Measuring the magnetization of a permanent magnet

B. Barman
Department of Computer Science, Engineering and Physics, University of Michigan-Flint, Flint, Michigan 48502

A. Petrou
Department of Physics, University at Buffalo, The State University of New York, Buffalo, New York 14260

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The effect of an external magnetic field ***B*** on magnetic materials is a subject of immense importance. The simplest and oldest manifestation of such effects is the behavior of the magnetic compass. Magnetization ***M*** plays a key role in studying the response of magnetic materials to ***B***. In this paper, an experimental technique for the determination of ***M*** of a permanent magnet will be presented. The proposed method discusses the effect of ***B*** (produced by a pair of Helmholtz coils) on a permanent magnet, suspended by two strings and allowed to oscillate under the influence of the torque that the magnetic field exerts on the magnet. The arrangement used Newton's second law for rotational motion to measure ***M*** via graphical analysis.

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I. INTRODUCTION

Magnetism has been intriguing mankind for centuries now. A magnet's ability to influence magnetic materials, from a distance, mesmerized numerous inquisitive minds of the past. The magnetic compass, used across all continents, is such a device, acting under the influence of Earth's magnetic field.[1] With the advancement of science, particularly after the discovery of the atomic model, magnetism at the microscopic scale came to the fore. Hans-Christian Oersted's discovery of a current-carrying wire producing a magnetic field added a whole new dimension, leading to the advent of a technological revolution in the field of electromagnetism.[2] With the exploration of newer magnetic materials, exhibiting paramagnetism, ferromagnetism, and diamagnetism, it became essential to develop techniques to measure magnetization in these. While many experimental methods have been proposed, along with the existence of high tech equipment for research, a basic design requirement for educational purposes has eluded us so far.[3,4] In this paper, we present an experimental technique to determine the magnetization of a permanent magnet using readily available lab instruments namely as a pair of Helmholtz coils, an ammeter, a power supply, and a stopwatch. While this technique is applicable for both introductory physics (calculus and algebra based) and undergraduate Physics/Engineering majors, the extent of exploration rests with the instructor. As a quick in-class demo, an introductory class can observe the effect of an external magnetic field on the oscillation period of a permanent magnet without determining any value of magnetization. In an advanced lab setting, students can explore numerical values of magnetization of the four types of permanent magnets, viz., neodymium iron boron (NdFeB), samarium cobalt (SmCo), alnico, and ceramic or ferrite magnets.

II. THEORY

A. Magnetic moment

It is well known that in a wire loop with area A, in which a current i is flowing, we define a vector known as the "magnetic moment" (symbol $\vec{\mu}$), perpendicular to the plane of the loop.[1,5–7] The magnitude μ is given by the following equation:

$$\mu = iA. \tag{1}$$

If we place this loop in a uniform magnetic field $\vec{B}$ at an angle θ with $\vec{\mu}$, as shown in Fig. 1(a), we have a torque acting on the loop whose magnitude τ is given by the following equation:

$$\tau = \mu B \sin\theta. \tag{2}$$

If the loop is allowed to move, it will rotate under the action of the torque in such a way that $\vec{\mu}$ becomes parallel with the magnetic field $\vec{B}$, as shown in Fig. 1(b). Indeed, in the configuration of Fig. 1(b), we have that $\theta = 0$ for which Eq. (2) shows that $\tau = 0$ and all movement stops. Thus, the magnetic moment is defined so that the torque exerted on a current carrying loop, when placed in an external magnetic field, is proportional to μ.[8]

B. Magnetization

For our experimental discussion, we consider a cylindrical permanent magnet (composed of iron atoms) of radius R and height l, as shown in Fig. 2. In each atom, its electrons move around the nucleus on orbits similar to that shown in Fig. 1. Thus, each iron atom behaves like a microscopic loop with magnetic moment $\vec{\mu}_{Fe}$. In iron and other ferromagnetic materials, all the magnetic moment vectors are aligned and the net magnetic moment μ of the magnet is equal to $N\mu_{Fe}$, where N is the number of iron atoms in the magnet. The magnetization M of the magnet, with a volume V is defined as[5,9]

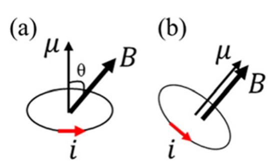

Fig. 1. (a) Magnetic moment of a current carrying loop at an angle θ with respect to $\vec{B}$ and (b) $\vec{\mu} \parallel \vec{B}$.

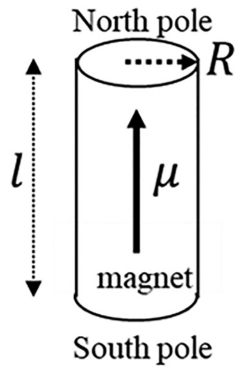

Fig. 2. Cylindrical permanent magnet of radius R and height l.

$$M = \frac{\mu}{V}. \tag{3}$$

For the cylindrical magnet of Fig. 2, the volume is given by the following equation:

$$V = \pi R^2 l. \tag{4}$$

C. Magnetic field at the center of a pair of Helmholtz coils

The schematic of a pair of Helmholtz coils is shown in Fig. 3. Each has a diameter D and consists of N turns of wire. They are positioned so that they have a common axis, and the distance between the coil centers is adjusted so that it is equal to half the diameter D. Under these conditions, the Helmholtz coils generate a magnetic field along the common axis of the coils, which is uniform in the vicinity of the mid-point between the coil centers. The magnetic field B is given by the following equation:[10,11]

$$B = \frac{16\mu_0 N i}{D\sqrt{125}}. \tag{5}$$

Here, the current i is measured in Amps, B is measured in tesla, $D = 0.21\,m$, $N = 119$, and μ_0 is a constant equal to $1.256 \times 10^{-6}\,T.m/A$.

III. DISCUSSION: OSCILLATION OF A MAGNET IN A MAGNETIC FIELD

A permanent magnet is suspended, in a stirrup like arrangement (Fig. 4), so that the magnet points in the

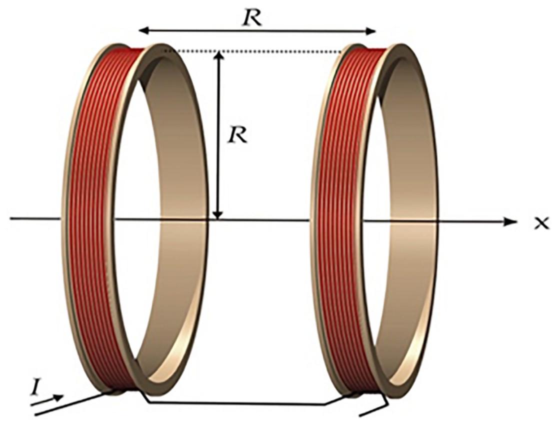

Fig. 3. Helmholtz coils of diameter D, consisting of N turns. Image copyright: https://www.emworks.com/application/numerical-simulation-of-helmholtz-coil.

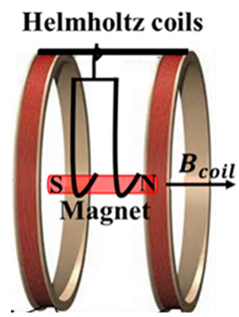

Fig. 4. Cylindrical permanent magnet suspended from strings.

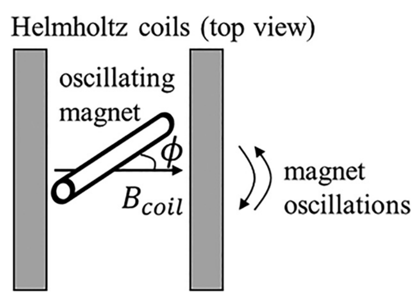

Fig. 5. Top view of the cylindrical magnet oscillating between the Helmholtz coils.

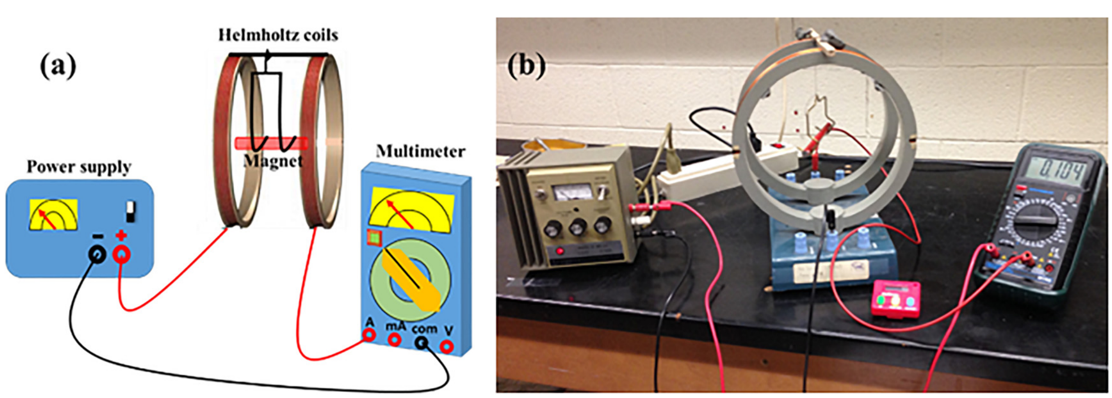

Fig. 6. Pictorial representation of the experimental set-up. (a) Schematic and (b) actual working rendition.

north-south direction. The magnetic field $\vec{B}$ generated by the Helmholtz coils is in the same direction such that the Earth's magnetic field does not influence our measurement of magnetization. This is an important criterion which has to be kept in consideration while performing this experiment. If we now consider Fig. 5, which is a top view of Fig. 4, displace the magnet by a small angle ϕ_0 from the direction of $\vec{B}$, and release it from rest, the magnet will oscillate around the equilibrium position as indicated by the curved arrows shown in Fig. 5. The time it takes for the magnet to swing from left to right and back to the starting position is called the period of oscillation (symbol T). By applying Newton's second law of rotational motion, we can write (using the small angle approximation)

$$\tau = I\frac{d^2\phi}{dt^2} = C\phi + \mu B\sin\phi \approx [C + \mu B]\phi, \tag{6}$$

where ϕ is the angle that the magnet axis forms with $\vec{B}$ at any instant of time t. Here, C is the torsional constant that describes the mechanical properties of the supporting strings and μ is the magnetic moment of the magnet. I is the moment of inertia (in our configuration) of the cylindrical magnet of mass m and is given by the following equation:[12]

$$I = \frac{1}{12}ml^2 + \frac{1}{4}mR^2. \tag{7}$$

The solution to the 2nd order differential equation in ϕ is given by

$$\phi = \phi_0\cos\left(\frac{2\pi}{T}t\right). \tag{8}$$

The quantity $2\pi/T$ is given by the following equation, which follows from Eqs. (6) and (8):

$$\frac{2\pi}{T} = \sqrt{\frac{C + \mu B}{I}}. \tag{9}$$

IV. EXPERIMENTAL PROCEDURE

A pictorial (schematic and actual) representation of the experimental set-up is shown in Fig. 6. The permanent magnet shown in Fig. 4 has been exaggerated to show the details of the experimental arrangement, while it is well known that the magnetic field is uniform only within a very small region between the centers of the coils. Hence, the choice of length for the permanent magnet is paramount to determining M precisely. More significantly, the distance between the coils should be equal to the radius in order to achieve this uniformity.[13,14] The period of 10 oscillations was measured as a function of the current in the Helmholtz coils; the resulting data were linearized using Eq. (9) and plotted. A least-squares fit was used to plot the data as shown in Fig. 7. In order to improve the accuracy of these measurements, it is important that the permanent magnet oscillates in a horizontal plane, keeping the small-angle approximation in mind. Any vertical mode of oscillation would introduce a considerable amount of error in the measurement of the magnetization. Vernier calipers were used to measure the radius R and the length ℓ of the magnet, while a balance was used to measure the mass m of the magnet in kilograms.

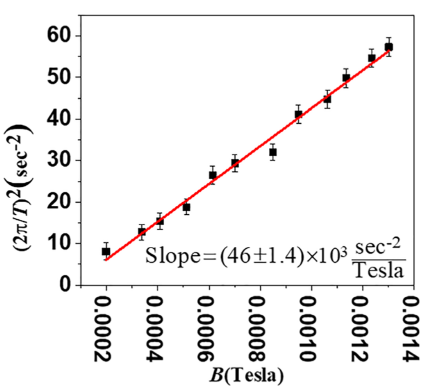

Fig. 7. Plot of $(2\pi/T)^2$ as a function of B.

Table I. Summary of the experimental parameters as well as the calculated value of M.

m (kg)	R (m)	l (m)	I (kg m^2)	μ(A m^2)	M(A m^{-1})
0.0511	0.0046	0.125	6.681×10^{-5}	3.047	$(37\pm1.2) \times 10^4$

V. EXPERIMENTAL RESULTS AND DISCUSSION

The experimental data were plotted with B on the x-axis and $(2\pi/T)^2$ on the y-axis as shown in Fig. 7 followed by modeling using Eq. (9) to calculate the magnetic moment μ. Having determined the slope (and the corresponding magnetic moment μ), together with volume V, we were able to determine the magnetization M of the magnet. We have summarized our experimental parameters as well as the calculated value of M in Table I. While the slope accounted for an uncertainty of 3%, the measurement of the moment of inertia carried an uncertainty of 0.25%. Thus, the overall uncertainty (3%) in M is attributed approximately to the uncertainty in the slope (3%), resulting from the non-uniformity of the magnetic field produced by the pair of Helmholtz coils over the physical dimensions of the permanent magnet. Upon knowing the composition (if it is an alloy) of the permanent magnet, one could compare it with certain known values of M reported in numerous textbooks as well as handbooks of physical parameters.[7,15] Our experimentally determined value of M indicates it to be a Mn ferrite alloy (saturation magnetization 39×10^4 A.m^{-1}) as reported in Ref. 7.

VI. CONCLUSION

We described a simple and cost-effective way of measuring the magnetization of a permanent magnet in an undergraduate laboratory setting suitable for both introductory classes and undergraduate physics majors. We used Newton's second law for rotational motion and verified the validity of our equations by performing an experiment using a cylindrical magnet suspended between a pair of Helmholtz coils. We also showed the data analysis technique, via graphical representation, required to calculate the value of the magnetization for a particular specimen.

ACKNOWLEDGMENT

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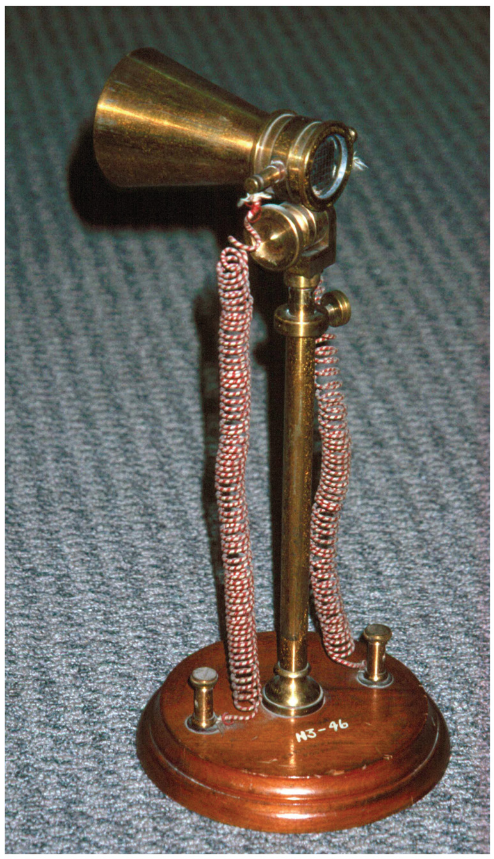

Differential Thermopile

At the heart of this instrument is a series of bismuth and antimony bars, laid alternately side by side and soldered together in pairs at their ends. This forms a series of thermoelectric junctions connected in series so that the total voltage output of a single junction is multiplied by the number of junctions. Usually there is a second horn on the right hand side to collect thermal radiation coming from that side. The device's output is the difference of the effect of thermal radiation falling on both sides of the junctions. It is in the apparatus collection of Washington and Lee University in Virginia. (Picture and text by Thomas B. Greenslade, Jr., Kenyon College)