Virtues of CI

Ontological innocence You can undertake two commitments, once to object x and once to object y; or you could commit yourself to them all at once by committing yourself to the mereological fusion of x and y. It's the same commitment either way. So once you have committed to some things, commitment to objects composed of those things is not a *further* commitment. (Cf. Lewis, *Parts of Classes*.)

Explanation of part-whole relation We may be able to explain why the part-whole relation behaves as it does, e.g., why it is transitive. We might be able to reduce the part-whole relation to something else. Perhaps for x to be a part of y is just for it to be the case that x and y compose y; which is to say, according to CI, that x and y just are y. (Cf. Sider, "Parthood".)

Lewis on the limitations of the "analogy"

> In the first place, I know of no way to generalize the definition of ordinary one-one identity in terms of plural quantification. We know that x and y are identical iff, whenever there are some things, x is one of them iff y is one of them. But if y is the fusion of the xs, then there are some things such that each of the xs is one of them and y is not; and there are some things such that y is one of them but none of the xs is. And in the second place, even though the many and the one are the same portion of Reality, and the character of that portion is given once and for all whether we take it as many or take it as one, still we do not really have a generalized principle of indiscernibility of identicals. It does matter how you slice it—not to the character of what's described, of course, but to the form of the description. What's true of the many is not exactly what's true of the one. After all they are many while it is one. The number of the many is six, as it might be, whereas the number of the fusion is one. And the singletons of the many parts are wholly distinct from the singleton of the one fusion. That is how we can have set theory.
>
> David Lewis, *Parts of Classes*

Yi's argument against CI (adapted)

(1)	J_l and J_r compose John.	Premise
(2)	Composition is identity.	Suppose for *reductio*
(3)	J_l and J_r = John	From (1) and (2)
(4)	John is one of John and Paul.	(logical truth)
(5)	John is one of J_l and J_r and Paul.	From (3) and (4)
(6)	Paul is composed of P_l and P_r.	Premise
(7)	Paul = P_l and P_r.	From (2) and (6)
(8)	John is one of J_l and J_r and P_l and P_r.	From (5) and (7)
(9)	Every one of J_l and J_r and P_l and P_r is a half-man.	Premise.
(10)	John is a half-man.	From (8) and (9)

Collective vs. distributive plural predication

John, Paul, and George sang.

John, Paul, and George each sang.

John, Paul, and George sang together.

John sang and Paul sang and George sang.

P and P $\rightarrow$ Q entail Q.

What goes wrong with the argument; first pass

(3a) J_l and J_r are identical with John.
(3b) J_l and J_r are each identical with John.
(3c) J_l and J_r are, together (collectively), identical with John.

Multigrade relations

On the left are representations of the "multiple-arity" conception; on the right are representations of the "fixed-arity" conception.

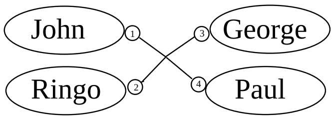
John fights Paul.

$\mathsf{Fight}(\underbrace{\mathsf{john}}, \underbrace{\mathsf{paul}})$

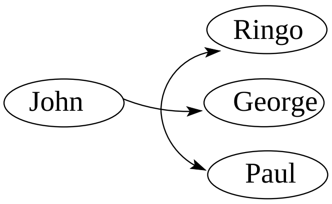
$\mathsf{Fight}(\underbrace{\mathsf{john}}, \underbrace{\mathsf{paul}})$

John fights Ringo, George and Paul.

$\mathsf{Fight}(\underbrace{\mathsf{john}}, \underbrace{\mathsf{ringo}}, \underbrace{\mathsf{george}}, \underbrace{\mathsf{paul}})$

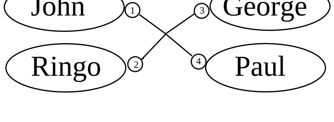


$\mathsf{Fight}(\underbrace{\mathsf{john}}, \underbrace{\mathsf{ringo}, \mathsf{george}, \mathsf{paul}})$

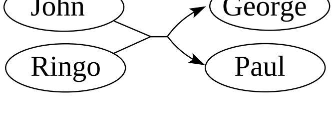


John and Ringo fight George and Paul.

$\mathsf{Fight}(\underbrace{\mathsf{john}}, \underbrace{\mathsf{ringo}}, \underbrace{\mathsf{george}}, \underbrace{\mathsf{paul}})$



$\mathsf{Fight}(\underbrace{\mathsf{john}, \mathsf{ringo}}, \underbrace{\mathsf{george}, \mathsf{paul}})$



'is one of' is not a relational expression

Observation 1:

> Each of John and Paul has written a hit song.

and

> One of John and Paul has written a hit song.

are parallel constructions.

Observation 2:

> John is one of John and Paul.

is equivalent with

> John is identical with one of John and Paul.

or, to put the quantifier in front:

> One of John and Paul is such that John is identical with it.

Observation 3:

> Yoko loves one of John and Paul.

is equivalent with

> It is not the case that: each of John and Paul is not loved by Yoko.

Observation 4:

> John is one of John and Paul.

is equivalent with

> It is not the case that: each of John and Paul is non-identical with John.

A formal plural language:

$$\text{Terms: } \left\{ \begin{array}{ll} \text{Singular terms:} & \left\{ \begin{array}{ll} \text{Constants:} & \mathsf{a, b, c, \ldots} \\ \text{Variables:} & \mathsf{x, y, z, \ldots} \end{array} \right. \\ \text{Plural terms:} & \left\{ \begin{array}{ll} \text{Constants:} & \mathsf{aa, bb, cc, \ldots} \\ \text{Variables:} & \mathsf{xx, yy, zz, \ldots} \\ \text{Lists:} & \mathsf{a + x, a + x + xx, \ldots} \end{array} \right. \end{array} \right.$$

$$\text{Quantifiers: } \left\{ \begin{array}{ll} \text{Absolute:} & \exists, \forall \text{ (can bind all variables)} \\ \text{Relative:} & \left\{ \begin{array}{ll} \forall(\_\_,\_\_) \text{ “Each of”:} & \text{e.g., } \forall(\mathsf{x, xx})\mathsf{F}(\mathsf{x}) \\ & \text{“Each of } \mathsf{xx} \text{ is } \mathsf{F}\text{”} \\ \exists(\_\_,\_\_) \text{ “One of”:} & \text{e.g.: } \exists(\mathsf{x, xx})\mathsf{F}(\mathsf{x}) \\ & \text{“One of } \mathsf{xx} \text{ is } \mathsf{F}\text{”} \end{array} \right. \end{array} \right.$$

(The first blank of the relative quantifier is to be filled with a singular variable, while the second can contain any term.)

$$\text{Predicates: } \left\{ \begin{array}{ll} \text{Non-logical:} & \mathsf{F, G, R} \ldots \text{ (each with a fixed arity, } n\text{,} \\ & \text{saturated by any } n \text{ terms to form an} \\ & \text{atomic wff)} \\ \text{Logical:} & = \text{ (2-place)} \end{array} \right.$$

Propositional connectives: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$

Abbreviations:[1]		
	$\sqsubseteq$ (is part of)	e.g., $\mathsf{x} \sqsubseteq \mathsf{y}$ abbreviates $\mathsf{x} + \mathsf{y} = \mathsf{y}$
	$\leq$ (are among)	e.g., $\mathsf{xx} \leq \mathsf{yy}$ abbr. $\forall(\mathsf{x, xx})\exists(\mathsf{y, yy})\mathsf{x} = \mathsf{y}$
	$\equiv$	$\mathsf{xx} \equiv \mathsf{yy}$ abbr. $\mathsf{xx} \leq \mathsf{yy} \wedge \mathsf{yy} \leq \mathsf{xx}$

[1]Here, and elsewhere, I omit the qualifications needed to avoid “variable collisions”.

Examples:

John and Paul are each singing.	$\mathsf{F}(\mathsf{a}) \wedge \mathsf{F}(\mathsf{b})$
Each of John and Paul is singing.	$\forall \mathsf{x}(\mathsf{x}, \mathsf{a}+\mathsf{b})\ \mathsf{F}(\mathsf{x})$
John and Paul are singing together.	$\mathsf{F}(\mathsf{a}+\mathsf{b})$
Some thing is singing.	$\exists \mathsf{x} \mathsf{F}(\mathsf{x})$
Some things (at least two) are singing together.	$\exists \mathsf{xx}(\exists(\mathsf{x}, \mathsf{xx})\exists(\mathsf{y}, \mathsf{xx})\mathsf{x} \neq \mathsf{y}\ \wedge \mathsf{F}(\mathsf{xx}))$
Each of xx entails z	$\forall(\mathsf{x}, \mathsf{xx})\mathsf{R}(\mathsf{x}, \mathsf{z})$
xx together entail z, but none of them (alone) entails z	$\mathsf{R}(\mathsf{xx}, \mathsf{z}) \wedge \neg\exists(\mathsf{x}, \mathsf{xx})\mathsf{R}(\mathsf{x}, \mathsf{z})$
There are some critics who admire only one another	$\exists \mathsf{xx} \forall(\mathsf{x}, \mathsf{xx})(\mathsf{F}(\mathsf{x}) \wedge \forall \mathsf{y}(\mathsf{R}(\mathsf{x}, \mathsf{y}) \rightarrow \exists(\mathsf{z}, \mathsf{xx})\mathsf{y} = \mathsf{z}))$

Valid versions of substitutivity:

$$\frac{\mathsf{t} = \mathsf{s} \quad \phi(\mathsf{t})}{\phi(\mathsf{s})}$$

where t and s are singular terms, and $\phi(\mathsf{s})$ arises from $\phi(\mathsf{t})$ by replacing an occurrence of t in $\phi(\mathsf{t})$ with s.

$$\frac{\mathsf{tt} = \mathsf{ss} \quad \phi(\mathsf{tt})}{\phi(\mathsf{ss})}$$

where tt and ss are plural (or singular) terms, and $\phi(\mathsf{ss})$ arises from $\phi(\mathsf{tt})$ by replacing an occurrence of tt in $\phi(\mathsf{tt})$ *that is not within a second argument of a relative quantifier* with ss.

$$\frac{\mathsf{tt} \equiv \mathsf{ss} \quad \phi(\mathsf{tt})}{\phi(\mathsf{ss})}$$

where tt and ss are plural (or singular) terms, and $\phi(\mathsf{ss})$ arises from $\phi(\mathsf{tt})$ by replacing an occurrence of tt in $\phi(\mathsf{tt})$ with ss.

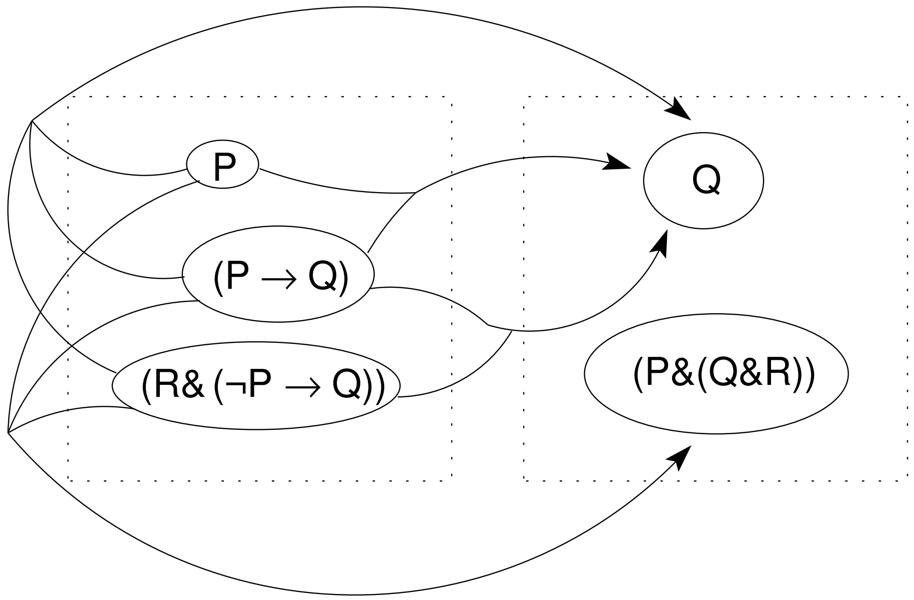
Atomism: Everything is composed of atoms—i.e., of things that have nothing but themselves as parts.

Logic quiz

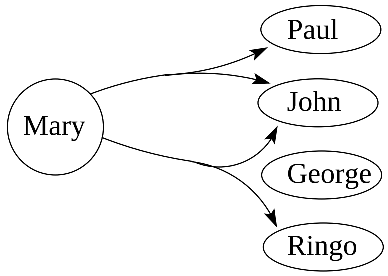
How many times is a sentence on the right entailed by sentences on the left? That is, how many instances of entailment (from left box to right box)?

P	Q
$(P \rightarrow Q)$	
$(R \& (\neg P \rightarrow Q))$	$(P \& (Q \& R))$

Diagram of holdings of entailment from sentences in left box to single sentences in right box:



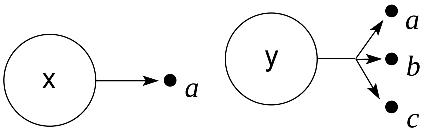
John and Paul together carried Mary, and John and Ringo together carried Mary, and no one else carried Mary. (So, Mary bears the "was carried by" relation exactly twice; but there is no person to whom she bears that relation.)



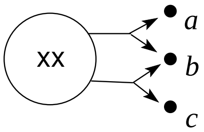
Sub-realist

Semi-formal presentation of Sub-realist semantic values relative to an assignment relation:

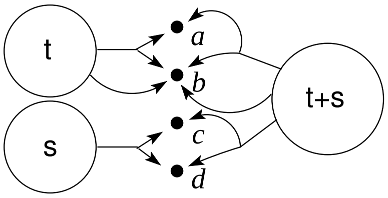
Singular terms t — t refers once, to some atom or some atoms collectively



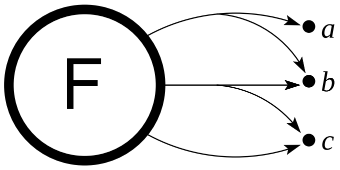
Plural terms t — t refers multiple times (or just once), each time to some atom or some atoms collectively



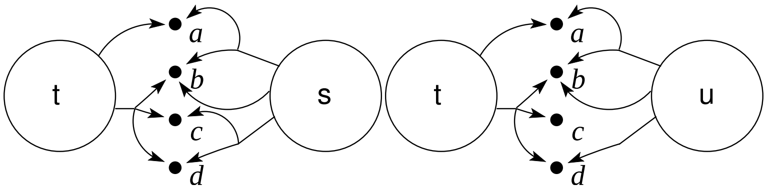
Lists t + s — if t refers to an atom or some atoms, so does t + s; if s refers to an atom or some atoms, so does t + s; and t + s refers to nothing else



Predicates F — for a one-place predicate F, F refers multiple times, each time to some atom or some atoms collectively



t = s — satisfied just in case every atom among some atoms referred to by t is among some atoms referred to by s, and vice-versa



Atomic wffs $\mathsf{F}(\mathsf{t})$	if t is singular, $\mathsf{F}(\mathsf{t})$ is satisfied just in case the atom or atoms collectively referred to by t are collectively referred to by F; if t is plural, consider the atoms (call them "t-ATOMS") such that: every atom that is one of some atoms that t refers to is one of t-ATOMS, and no other atoms are among t-ATOMS. $\mathsf{F}(\mathsf{t})$ is satisfied just in case F refers to, collectively, t-ATOMS.
$\exists(\mathsf{x},\mathsf{t})\phi(\mathsf{x})$	satisfied just in case there is some atom or are some atoms collectively referred to by t such that $\phi(\mathsf{x})$ is satisfied on the assignment relation just like the current one except that it assigns x to them.

Classical Atomistic Mereology

Let $\ulcorner \mathsf{x} \circ \mathsf{y} \urcorner$ abbreviate $\ulcorner \exists \mathsf{z}(\mathsf{z} \sqsubseteq \mathsf{x} \wedge \mathsf{z} \sqsubseteq \mathsf{y}) \urcorner$.
We may then formulate Classical Atomistic Mereology as:

Reflexivity $\forall \mathsf{x}\, \mathsf{x} \sqsubseteq \mathsf{x}$

Transitivity $\forall \mathsf{x} \forall \mathsf{y} \forall \mathsf{z}((\mathsf{x} \sqsubseteq \mathsf{y} \wedge \mathsf{y} \sqsubseteq \mathsf{z}) \rightarrow \mathsf{x} \sqsubseteq \mathsf{z})$

Atomism $\forall \mathsf{x} \exists \mathsf{y}(\forall \mathsf{z}(\mathsf{z} \sqsubseteq \mathsf{y} \rightarrow \mathsf{z} = \mathsf{y}) \wedge \mathsf{y} \sqsubseteq \mathsf{x})$

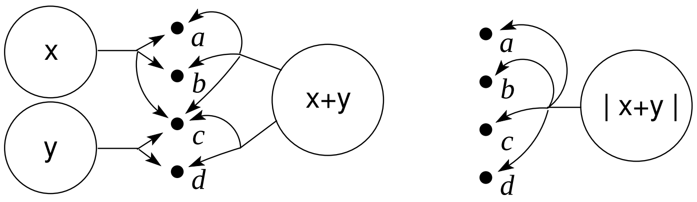
Composition $\forall \mathsf{ww} \exists! \mathsf{x}(\ \forall(\mathsf{y}, \mathsf{ww})(\mathsf{y} \sqsubseteq \mathsf{x}) \wedge \forall \mathsf{y}(\mathsf{y} \sqsubseteq \mathsf{x} \rightarrow \exists(\mathsf{z}, \mathsf{ww})(\mathsf{y} \circ \mathsf{z}))\ )$

Terms for fusions

For any term t, let $|\mathsf{t}|$ refer once, and refer to each thing that is among some things t refers to, and refer to nothing else.

Example

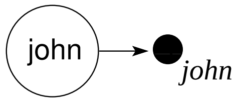
If x refers to *a*, *b*, and *c* (together) and y refers to *c* and *d* together, then $\mathsf{x}+\mathsf{y}$ refers once to *a*, *b*, and *c*, and once to *c* and *d*; $|\mathsf{x}+\mathsf{y}|$ refers once, to all four things together.



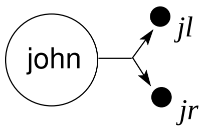
Realist

Three correct ways of looking at the fact that 'john' refers to John:

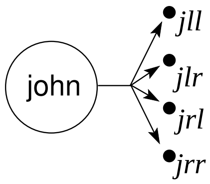
'john' refers to John



'john' refers to Johnleft and Johnright (collectively)



'john' refers to Johnleftleft, Johnleftright, Johnrightleft, and Johnrightright (collectively)



A bit of Realist semantics:

Singular terms each refer once, to one thing (which may be identical with many things, taken together); plural terms each refer once or more than once, each time to one thing (which may be identical with many things, taken together).

t = s is satisfied just in case the thing or things referred to by t, taken together, are identical with the thing or things referred to by s, taken together.

Oddities

> The students gathered on the South Lawn. Therefore, there is something that gathered on the South Lawn.
>
> John and Paul are bandmates. Therefore, Johnleft, Paulright, Paulleft, and Johnright are bandmates.

References

[1] David Lewis. *Parts of Classes*. Blackwell, 1991.

[2] Theodore Sider. Parthood. Forthcoming in a volume of *Topics in Contemporary Philosophy*.

[3] Byeong-Uk Yi. Is mereology ontologically innocent? *Philosophical Studies*, 93(2), February 1999.