times 10^3\,\Omega)}{(220 \times 10^3\,\Omega) + (20 \times 10^3\,\Omega)} = 1.67\,V. \tag{19}$$

It is seen that this circuit is pretty well designed because $V_C > V_B > V_E$ and the DC-level of the output voltage $V_{out}^{DC} = V_C$ is roughly centered. Centering is important to avoid AC-voltage clipping, meaning that, when the AC input is turned on, we will be able to tolerate an output amplitude of roughly 7.7 V without going below V_B or above V_{CC}.

4.4 Temperature stability

Although it is hard to put exact numbers on it, here is why the emitter resistor helps with temperature stability (see Fig. 8).

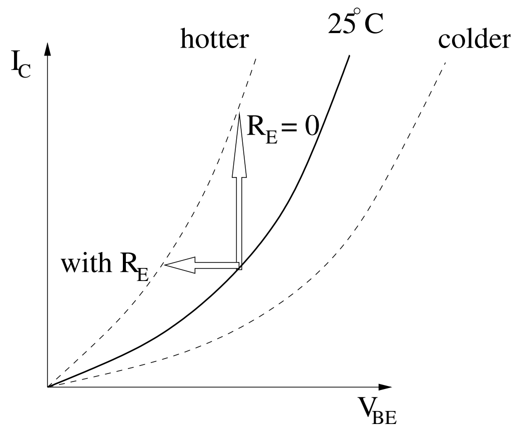

Figure 8: Effect of temperature rise for grounded emitter amplifier and amplifier with emitter resistor

The grounded emitter amplifier ($R_E = 0$) is unstable because in this case the emitter is connected to ground, $V_E = 0$ is fixed, and V_B is also fixed due to the presence of the input stage. Therefore, V_{BE} is roughly constant but, because of the temperature rise, the same voltage V_{BE} now leads to a much larger current I_C. This temperature induced increase of I_C can easily lower the collector voltage V_C to a level close to V_B, making the transistor useless (if $V_C < V_B$ the transistor is not forward biased anymore and won't work).

The common emitter amplifier with $R_E \gg r_e$ remedies this problem because now we establish negative feedback. Again V_B is roughly fixed but now V_E is not.

- T rises and I_C begins to grow.
- The emitter voltage grows due to Ohm's law ($V_E = R_E I_E = R_E I_C$).
- V_{BE} decreases, therefore turning-off/decreasing I_C (Ebers-Moll equation).

In the idealized case this negative feedback leads to a fixed I_C and therefore stable bias voltages.

5 AC voltages: What is the gain?

The objective of this section is to obtain expressions and values for the voltage gain for two cases, the common emitter amplifier with an emitter resistor that is bypassed with a capacitor, C_E, and the common emitter amplifier with an emitter resistor that is not bypassed (without the capacitor). To arrive at the equivalent AC-voltage diagram, we use the following rules and approximations

1. $\beta_{ac} \to \infty \Rightarrow i_C = i_E, \text{ since } i_B = 0$
2. Capacitance impedances goe to zero (*i. e.* $\omega \to \infty \Rightarrow Z_C = 0$), *i.e.* a capacitor acts like a wire.
3. A DC source also becomes a short in an ac equivalent circuit.
4. Use the improved transistor model, *i.e.* the equivalent diagram Fig.3b.

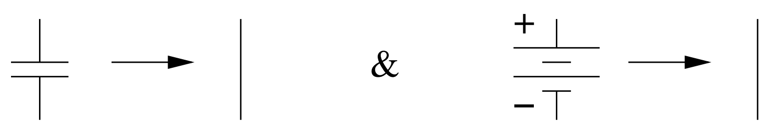

Figure 9: AC analysis equivalent diagrams.

5.1 AC gain WITH capacitor

If we use these rules on the circuit diagram for our common-emitter amplifier with bypassed emitter resistor (Fig. 4b) we obtain what is shown in Fig. 10b

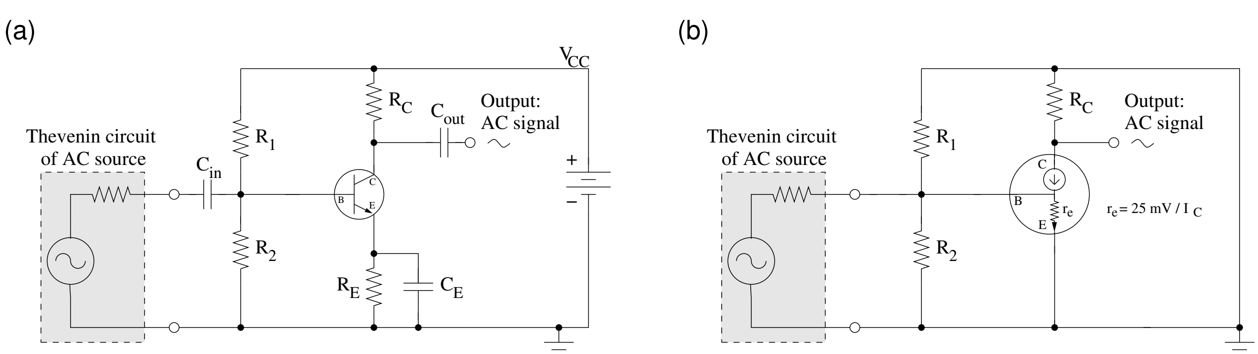

Figure 10: (a) Reproduction of Fig. 4b (b) Equivalent diagram for AC analysis. Only the capacitor in parallel with the emitter resistor is considered.

Note, that where in a DC-circuit we have the supply voltage V_{CC}, in the AC-equivalent circuit we have ground. More importantly, it is seen that the circuit, from the perspective of AC-voltages, looks like a grounded common-emitter amplifier. This is because the bypass capacitor C_E acts like a wire at high frequencies, essentially no current will flow through the emitter resistor R_E, and we can remove R_E from the diagram. At this stage it should become clear why we need to consider the improved transistor model to analyze this circuit. Recall, that the simple model makes the absurd prediction that a grounded common-emitter amplifier has infinite AC-voltage gain.

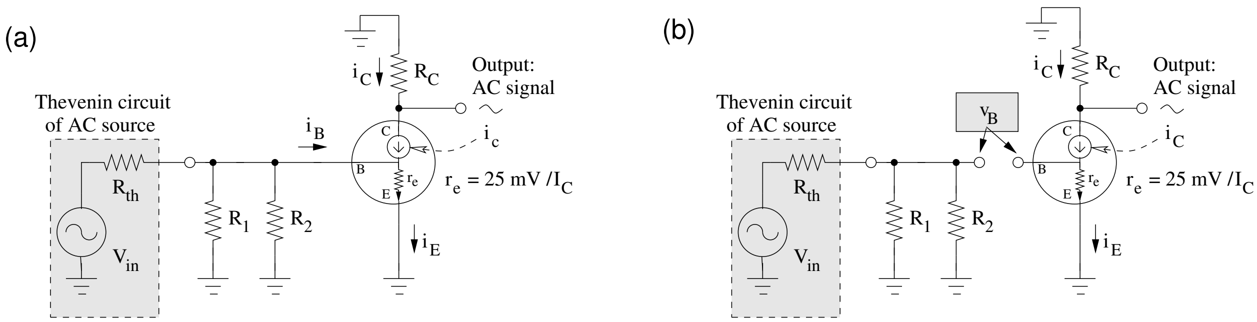

Figure 11: (a) A different rendering of Fig. 10b (b) Separation of input stage and transistor stage $i_B = 0$.

To further simplify the analysis we will take advantage of the fact that the base current i_B is tiny compared to the collector and emitter current and simply set $i_B = 0$ (rule 1). As a consequence of this approximation the input stage can be disconnected from the transistor stage as is shown in Fig. 11, where, in Fig. 11b, the input stage is shown to the left and the stage containing the transistor to the right. How should we interpret this step of disconnecting the two parts of the circuits? All we are saying here is that we claim that the transistor stage will not put any load on the input stage. In other words, there is no appreciable flow of *current* from one stage to the other and, consequently, the input stage is not at all influenced by what comes after it. However, the *voltage* $v_B(t)$ is the same on both sides of the gap that we created in the equivalent circuit schematic. In essence, in this approximation, we can analyze the input stage and the stage containing the transistor separately. Note, however, that for this to be a good approximation the output impedance of the input stage has to be small compared to the input impedance of the transistor stage (otherwise the $i_B = 0$ assumption breaks down). One should always check that this condition is true (see Section 6)!

We will for now postpone this calculation and assume that R_1 and R_2 are chosen correctly. Our main aim is to calculate the voltage gain, which is defined as

$$g_V = \frac{|v_{out}|}{|v_{in}|} \tag{20}$$

For correctly chosen resistors R_1 and R_2 the AC voltage on the transistor's base is essentially equal to the input-signal voltage, $v_B = v_{in}$ (for details see appendix). To calculate v_{out} we first note that $v_{out} = v_C$ and that

$$0 - v_C = i_C R_C \qquad \Rightarrow \qquad v_{out} = -i_C R_C. \tag{21}$$

By assumption $i_B = 0$ and thus $i_C = i_E$. To calculate i_E, note that $v_E = 0$ for the case of the bypassed emitter and therefore

$$i_E = \frac{v_B}{r_e}. \tag{22}$$

As a result, we get for the output voltage

$$v_{out} = -i_C R_C = -i_E R_C = -\frac{R_C}{r_e} v_{in} \tag{23}$$

and therefore the ratio

$$\frac{v_{out}}{v_{in}} = -\frac{R_C}{r_e}, \tag{24}$$

where the minus sign indicates that the signal is inverted. Thus, the voltage gain is

$$g_V = \frac{R_C}{r_e} = \frac{I_C^{total}(\text{in mA})\ R_C}{25\,\Omega} \tag{25}$$

where $r_e = 25$ mV$/I_C^{total}$ is used for the second equality.

The gain does not depend on R_E at all, which is the whole point of bypassing the emitter resistor with a capacitor. The gain only depends on R_C and the dynamic emitter resistance r_e. Due to the r_e dependence, the gain varies as the collector current changes. In other words, the gain and the amplitude of the output voltage are not independent!

Example: $R_C = 20$ kΩ (see Fig. 7) and $I_C^{total} = .5$ mA (quiescent current).

$$g_V = \frac{(0.5)\,(20 \times 10^3\,\Omega)}{(25\,\Omega)} = 400. \tag{26}$$

5.2 AC gain WITHOUT capacitor

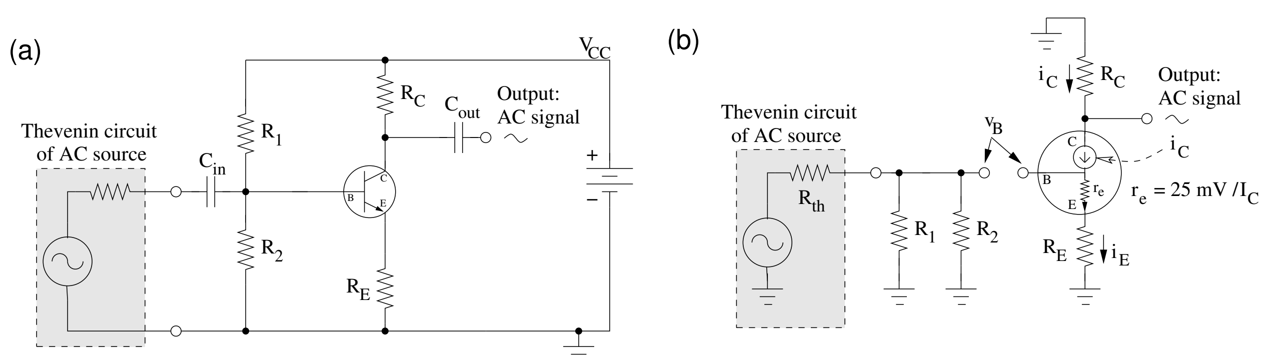

Figure 12: Without bypass capacitor: (a) Circuit diagram (b) AC-equivalent circuit

The AC-analysis of the common emitter amplifier without a bypass capacitor is entirely similar to the derivation of the gain when the capacitor is included. The main difference is that now the emitter resistor is not bypassed and therefore R_E is in series with r_e. Thus

$$g_V = \frac{R_C}{r_e + R_E} \tag{27}$$

Example: $R_C = 20$ kΩ, $R_E = 2$ kΩ (see Fig. 7) and $I_C^{total} = .5$ mA

$$g_V = \frac{(20 \times 10^3\,\Omega)}{(25\,\Omega/0.5) + (2 \times 10^3\,\Omega)} = 9.75. \tag{28}$$

6 Impedances and Thevenin's theorem

To calculate an *impedance* at a point, apply ΔV, find ΔI, and take the quotient. The problem with calculating the input or output impedance of a sub-circuit is that connected circuits might influence the apparent impedance. To get around this we have to divide up the overall circuit into sub-circuits for which the condition $Z_{out}^{previous\,stage} \ll Z_{in}^{next\,stage}$ holds (at least in principle). Then, for a sub-circuit, we make the following assumptions:

Input impedance - ideal voltage source and infinite load impedance (nothing is attached to the output);

Thevenin voltage - is the output voltage when nothing is attached to the output;

Output impedance - the output is shorted to ground and $Z_{th} = V_{th}/I_{short}$.

Example 1: To calculate the input impedance Z_{in} of a voltage divider (Fig. 13a), note that ΔV results in a current change $\Delta I = \Delta V/(Z_1 + Z_2)$. Therefore $Z_{in} = Z_1 + Z_2$.

Example 2: To calculate the Thevenin equivalent voltage of a voltage divider (Fig. 13b), assume that nothing is attached to the output and that the voltage source producing V_{in} is ideal (0 output impedance of the source). Then, a simple use of the voltage divider formula gives $V_{th} = Z_2/(Z_1 + Z_2)\,V_{in}$. To calculate the output impedance (or Thevenin equivalent impedance), Z_{th}, assume that the output is shorted to ground. Then the current $I_{short} = V_{in}/Z_1$ and Z_{th} is

$$Z_{th} = \frac{V_{th}}{I_{short}} = \frac{Z_2/(Z_1 + Z_2)\,V_{in}}{V_{in}/Z_1} = \frac{Z_1\,Z_2}{Z_1 + Z_2} = Z_1||Z_2.$$

So for the R_1, R_2 voltage divider in Fig. 7 the Thevenin voltage is $V_{th} = 20/(20+220) \cdot 20V = 1.67V$, which is of course what you would expect [see Eq.(15) and Eq.(19)]. The output impedance is $Z_{out} = R_{out} = (20 \cdot 220)/(20 + 220)$ k$\Omega = 18.3$ kΩ.

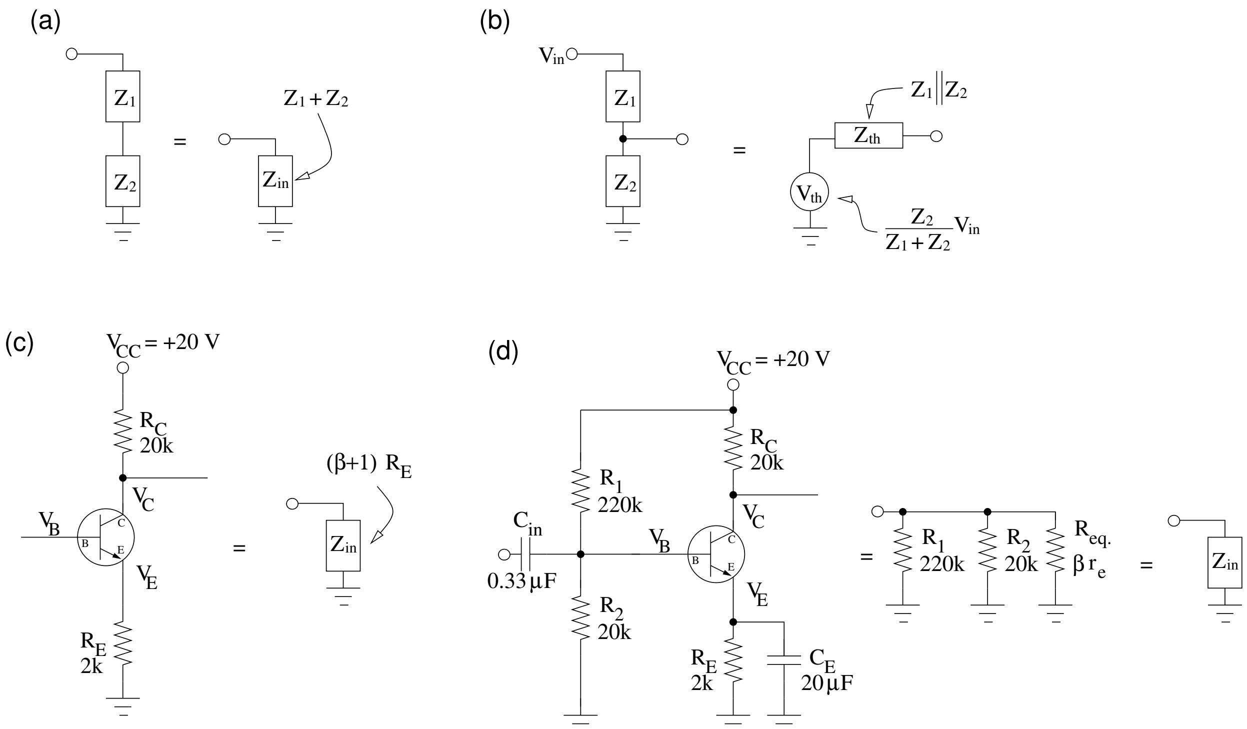

Figure 13: (a) Voltage divider input impedance (b) Voltage divider Output: Thevenin equivalent (c) Transistor-stage DC-input impedance of the common emitter amplifier (d) AC - input impedance of the grounded common-emitter amplifier

Example 3: We want to know (roughly) the *DC-input impedance of the transistor-stage* of our common-emitter amplifier with emitter resistor. In other words, we consider the circuit shown in Fig. 7 *without* the two resistors R_1 and R_2. This is shown in Fig. 13c. For this circuit the simple transistor model is sufficient. We will again assume an infinite load impedance.

A change in the DC-input voltage ΔV_B will lead to a change $\Delta V_E = \Delta V_B$, this in turn, will result in a changed emitter current $\Delta I_E = \Delta V_E / R_E$. Then, with $\beta I_B = I_C$ and $I_E = I_B + I_C = (\beta + 1) I_B$, we obtain the input impedance

$$Z_{in} = \frac{\Delta V_B}{\Delta I_B} = \frac{\Delta V_B}{\Delta I_E/(\beta+1)} = (\beta+1) R_E \frac{\Delta V_B}{\Delta V_E} = (\beta+1) R_E \approx 200 k\Omega \qquad (\text{for } \beta = 100).$$

Comparing the DC-voltage output impedance of the voltage divider (18.3 kΩ) to the input impedance of the common emitter amplifier, we see that $Z_{in} \sim 10 \cdot Z_{out}$. A factor of 10 is good enough for a circuit design where we are happy with a 10% accuracy. Recall, we want $Z_{in} \gg Z_{out}$ (Z_{out} of previous stage) to make sub-circuits independent of each other.

Example 4: The *output impedance of the AC-input stage* in Fig. 11b. Assuming $R_{th} \ll R_1, R_1 \approx R_2$, one obtains $V_{th} \approx V_{in}$ and $Z_{th} \approx R_{th}$. I'll leave the details as an exercise.

Example 5: The *AC - input impedance of the transistor stage* in Fig. 11b (AC-voltages). In this case we have a grounded common-emitter amplifier. For this case we need the improved model. Since $v_E = 0$ always (the emitter pin is grounded), $v_B = v_{BE}$ holds. From Eqs. (2)-(4) we have that $r_e\, i_E = v_{BE} = v_B$. With $i_E = (\beta+1) i_B$ we obtain

$$\begin{aligned} Z_{in} &= \frac{v_B}{i_B} = \frac{v_B}{i_E/(\beta+1)} = (\beta+1)\, r_e \approx \beta\, r_e, \\ Z_{in} &= \beta \frac{25\,\Omega}{I_C(\text{in mA})} \approx \frac{25\,\text{k}\Omega}{I_C(\text{in mA})} \qquad (\text{for } \beta = 100). \end{aligned}$$

For our example circuit operating under quiescent conditions (no AC-input signal, or, if you want, zero amplitude AC-input signal) we calculated that $I_C \sim 0.5$ mA [see Eq. (17)], yielding an input impedance

$$Z_{in} = R_{in} = 50\text{k}\Omega.$$

Based on Example 4 and Example 5, we can see that the output impedance of the input stage $Z_{th} \sim R_{th}$ is small compared to the input impedance of the transistor stage, $Z_{in} = 50\,\text{k}\Omega$, for reasonable values of R_{th}. For example, the function generator has $R_{th} = 50\,\Omega$, which means that the output impedance of the input stage is by a factor of 1000 smaller than Z_{in}. In this case we clearly can treat the two stages separately, as we have done in our derivation of the AC-gain.

Example 6: The *AC - input impedance of the common-emitter amplifier*. In this case we have to consider both the transistor stage and the two biasing resistors R_1 and R_2. As is shown in Fig. 13d, we can view this circuit as a parallel combination of the resistors R_1 and R_2 and the equivalent input resistance that we have calculated in the previous example, $R_{eq} = R_{in} = 50\text{k}\Omega$.

$$Z_{in}^{-1} = R_1^{-1} + R_2^{-1} + R_{eq}^{-1} = (220\text{k}\Omega)^{-1} + (20\text{k}\Omega)^{-1} + (50\text{k}\Omega)^{-1} \quad \rightarrow \quad Z_{in} = 13\text{k}\Omega$$

A Appendix

A.1 DC analysis based on the simple transistor model with β=100

β is finite ($e.\ g.\ \beta = 100$)

Remember, as before, we first have to solve for I_E, I_B, and I_2 and then use the obtained values for the currents to compute the desired voltages V_B and V_C. Let us reproduce here Eqs. (10-12) but written in matrix form:

$$\begin{pmatrix} V_{CC} \\ V_{CC} - V_{BE} \\ 0 \end{pmatrix} = \begin{pmatrix} R_1 + R_2 & R_1 & 0 \\ R_1 & R_1 & R_E \\ 0 & \beta + 1 & -1 \end{pmatrix} \begin{pmatrix} I_2 \\ I_B \\ I_E \end{pmatrix} \tag{29}$$

Formally the solution is then given by

$$\begin{pmatrix} I_2 \\ I_B \\ I_E \end{pmatrix} = \begin{pmatrix} R_1 + R_2 & R_1 & 0 \\ R_1 & R_1 & R_E \\ 0 & \beta + 1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} V_{CC} \\ V_{CC} - V_{BE} \\ 0 \end{pmatrix} \tag{30}$$

$$= \begin{pmatrix} 240k\Omega & 220k\Omega & 0 \\ 220k\Omega & 220k\Omega & 2k\Omega \\ 0 & 101 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 20\,V \\ 19.4\,V \\ 0 \end{pmatrix} \tag{31}$$

The real work is of course to actually invert the matrix, which you may do using your in-depth knowledge of linear algebra or, alternatively, using your favorite mathematical computer software. Using Matlab, I obtain:

$$I_2 = 79\,\mu\text{A}, \tag{32}$$
$$I_B = 4.84\,\mu\text{A}, \tag{33}$$
$$I_E = 0.489\,\text{mA}. \tag{34}$$

Eq. (7), $V_B = I_2 R_2$, provides;

$$V_B = 79 \times 10^{-6}\,A \times 20 \times 10^3\,\Omega = 1.58\text{V}.$$

Eq. (8) gives

$$V_C = V_{CC} - (I_E - I_B)\,R_C = 20\,V - (0.489 \times 10^{-3}\,A - 4.84 \times 10^{-6} A)(20 \times 10^3\,\Omega) = 10.3\,\text{V}.$$

To conclude, let us calculate the percentage differences of V_C and V_B in the two approximations, where one approximation assumes $\beta \to \infty$ and the other a more realistic value of $\beta = 100$. Both approximations are based on the simple transistor model, where the transistor is understood as a current amplifier.

$$\text{Relative difference in } V_C = \frac{10.3\,\text{V} - 9.3\,\text{V}}{10.3\,\text{V}} = 8.7\%.$$
$$\text{Relative difference in } V_B = \frac{1.58\,\text{V} - 1.67\,\text{V}}{1.58\,\text{V}} = 5.7\%$$