

- I will not be at Reed this Thursday (12/4)
- There will be a lecture next week (12/8)
- LV Assginment 4 is due on Friday
- Final Project: Wed. 12/10 [Friday 12/19]
- I am there to help with the TE device/ PID VI

10	11/3 - 11/7	(VIII) MathScript, Data Acquisition (LV2)	E: Ch. 3 - 4	LV Assignment 1 due*	
11	11/10 - 11/14	No Lecture	E: Ch. 5 - 8	LV Assignment 2 due*	
12	11/17 - 11/21	(IX) Data Files; Shift Reg.; Case & Sequence Structure (LV3)	E: Ch. 9 - 10		
13	11/24 - 11/26	(X) Curve Fitting and FFT; Digital Thermometer (LV4)		Wed.: LV Assignment 3 due [#] ; Makeup Lab	Thu. Thanksgiving
14	12/1 - 12/5	(XI) PID Temperature Control (LV5)	E: Ch. 12 E: Appendix I	LV Assignment 4 due [#]	
15	12/8 - 12/10	Last Lecture		LV Assignment 5 due ^{\$} Lab open during reading/final week	
				regular time	Thu. lab meets on Tuesday

* The **reports** are due 11:00 (11 AM) on the **day of your lab**.

The **LV assignment** write-ups are due on **Friday** at 11 AM (assignment 3 & 5 are exceptions).

\$ The **Final Project (LV 5)** is has to be turned in by **12/19**, 12/10 is recommended.

- The lab tickets are due at the beginning of your lab, i.e. 13:10.

- The next week's assignment will be given out in lab lecture the week of the lab.

S: R. E. Simpson

E: John Essick

II) This week's assignment

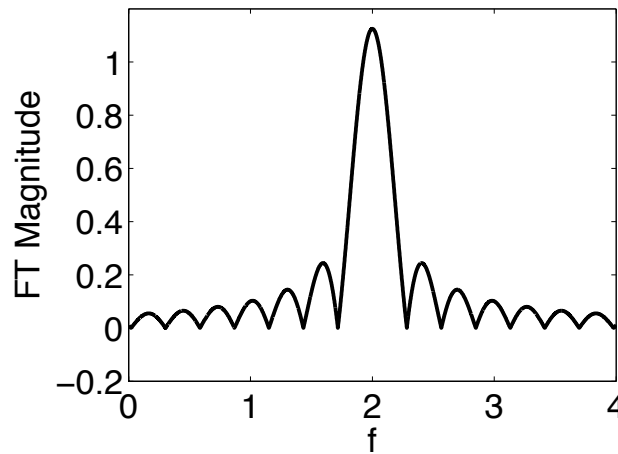
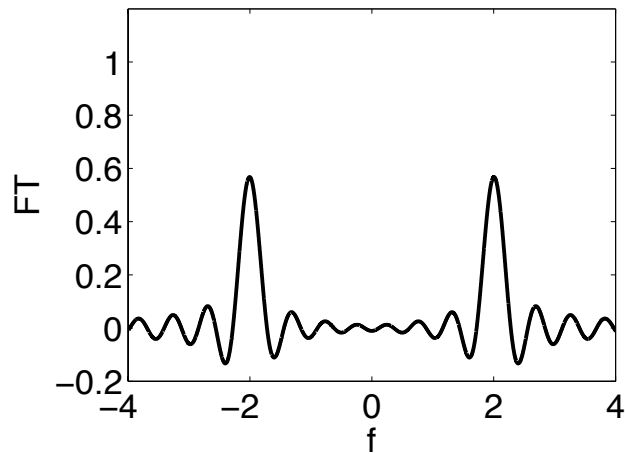
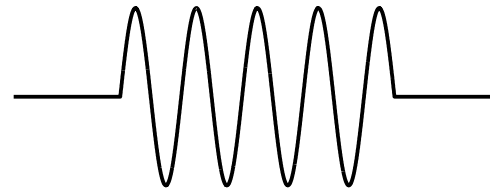
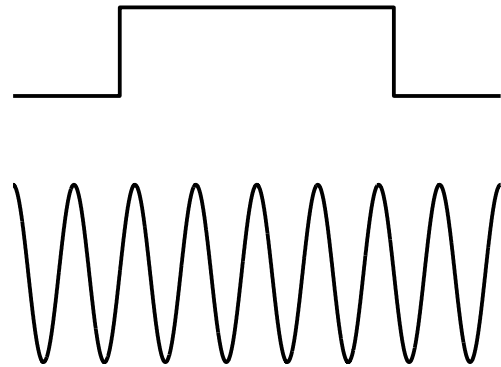
FFT

Curve Fitting

Temperature Measurement using a Thermistor

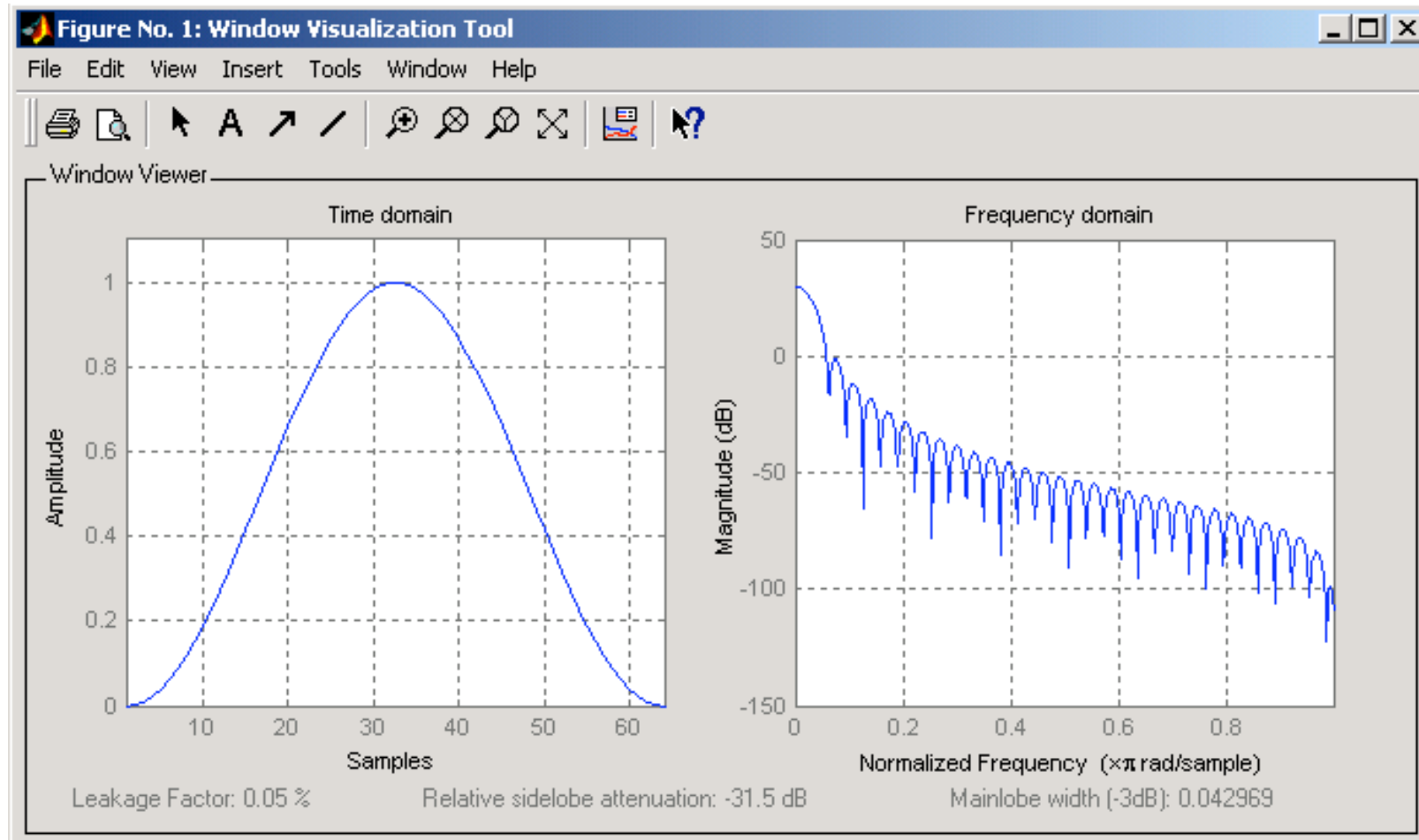
Fourier Analysis - Part 3

Reality I: Finite Time Series \Rightarrow Spectral Leakage



High Resolution
Low Dynamic Range

Homework - Hann Window:



$$\text{FT} \quad \hat{x}(f) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} x(t) e^{-i 2\pi f t} dt$$

Reality II: Discrete Data \Rightarrow Periodic Spectrum

$$\text{Sampling Rate: } f_s = 1/\Delta t \quad t_j = j\Delta t \quad j = 0, 1, 2, \dots, N - 1$$

Discrete Time Fourier Transform:

$$\text{DTFT} \quad \hat{X}(f) = \sum_{j=0}^{N-1} X(t_j) e^{-i 2\pi f t_j}$$

$$\text{Periodic Spectrum:} \quad \hat{X}(f + k f_s) = \hat{X}(f)$$

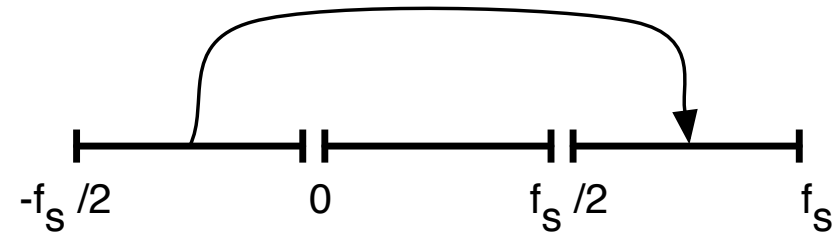
$$e^{-i 2\pi (f + k f_s) t_j} = e^{-i (2\pi f t_j + 2\pi k f_s j / f_s)} = e^{-i 2\pi f t_j} e^{-i 2\pi k j} = e^{-i 2\pi f t_j}$$

Discrete Fourier Transform (Fast Fourier Transform)

DFT (FFT)

$$\hat{X}(f_k) = \sum_{j=0}^{N-1} X(t_j) e^{-i 2\pi f_k t_j} \quad k = 0, \dots, N - 1$$

$$f_k = k \Delta f = k \frac{1}{N\Delta t}$$



FFT

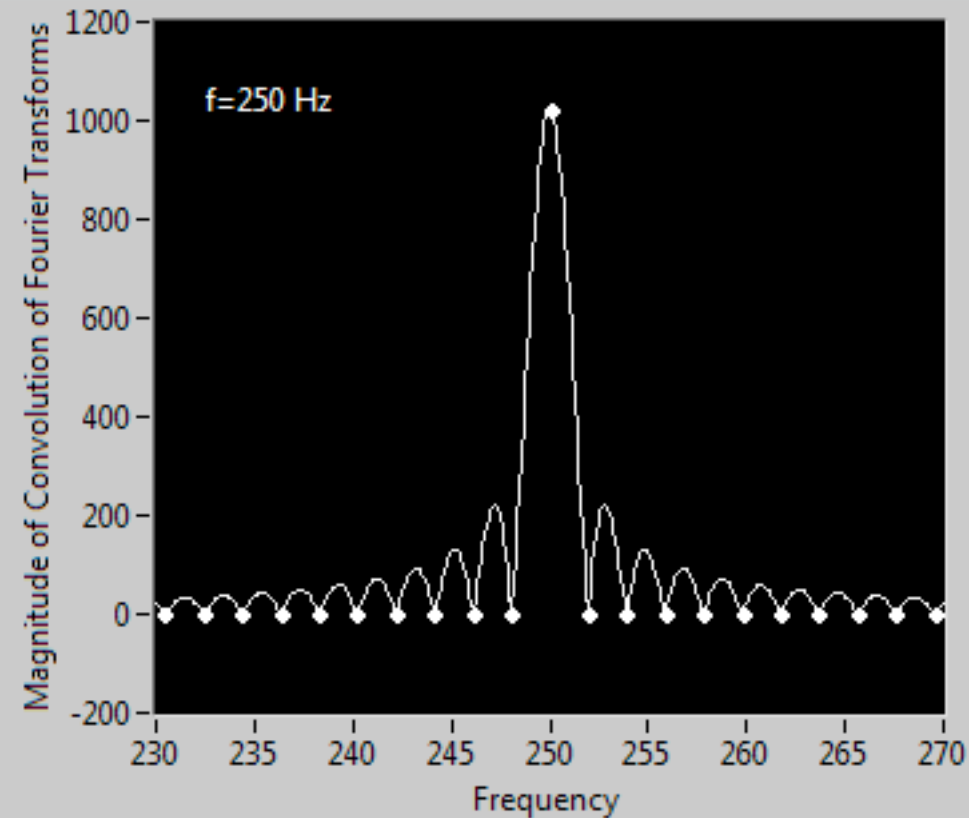
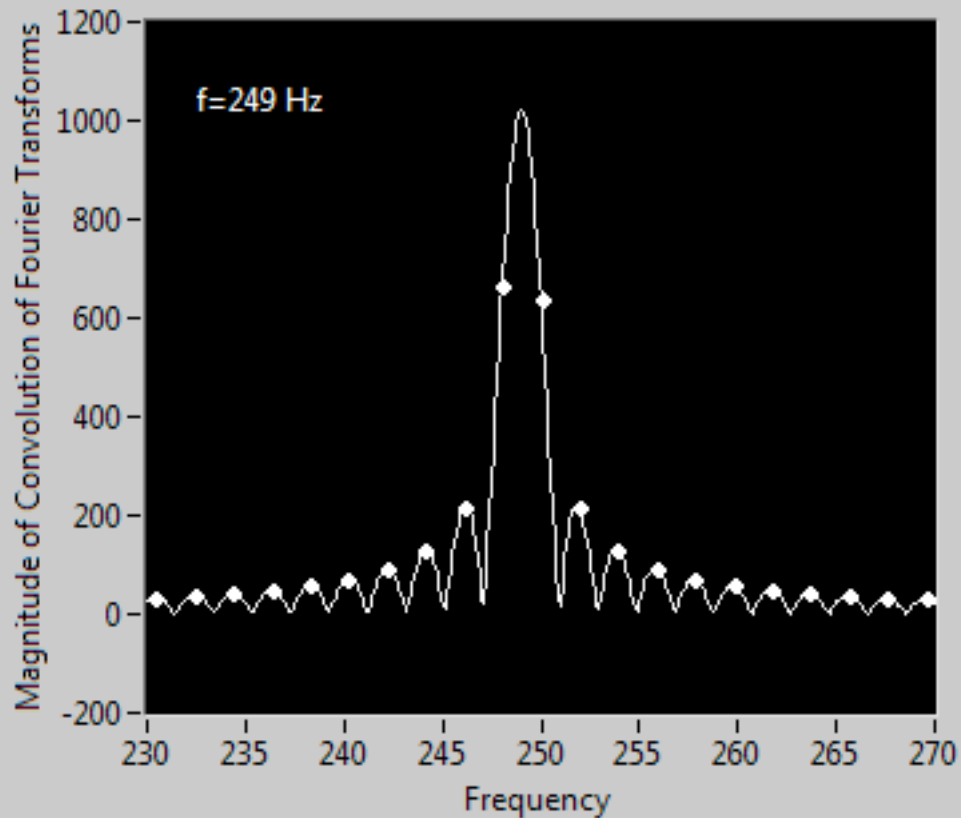
$N \log_2 N$ (vs. N^2) for $N = 2^m$ with m integer

one kilosample	2^{10}	\rightarrow	100 ×
one Megasample	2^{20}	\rightarrow	50000 ×
one hundred Megasample	2^{27}	\rightarrow	5 Million ×

Discrete Fourier Transform (Fast Fourier Transform)

Typically:
Leakage + Sampling

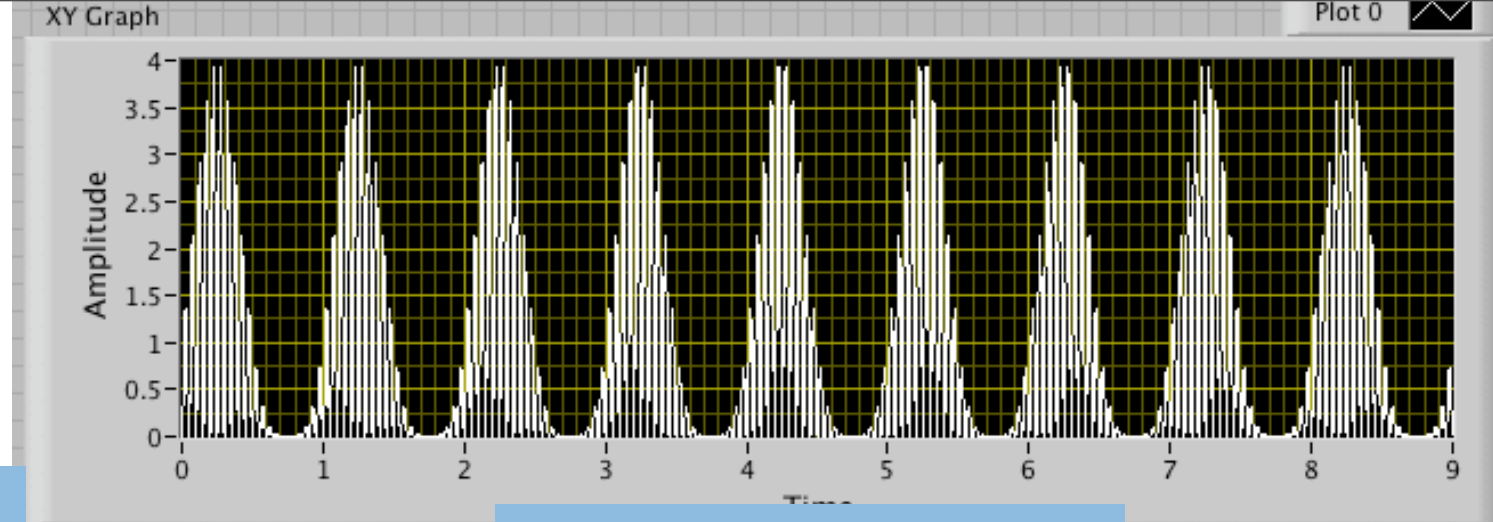
Exact for periodic functions with
period $T = N\Delta t$



Fourier Analysis

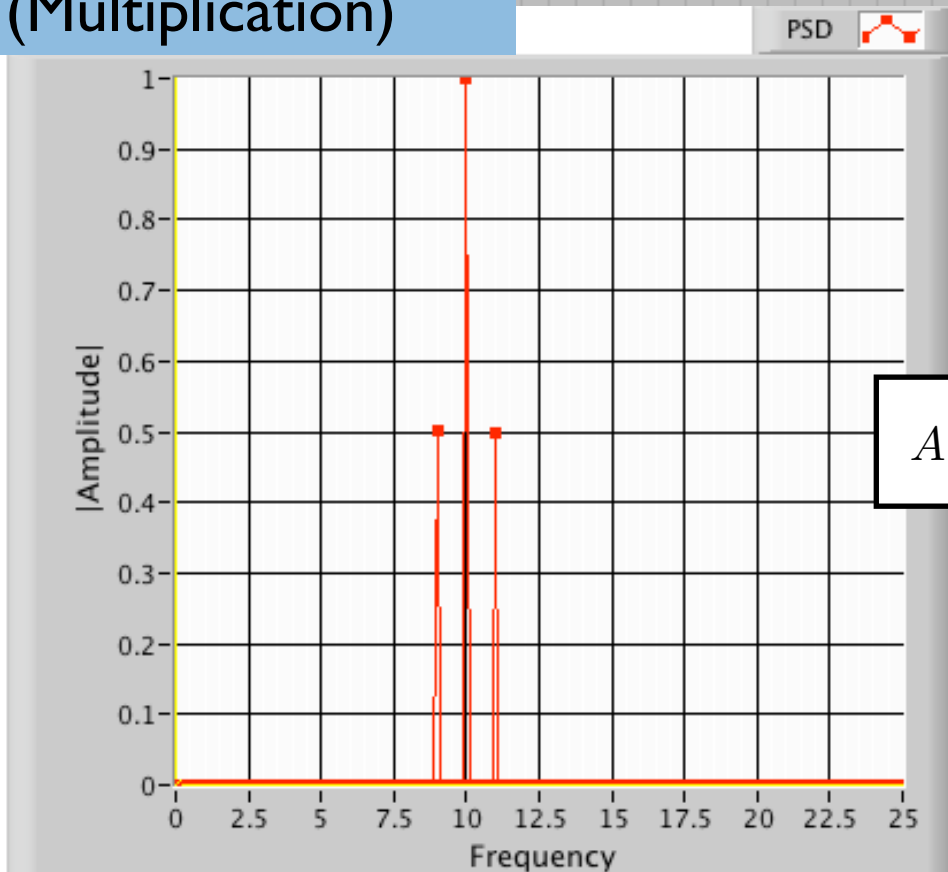
Time Domain	Transform Type	Frequency Domain
$x(t)$ continuous, non-periodic	Fourier Transform	$\hat{x}(f)$ continuous, non-periodic
$x(t)$ continuous, periodic	Fourier Series	$\hat{x}(f_k)$ discrete, non-periodic
$x(t_j)$ discrete, non-periodic	Discrete Time Fourier Transform	$\hat{x}(f)$ continuous, periodic
$x(t_j)$ discrete, periodic	Discrete Fourier Transform (Fast Fourier Transform)	$\hat{x}(f_k)$ discrete, pe- riodic

Homework:

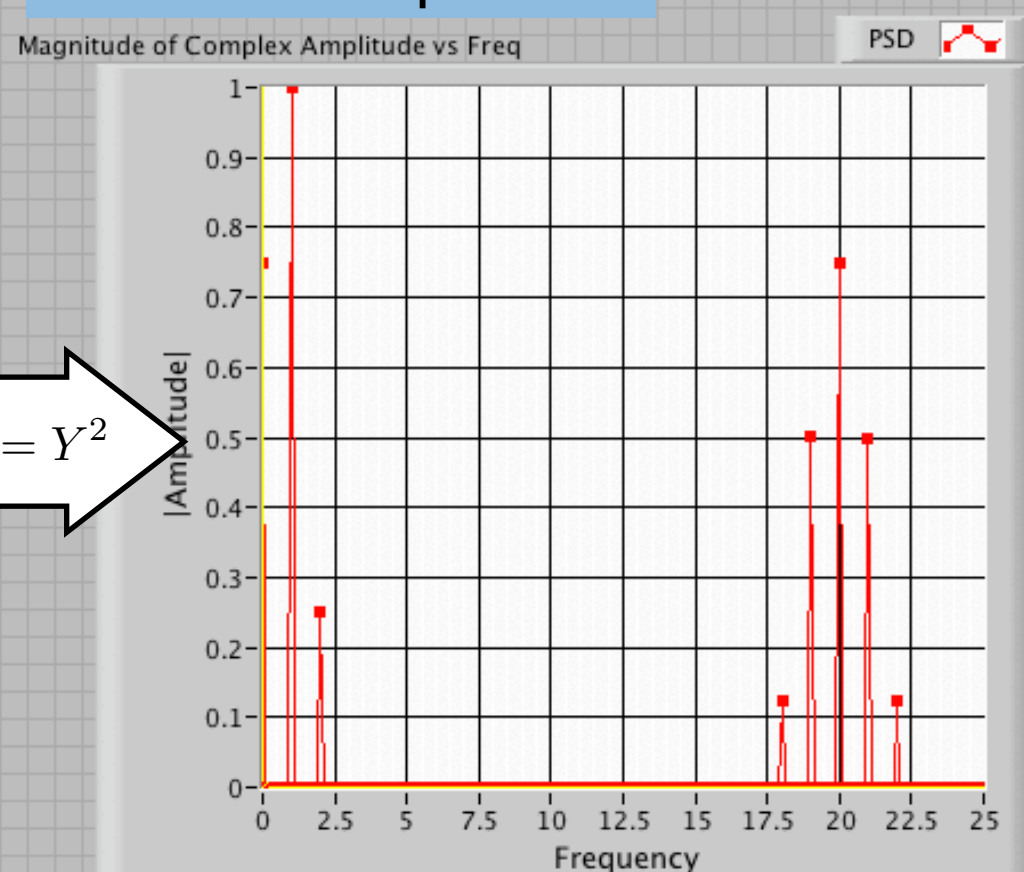


Amplitude Modulation
(Multiplication)

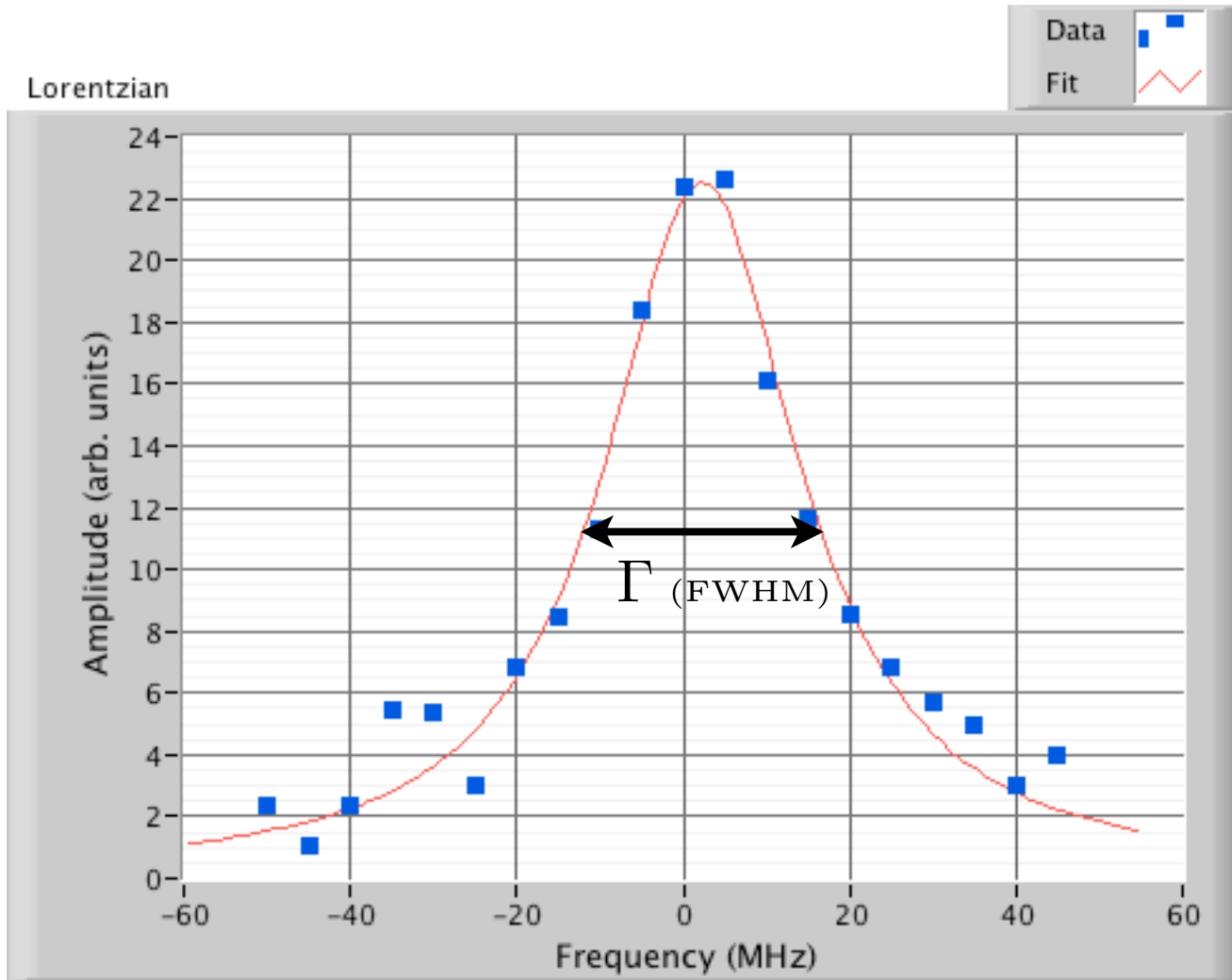
Another Multiplication



$A = Y^2$



Lorentzian: $L(f) = \frac{A}{\pi} \frac{\Gamma/2}{(f - f_0)^2 + (\Gamma/2)^2}$



$^{87}\text{Rb D}_2 (5^2\text{S}_{1/2} \rightarrow 5^2\text{P}_{3/2})$

$f_0 = 384.230484 \text{ THz}$

$\lambda = 780.2412 \text{ nm}$

$\tau = ?$

$$\mathcal{F}_{f,t}^{-1} \left[\frac{A}{\pi} \frac{\Gamma/2}{(f - f_0)^2 + (\Gamma/2)^2} \right] = A e^{-i(2\pi f_0 t) - \Gamma\pi|t|}$$

Thermistor - Physics

Thermal activation:

$$n = n_0 \exp \left[-\frac{\Delta E}{kT} \right]$$

-> resistance decreases with Temp.

Lorentz force:

$$\mathbf{F} = q (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Drude model: $m \frac{d\langle \mathbf{v} \rangle}{dt} = q\mathbf{E} - \frac{m}{\tau} \langle \mathbf{v} \rangle$

$$\frac{d\langle \mathbf{v} \rangle}{dt} = 0 \Rightarrow \langle \mathbf{v} \rangle = \frac{q\tau}{m} \mathbf{E}$$

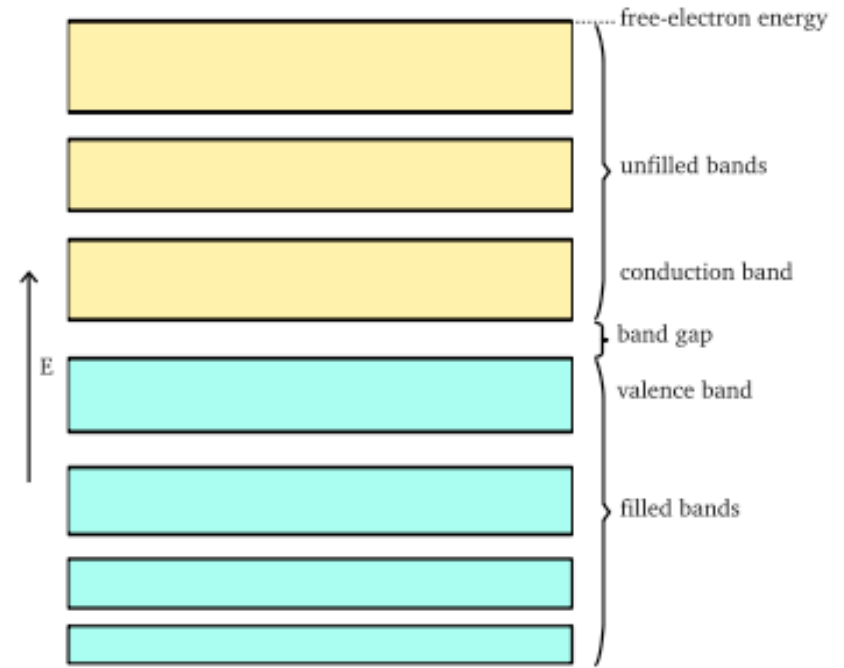
Current density: $\mathbf{j} = \sigma \mathbf{E} = nq\langle \mathbf{v} \rangle = \frac{nq^2\tau}{m} \mathbf{E}$

Resistivity: $\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$

$$\rho = \frac{m}{n_0 e^2 \tau} e^{\frac{\Delta E}{kT}} = \rho_0 e^{\frac{\Delta E}{kT}}$$

Resistance: $R = \rho \frac{L}{A} = R_0 e^{\frac{\Delta E}{kT}}$

Semiconductor Band Structure



Thermistor - Physics

$$R = \rho \frac{L}{A} = R_0 e^{\frac{\Delta E}{kT}}$$

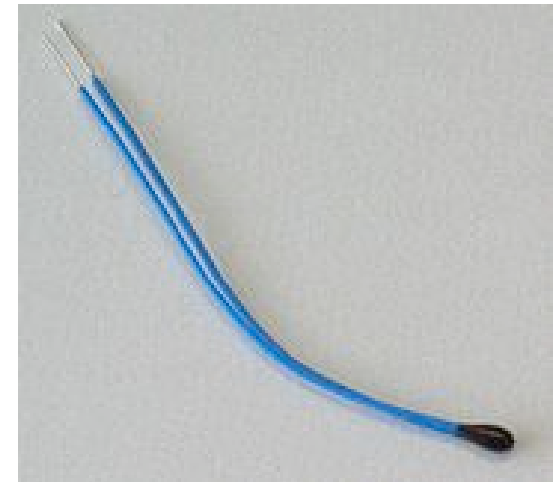
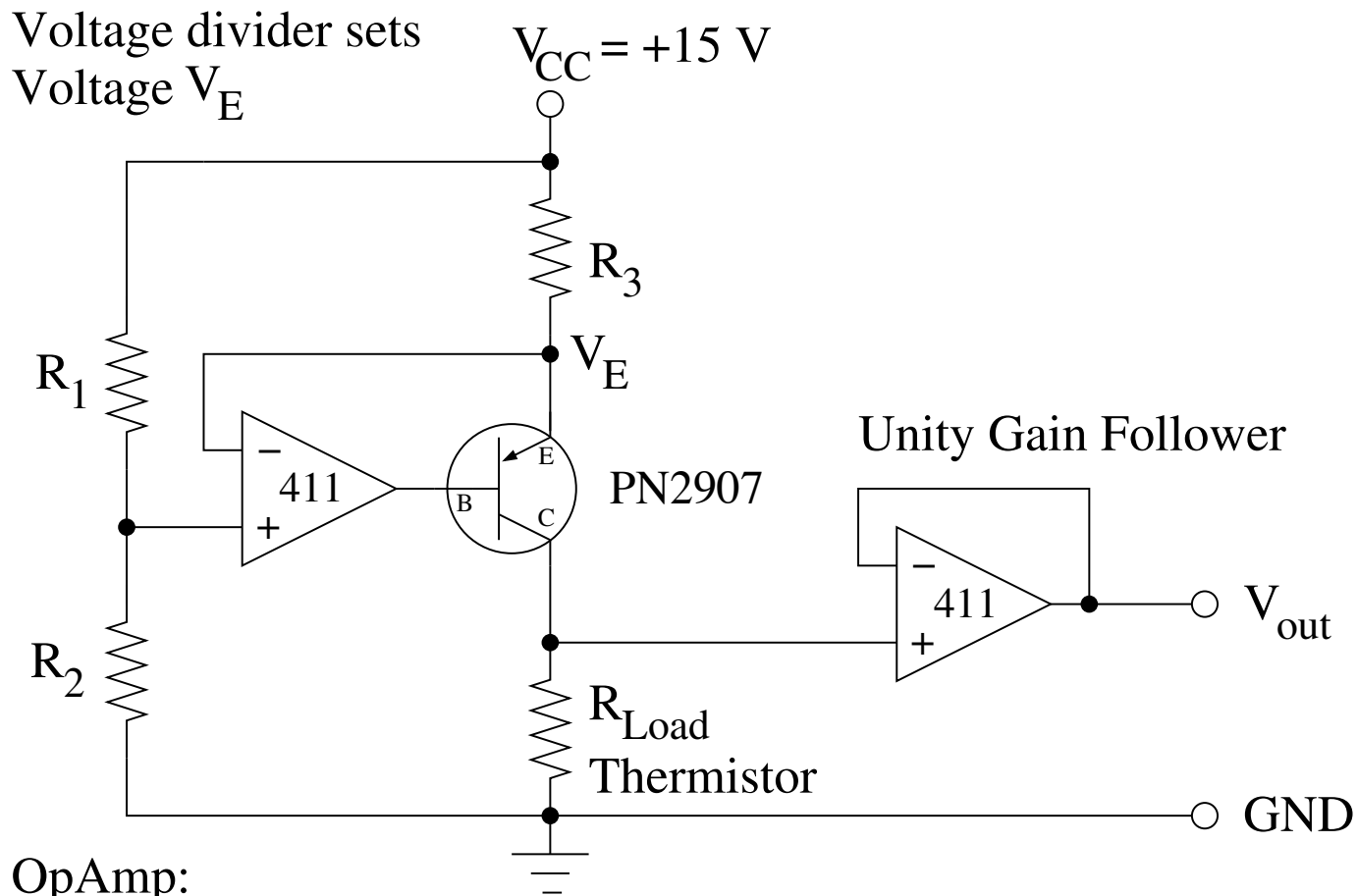
$$\ln(R) = \ln(R_0) + \frac{\Delta E}{kT}$$

Beta Formula:

$$\frac{1}{T} = A + B \ln(R) \quad A = \frac{-k}{\Delta E} \ln \left(\frac{mL}{An_0 e^{2\tau}} \right) \quad B = \frac{k}{\Delta E}$$

Steinhart - Hart Equation:

$$\frac{1}{T} = A + B \ln(R) + D \ln(R)^3$$



OpAmp:
 Decouples Voltage Divider
 Sets base voltage such that emitter voltage is constant

$$I = 0.1\text{ mA}, \text{ e.g. } R_1 = R_2 = 10\text{k}\Omega \quad R_3 = 75\text{k}\Omega$$

Report the measured current through the thermistor

Fit Thermistor Data to get coefficients
of Steinhart-Hart equation.

Read thermistor voltage N times

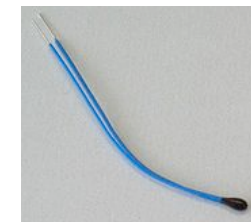
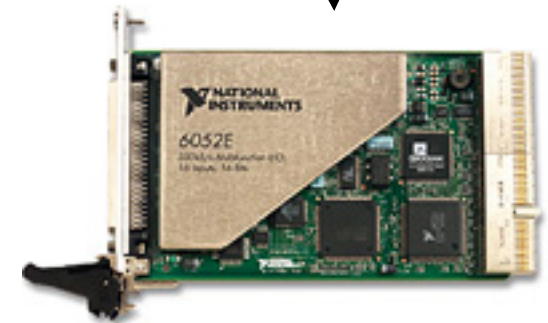
Find average voltage V

Calculate thermistor resistance $R = V/I$

Calculate temperature via Steinhart-
Hart Equation

Display temperature in $^{\circ}\text{C}$

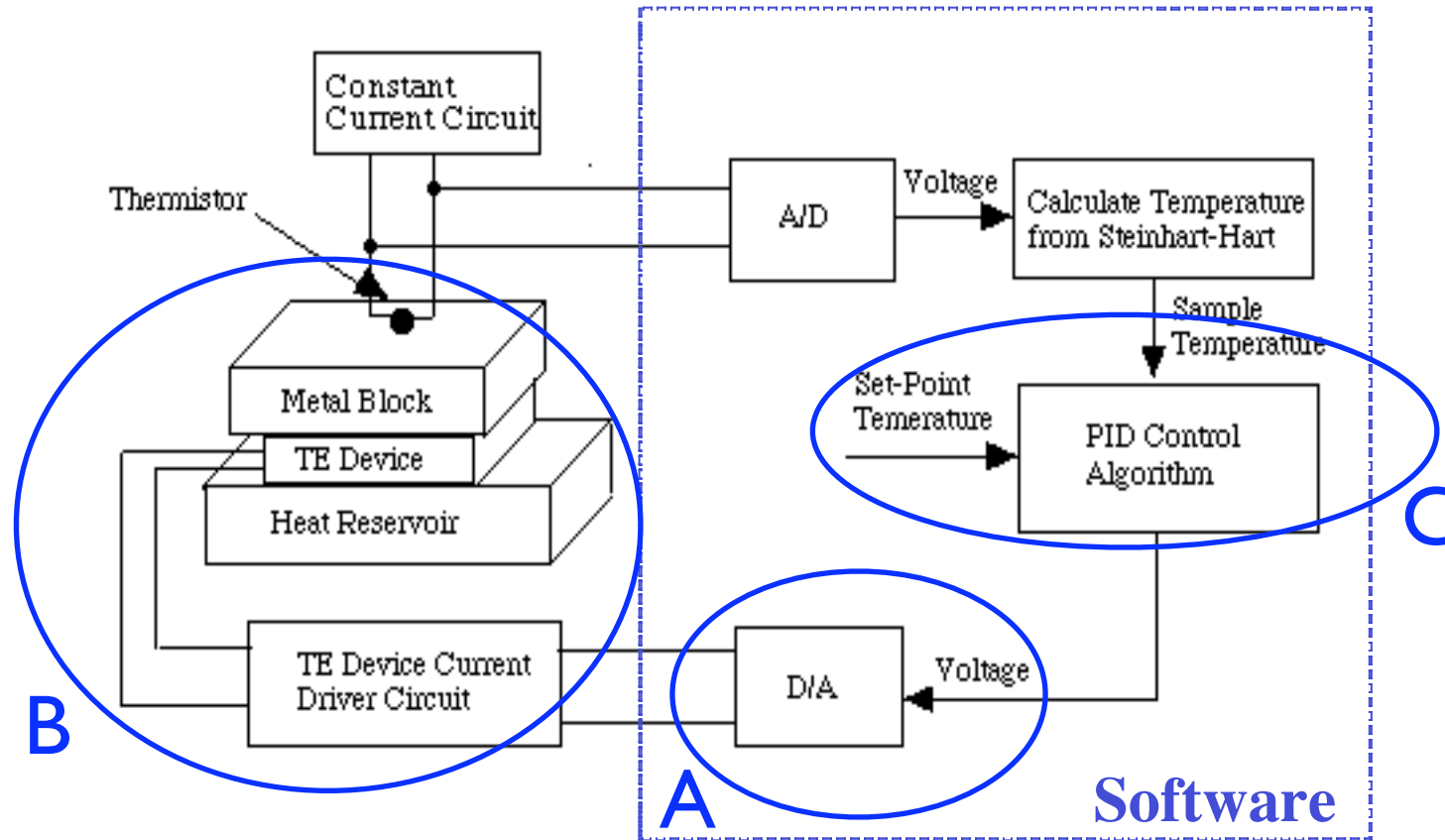
Wait, then repeat



III) Final Project

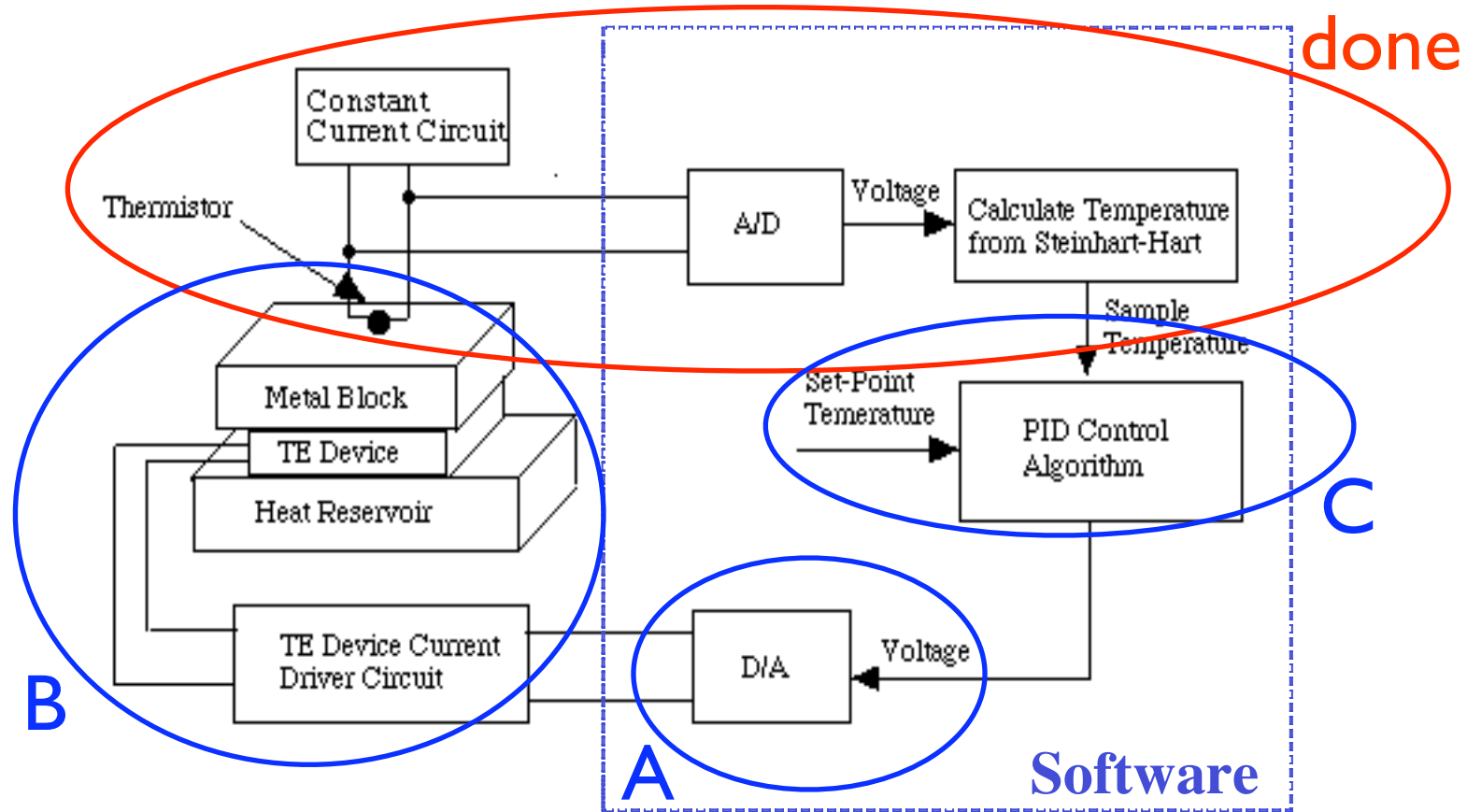
PID-Temperature Controller

TEMPERATURE CONTROL PROJECT



A) Use DAQ Assistant

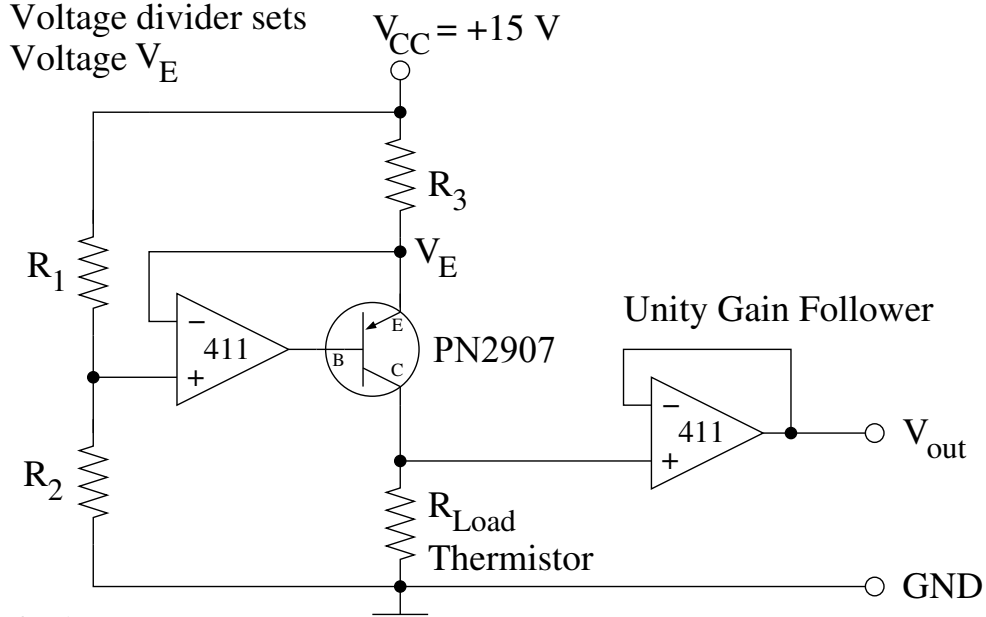
TEMPERATURE CONTROL PROJECT



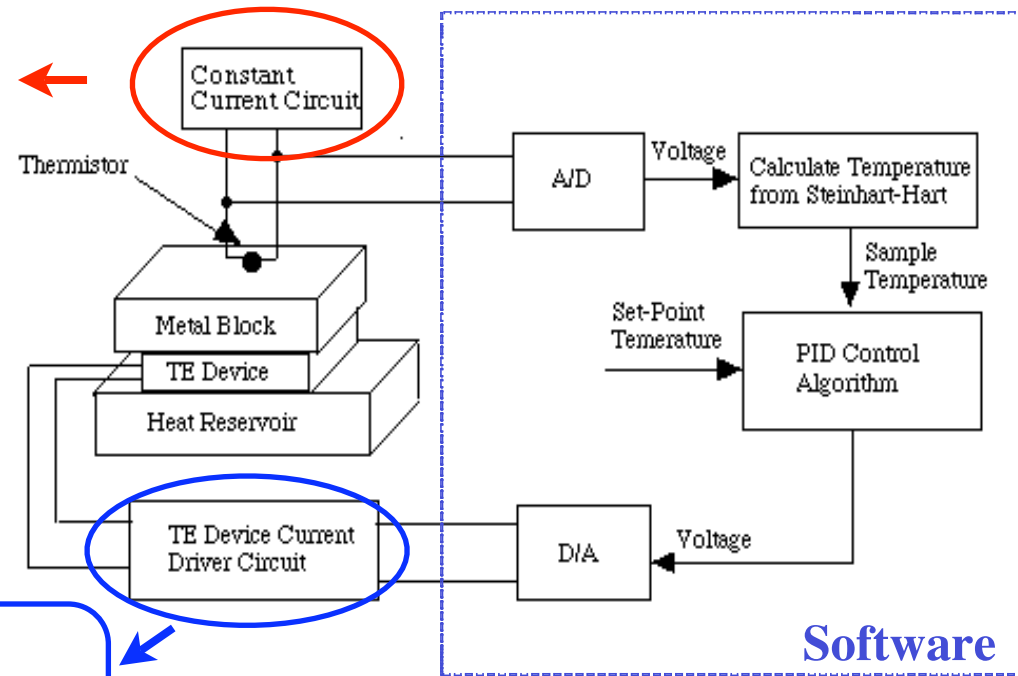
A) Use DAQ Assistant

B) Analog Hardware

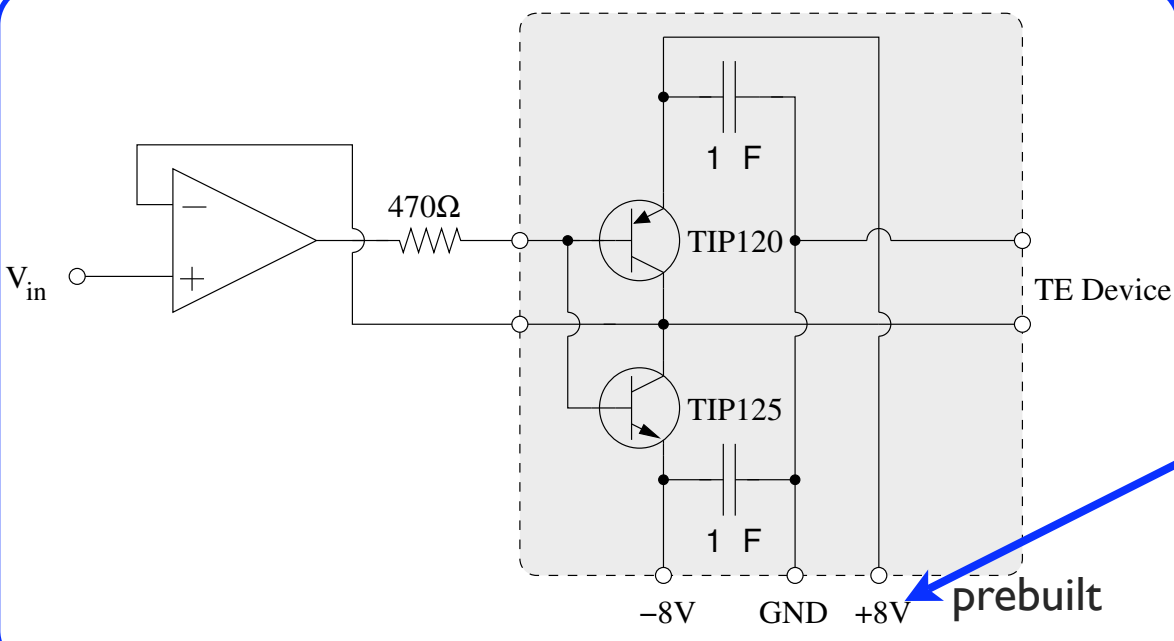
Voltage divider sets Voltage V_E



OpAmp:
Decouples Voltage Divider
Sets base voltage such that emitter voltage is constant

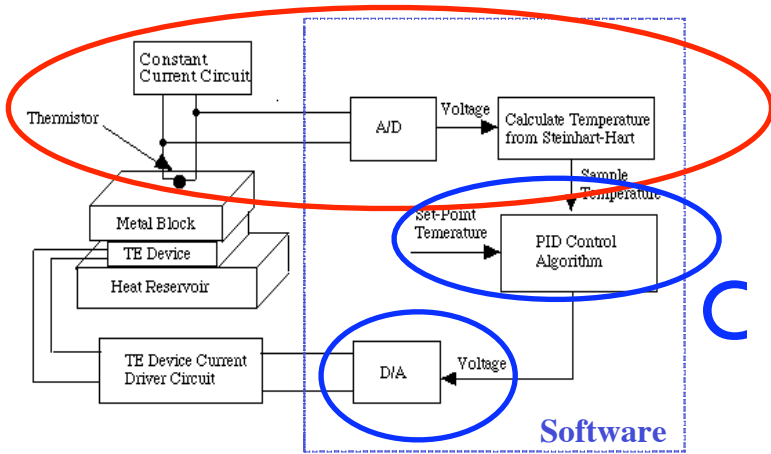


Software

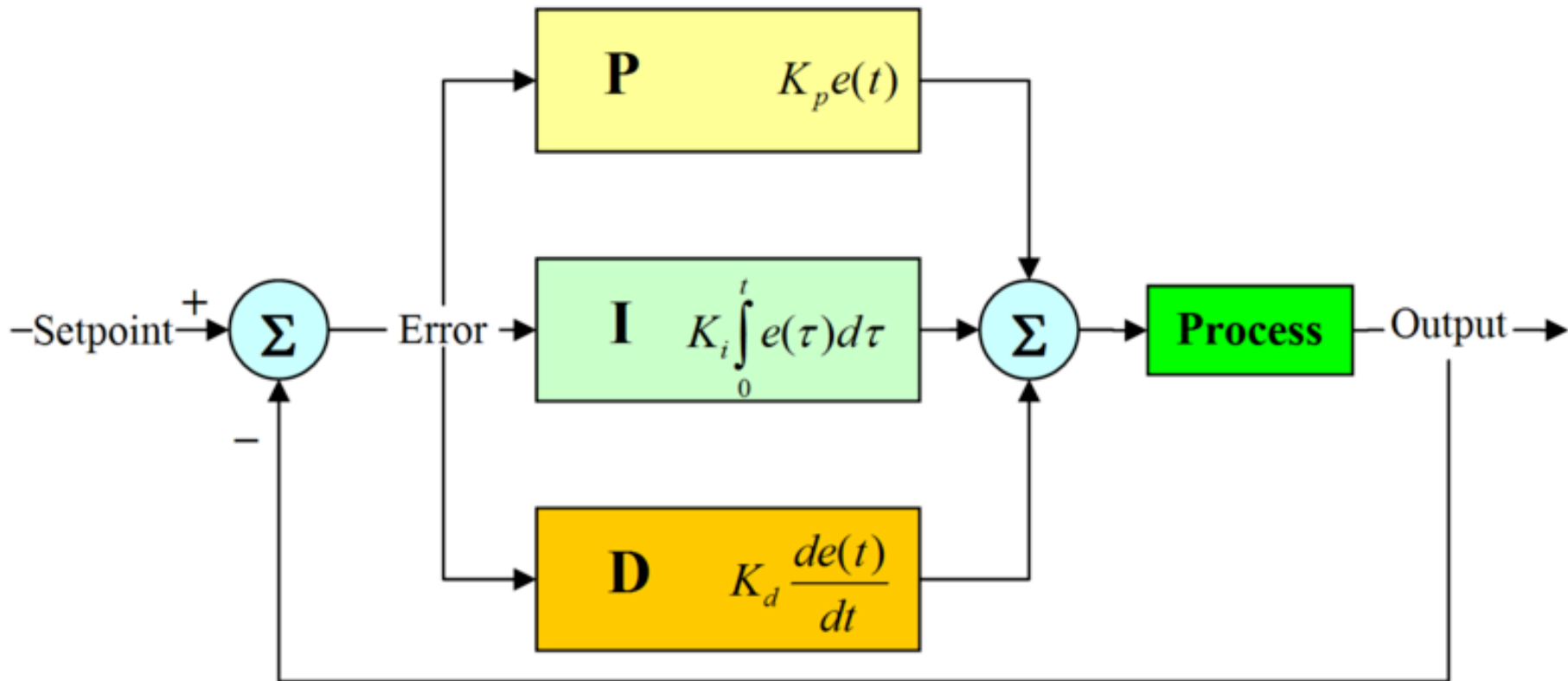


PS280 Power supply

prebuilt



C) PID Control



Example: A very very simple model

$$\frac{d\Theta}{dt} = -\gamma \Theta + u(t) \quad \Theta(t) = T_{plate} - T_{air} \quad u(t) = control$$

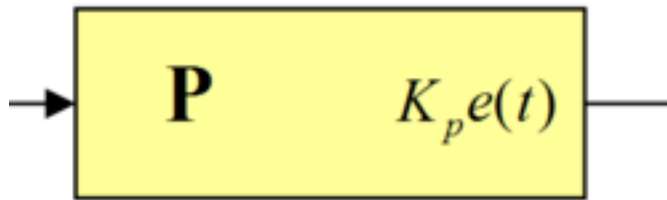
Consider constant control

$$\text{If } u(t) = K \Theta_0 = \text{const.} \quad \Theta(t) = \frac{K \Theta_0}{\gamma} (1 - e^{-\gamma t}) \quad \Theta(\infty) = \frac{K}{\gamma} \Theta_0$$

Consider **P** - proportional control

$$\begin{aligned} \text{If } u(t) = K_P (\Theta_0 - \Theta) \quad \frac{d\Theta}{dt} &= -\gamma \Theta + K_P (\Theta_0 - \Theta) \\ &= -(\gamma + K_P) \Theta + K_P \Theta_0 \end{aligned}$$

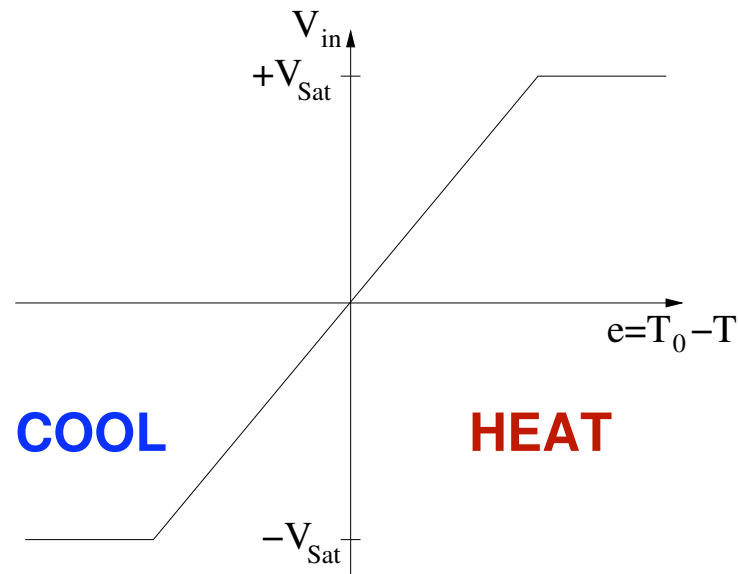
$$\Theta(\infty) = \frac{K_P}{\gamma + K_P} \Theta_0 \neq \Theta_0$$



P - proportional control

- Proportional to deviation from setpoint
- Considers only present value
- Big K_P - rapid control, potentially unstable
- Overshoot typical
- Always has offset (small for K_P big)

- Saturation of feedback



Consider **I** - integral control

$$\frac{d\Theta}{dt} = -\gamma \Theta + u(t) \quad \Theta(t) = T_{plate} - T_{air} \quad u(t) = K_i \int_0^t (\Theta_0 - \Theta) dt$$

Differentiate:

$$\frac{d^2\Theta}{dt^2} + \gamma \frac{d\Theta}{dt} + K_i \Theta = K_i \Theta_0$$

$$\Theta(\infty) = \Theta_0$$

Same for **PI** - proportional-integral control

$$\frac{d\Theta}{dt} = -\gamma \Theta + u(t) \quad u(t) = K_p(\Theta_0 - \Theta) + K_i \int_0^t (\Theta_0 - \Theta) dt$$

$$\frac{d^2\Theta}{dt^2} + (K_p + \gamma) \frac{d\Theta}{dt} + K_i \Theta = K_i \Theta_0 \quad \rightarrow \quad \Theta(\infty) = \Theta_0$$

$$\rightarrow \boxed{\mathbf{I} \quad K_i \int_0^t e(\tau) d\tau} \rightarrow$$

I - integral control

- Proportional to integral of deviation from setpoint
- Eliminates offset error of P-control
- Considers past values
- Big K_i - rapid rise time, increased overshoot

$$\rightarrow \boxed{\mathbf{D} \quad K_d \frac{de(t)}{dt}} \rightarrow$$

D - differential control

- Proportional to derivative of deviation from setpoint
- Reduces overshoot of PI controller
- Predicts future values

PID - control for our very very simple model

$$\frac{d\Theta}{dt} = -\gamma \Theta + u(t) \quad \Theta(t) = T_{plate} - T_{air} \quad u(t) = control$$

No control $\frac{d\Theta}{dt} + \gamma\Theta = 0$

PID control

$$\frac{d\Theta}{dt} + \gamma\Theta = K_p(\Theta_0 - \Theta) + K_i \int_0^t (\Theta_0 - \Theta) dt + K_d \frac{d(\Theta_0 - \Theta)}{dt}$$

$$E = \Theta_0 - \Theta = (T_{plate}^{goal} - T_{air}) - (T_{plate} - T_{air}) = T_{plate}^{goal} - T_{plate}$$

iteration n

Thermistor SubVI

Read and find average thermistor voltage $V_{\text{thermistor}}$

Calculate thermistor resistance $R_{\text{thermistor}}$

Calculate temperature T_n via Steinhart-Hart Equation
(A=? B=? D(or C)=?)

PID VI

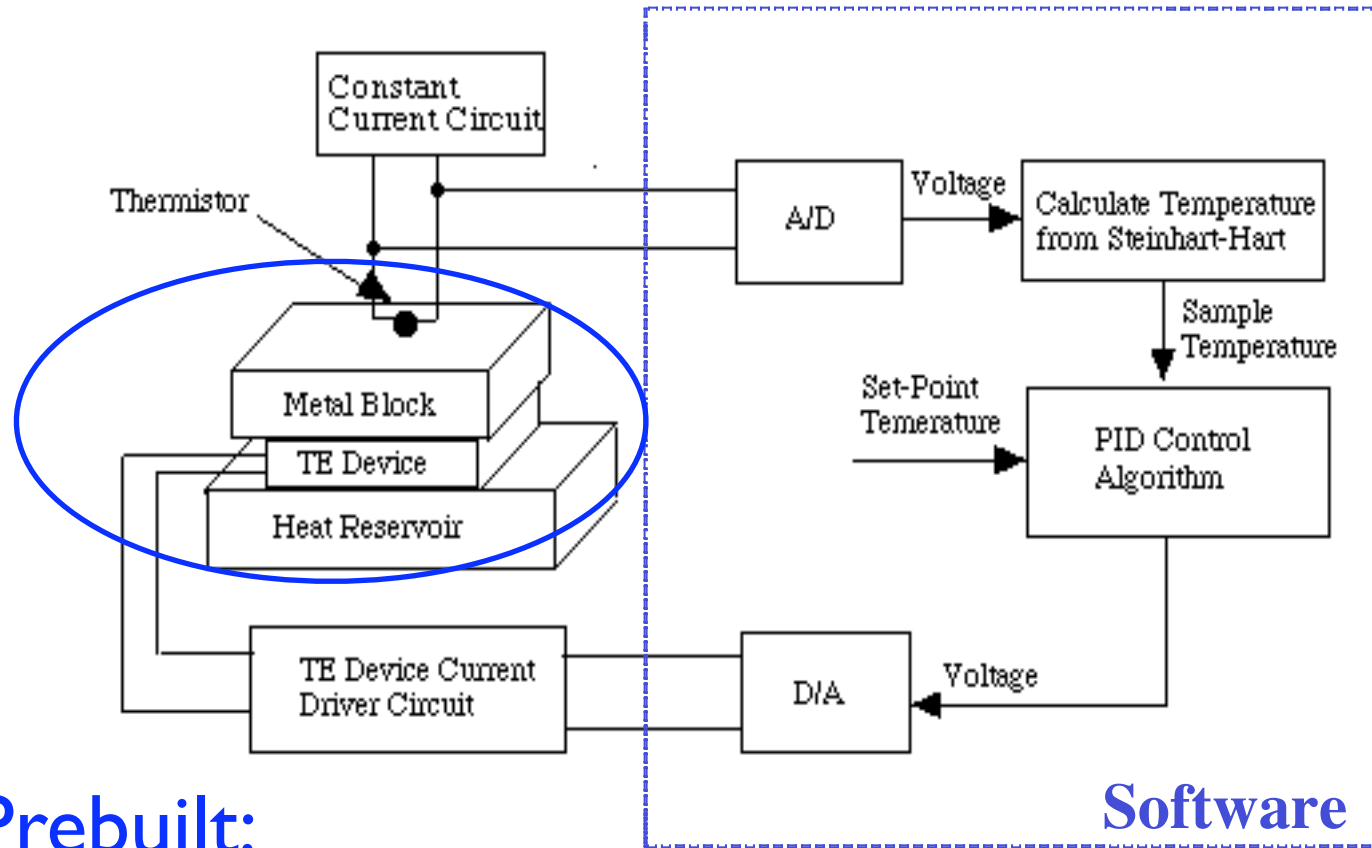
Calculate error : $E_n = T_{\text{set}} - T_n$

Calculate PID Control Voltage:

$$V_{PID} = K_p E_n + K_i \Delta t \sum_{m=0}^n E_m + K_d \frac{1}{\Delta t} (E_n - E_{n-1})$$

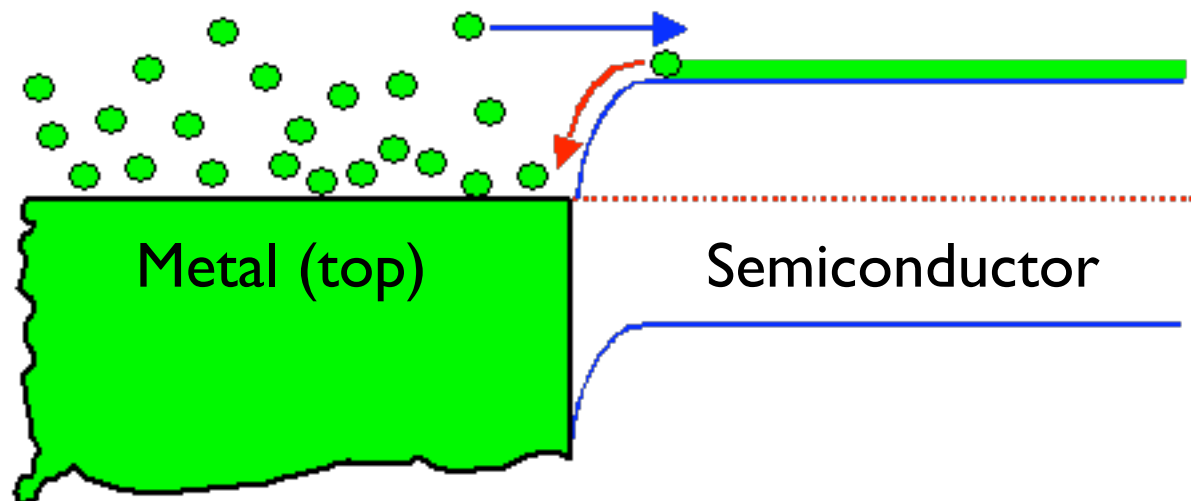
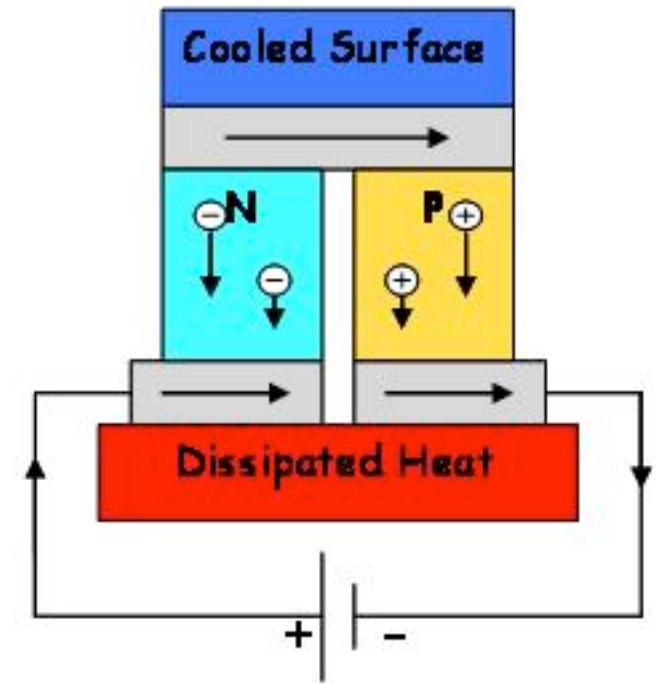
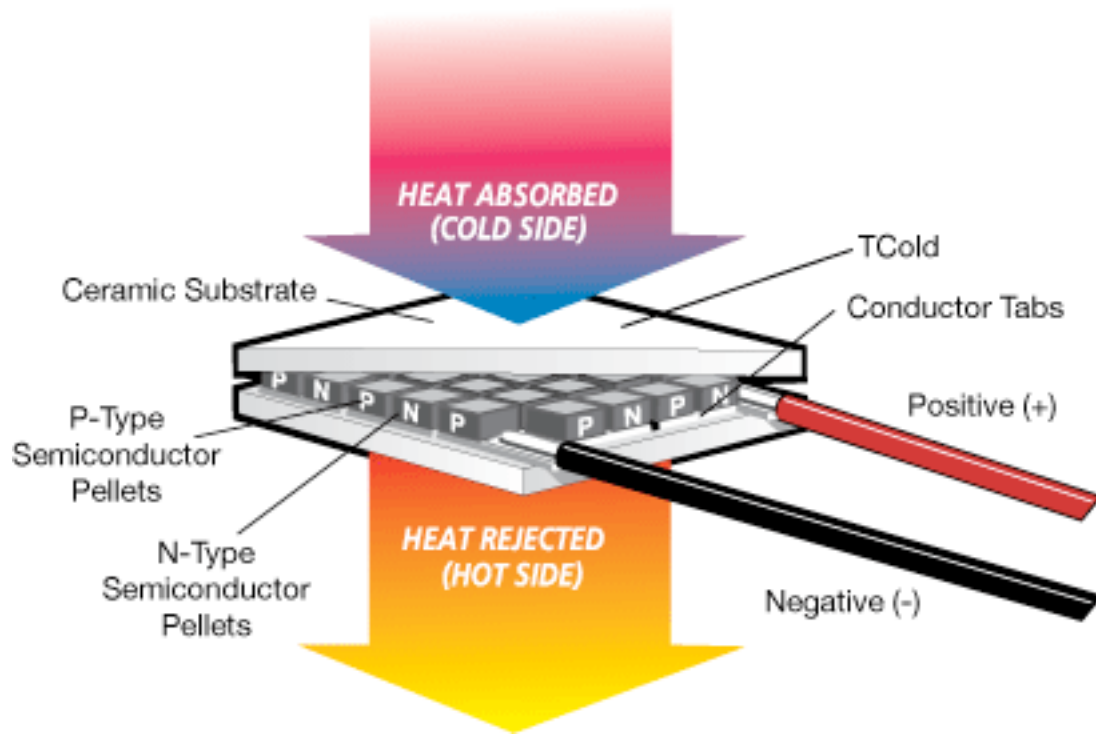
Write V_{PID}

TEMPERATURE CONTROL PROJECT



D - Prebuilt:
Thermoelectric
Element

D) Thermoelectric Device



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