Two defenses of Composition as Identity

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1 Introduction

The title of my paper is *Two defenses of Composition as Identity.* But my defenses today are limited in scope: I will only argue that the thesis that composition is identity is *coherent*; I will not argue that it is actually true. It's coherence is in question because of certain logical difficulties that confront it; I aim to solve those difficulties.

Now, what is the thesis that Composition is Identity? Roughly, it says that if some things compose some thing x, then those things *just are* x. It's not that the whole is just the sum of the parts—the whole literally *is* the parts.

Why is this an interesting idea? Among other virtues, it may support the claim that ontological commitment to composed entities is not really a further commitment, above and beyond commitment to the composers. As David Lewis puts it, Mereology is ontologically innocent. (LOOK HANDOUT)

You can undertake two commitments, once to object x and once to object y; or you could commit yourself to them all at once by committing yourself to the mereological fusion of x and y. It's the same commitment either way. So once you have committed to some things, commitment to objects composed of those things is not a *further* commitment.

A second virtue is that if composition as identity we may be able to explain why the part-whole relation behaves as it does, for example why it is transitive. We might be able to reduce the part-whole relation to something else. Perhaps for x to be a part of y is just for it to be the case that x and y compose y; which is to say, according to the thesis, that x and y just are y.

But on the face of it, the thesis that composition is identity seems wrong because there is a glaring difference between the composed thing, on the one hand, and the composers, on the other: namely, the composed thing is a single thing, and the composers are not a single thing; they are many things.

David Lewis, as I read him, actually gives up on Composition as Identity, and settles for the thesis that composition is *like* identity. He thus gives up, it seems to me, on the hope of proving the ontological innocence of mereology, and on the hope of explaining the part-whole relation in more basic terms. Let me introduce a further argument against composition as identity, adapted from the work of Byeong Yi. Yi's argument really gets at the core logical difficulty for CI.

2 Two arguments against CI

First, we need some names and things. Consider John, Paul, George, and Ringo. We will name the left half of John, Johnleft, and the right half Johnright. Similarly for Paulleft and Paulright.

An adaptation of Yi's argument goes like this:

(1)	J_l and J_r compose John.	Premise
(2)	Composition is identity.	Suppose for *reductio*
(3)	J_l and J_r = John	From (1) and (2)
(4)	John is one of John and Paul.	(logical truth)
(5)	John is one of J_l and J_r and Paul.	From (3) and (4)
(6)	Paul is composed of P_l and P_r.	Premise
(7)	Paul = P_l and P_r.	From (2) and (5)
(8)	John is one of J_l and J_r and P_l and P_r.	From (5) and (7)
(9)	Every one of J_l and J_r and P_l and P_r is a half-man.	Premise.
(10)	John is a half-man.	From (8) and (9)

Ted Sider gives an argument that if composition is identity, then plural quantification is in trouble. His argument turns on essentially the same issue as Yi's, but uses plural quantifiers and variables instead of names. My response to Yi's argument can be extended to Sider's as well; the way I deal with names and plural noun phrases extends fairly straightforwardly to plural variables under an assignment.

2.1 Collective plural predication

In a moment I will say how Yi's argument wrong. Before proceeding, however, I must draw your attention to a certain general aspect of plural language. Consider the sentence

> John, Paul, and George sang.

The sentence is ambiguous between two readings, commonly called the "distributive" and the "collective" readings. The distributive reading is brought out explicitly in the sentence

> John, Paul, and George each sang.

The collective reading, on the other hand, is brought out explicitly by the sentence

> John, Paul, and George sang together.

Note that the distributive reading makes the original sentence logically equivalent with

John sang and Paul sang and George sang.

Note that this does not logically imply the collective reading; if each of them sang, they may or may not have sang together.

So the distributive does not imply the collective. Does the collective imply the distributive? In this case, it seems that it does. If they sang together, then each of them sang. But this implication flows from the character of the verb; in general, the collective version of a plural predication *does not* imply the singular version.

Here is an example that shows this quite clearly.

P and $\mathsf{P} \rightarrow \mathsf{Q}$ entail Q.

Here, I am using the symbols P and $\mathsf{P} \rightarrow \mathsf{Q}$ and Q as names of symbolic sentences of a formal propositional language, and I mean to be speaking of tautological entailment. Here the collective reading of the sentence is true, but the distributive reading is false. It is not the case that *each of* P and $\mathsf{P} \rightarrow \mathsf{Q}$ entails Q. But the two together do.

The lesson we take away is that the two readings are generally, logically independent; neither implies the other. There may be logical connections that flow from the predicates: with the intransitive verb 'sang', the collective predication implies the distributive, and not vice-versa. With the relational predicate 'entails Q', the distributive implies the collective, but not vice-versa.

2.2 What goes wrong with the argument; first pass

Returning now to Yi's argument against CI, wee see an impotant ambiguity.

Notice that line (3) is not an English sentence. What English sentence might be a good translation of what is intended? Here are some candidates:

(3a) J_l and J_r are identical with John.
(3b) J_l and J_r are each identical with John.
(3c) J_l and J_r are, together (collectively), identical with John.

(3a) is a good translation, because it too is ambiguous; it could be read as either (3b) or (3c). (3b) is false, though if it were true, it would support the substitution step, the inference to line (5) of the argument. (3c) is true, I suggest, but does not support the inference to line (5).

3 Properties and Propositions

Of course I have to explain why, and I will. My two defenses give slightly different accounts of just what is going on here. But before I go on to the details, I need to put a little bit more in place.

I turn to a picture of properties, relations, and structured propositions that is compatible with the thesis that composition is identity.

Traditionally, we have thought of properties and relations as having a fixed arity, a fixed number of places. Properties have an arity of one, since they are had by a single thing, while most familiar relations have an arity of two: they relate one thing to another. This metaphysical picture fits nicely with the grammar of classical first-order languages, in which each predicative symbol has a fixed arity.

3.1 Multigrade relations?

Leonard and Goodman, in 1940, discussed what they called "multigrade" relations that seemed not to fit with this classical picture. Examples they give are "met with" and "are lodge-brothers". These appear to be satisfiable by arbitrarily many things, and hence, suggest that we should hold that the relation "met" is multigrade, meaning that it has no single arity, but is able to relate two things at once, or three things at once, or four things, etc.

3.2 Fixed arities for "multigrade" relations

This is not how I will understand "multigrade" relations, however. I think that the traditional conception is basically right: every property and relation has a fixed arity, a fixed number of blank spots; SINGS has one, ENTAIL has two. The multigrade nature of these relations consists in the fact that more than one thing can simultaneously fill any of the blank spots.

SEE HANDOUT FOR WHY

Reflection on all this quickly shows that any advocate of Composition as Identity should hold that all properties and relations are in principle "multigrade", even ones that before might not have seemed to have been. Since I am identical with some atoms, it must be that those atoms are jointly in the one place of the property of speaking. They, together, have the property of speaking.

Now the advocate of CI should not hold that everything that grammatically looks like a two-place relation is in fact a two-place relation, into each place of which any number of things can simultaneously go.

3.3 'is one of' is not a relational expression

Now not all expressions that look like they express two-place relations in my sense are not in fact to be thought of that way. Two important examples are "is one of" and "is (are) among".

For reasons that will become clear to you if you reflect on it, my defenses cannot regard these predicates as expressing relations, at least not typical fixed-arity relations. I think there are good arguments to be made for thinking that the expression "is one of" is a quantifer, on a par with "each of", but I don't have time to give them now.

3.4 Revisiting Yi's argument

But given this, we can now make a little more headway in examining the argument from Yi. The advocate of CI should say that line (3) is ambiguous; (3c) was the true reading, but it does not support the substitution step from (4) to (5). This is, in part, because (4) involves a quantifier—it does not simply express a relation between things.

Now I consider an objection: If (3c) is supposed to express an identity fact, and yet it does not support substitution of the terms asserted to be identical; don't we have a violation of the indiscernibility of identicals here?

Answer: No. The *expressions* 'Johnleft and Johnright' on the one hand, and 'John' on the other are not semantically identical. There are some contexts that are sensitive to the difference between the expressions, and hence they are not inter-substitutable *salva veritate*. Among those contexts are ones that involve the quantifier "is one of". Despite the semantic difference between these expressions, they can be used to express an identity fact, and you *can* substitute the relevant *entities*, one for the other, so to speak, in all facts. Identicals have all the same properties. What we get is that every property had by John—the man, not the expression—is had collectively by Johnleft and Johnright. Not "had by Johnleft and had by Johnright"! Rather, had by the two of them collectively. And conversely, anything that Johnleft and Johnright collectively do, John does. Any property they together possess, John possesses.

So the metaphysical version of Leibniz' Law is upheld. But why exactly do the linguistic versions fail—what is the semantic difference between the terms?

4 The two defenses

This question brings us to the details of the two defenses.

Besides giving their basic metaphysical theses, I will present the ways that the two defenses interpret a language with plural predication and quantification. I hope that by giving coherent accounts of a logically powerful formalization of a fragment of English, the defenses can ensure the logical coherence of Composition as Identity.

We will consider the semantics for the following formal language. It is an expansion of a standard first-order language, expanded to include plural terms and quantifiers.

$$\text{Terms: }\left\{\begin{array}{ll}\text{Singular terms:} & \left\{\begin{array}{ll}\text{Constants:} & \mathsf{a}, \mathsf{b}, \mathsf{c}, \ldots \\ \text{Variables:} & \mathsf{x}, \mathsf{y}, \mathsf{z}, \ldots\end{array}\right. \\ \\ \text{Plural terms:} & \left\{\begin{array}{ll}\text{Constants:} & \mathsf{aa}, \mathsf{bb}, \mathsf{cc}, \ldots \\ \text{Variables:} & \mathsf{xx}, \mathsf{yy}, \mathsf{zz}, \ldots \\ \text{Lists:} & \mathsf{a}+\mathsf{x}, \mathsf{a}+\mathsf{x}+\mathsf{xx}, \ldots\end{array}\right.\end{array}\right.$$

$$\text{Quantifiers: }\left\{\begin{array}{ll}\text{Absolute:} & \exists, \forall \text{ (can bind all variables)} \\ \\ \text{Relative:} & \left\{\begin{array}{ll}\forall(\_\_,\_\_)\text{ "Each of":} & \text{e.g., } \forall(\mathsf{x},\mathsf{xx})\mathsf{F}(\mathsf{x}) \\ & \text{"Each of } \mathsf{xx} \text{ is } \mathsf{F}\text{"} \\ \exists(\_\_,\_\_)\text{ "One of":} & \text{e.g.: } \exists(\mathsf{x},\mathsf{xx})\mathsf{F}(\mathsf{x}) \\ & \text{"One of } \mathsf{xx} \text{ is } \mathsf{F}\text{"}\end{array}\right.\end{array}\right.$$

(The first blank of the relative quantifier is to be filled with a singular variable, while the second can contain any term.)

Predicates:
- Non-logical: $\mathsf{F}, \mathsf{G}, \mathsf{R} \ldots$ (each with a fixed arity, n, saturated by any n terms to form an atomic wff)
- Logical: $=$ (2-place)

Propositional connectives: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$

Abbreviations:[1]		
	$\sqsubseteq$ (is part of)	e.g., $\mathsf{x} \sqsubseteq \mathsf{y}$ abbreviates $\mathsf{x} + \mathsf{y} = \mathsf{y}$
	$\leq$ (are among)	e.g., $\mathsf{xx} \leq \mathsf{yy}$ abbr. $\forall(\mathsf{x}, \mathsf{xx})\exists(\mathsf{y}, \mathsf{yy})\mathsf{x} = \mathsf{y}$
	$\equiv$	$\mathsf{xx} \equiv \mathsf{yy}$ abbr. $\mathsf{xx} \leq \mathsf{yy} \wedge \mathsf{yy} \leq \mathsf{xx}$

Examples:

John and Paul are each singing. Each of John and Paul is singing.	$\mathsf{F}(\mathsf{a}) \wedge \mathsf{F}(\mathsf{b})$ $\forall \mathsf{x}(\mathsf{x}, \mathsf{a} + \mathsf{b})\ \mathsf{F}(\mathsf{x})$
John and Paul are singing together.	$\mathsf{F}(\mathsf{a} + \mathsf{b})$
Some thing is singing.	$\exists \mathsf{x} \mathsf{F}(\mathsf{x})$
Some things (at least two) are singing together.	$\exists \mathsf{xx}(\exists(\mathsf{x}, \mathsf{xx})\exists(\mathsf{y}, \mathsf{xx})\mathsf{x} \neq \mathsf{y}\ \wedge \mathsf{F}(\mathsf{xx}))$
Each of xx entails z	$\forall(\mathsf{x}, \mathsf{xx})\mathsf{R}(\mathsf{x}, \mathsf{z})$
xx together entail z, but none of them (alone) entails z	$\mathsf{R}(\mathsf{xx}, \mathsf{z}) \wedge$ $\neg\exists(\mathsf{x}, \mathsf{xx})\mathsf{R}(\mathsf{x}, \mathsf{z})$
There are some critics who admire only one another	$\exists \mathsf{xx} \forall(\mathsf{x}, \mathsf{xx})(\mathsf{F}(\mathsf{x}) \wedge$ $\forall \mathsf{y}(\mathsf{R}(\mathsf{x}, \mathsf{y}) \rightarrow \exists(\mathsf{z}, \mathsf{xx})\mathsf{y} = \mathsf{z}))$

Note that when a plural term occurs directly as an argument of a predicate, including the identity predicate, we interpret the predication as collective. Distributive plural predication is represented through the use of the relative quantifiers. As a result, the traditional scheme of substitutivity of identicals will fail, for plural terms, though it will hold for singular terms. A weakened version will hold for plural terms, and unqualified substitutivity holds when plural terms are "equivalent". On the handout

[1] Here, and elsewhere, I omit the qualifications needed to avoid "variable collisions".

I've written down the valid versions of the substitutivity, but to see why they are valid will require a look at the semantics.

Valid versions of substitutivity:

$$\begin{array}{l} \mathsf{t} = \mathsf{s} \\ \phi(\mathsf{t}) \\ \hline \phi(\mathsf{s}) \end{array}$$

where t and s are singular terms, and $\phi(\mathsf{s})$ arises from $\phi(\mathsf{t})$ by replacing an occurrence of t in $\phi(\mathsf{t})$ with s.

$$\begin{array}{l} \mathsf{tt} = \mathsf{ss} \\ \phi(\mathsf{tt}) \\ \hline \phi(\mathsf{ss}) \end{array}$$

where tt and ss are plural (or singular) terms, and $\phi(\mathsf{ss})$ arises from $\phi(\mathsf{tt})$ by replacing an occurrence of tt in $\phi(\mathsf{tt})$ *that is not within a second argument of a relative quantifier* with ss.

$$\begin{array}{l} \mathsf{tt} \equiv \mathsf{ss} \\ \phi(\mathsf{tt}) \\ \hline \phi(\mathsf{ss}) \end{array}$$

where tt and ss are plural (or singular) terms, and $\phi(\mathsf{ss})$ arises from $\phi(\mathsf{tt})$ by replacing an occurrence of tt in $\phi(\mathsf{tt})$ with ss.

4.1 Sub-realist

I will call the first defense of CI the "Sub-realist" or "Atomist" defense. It assumes the principle of mereological atomism:

> **Atomism:** Everything is composed of atoms—i.e., of things that have nothing but themselves as parts.

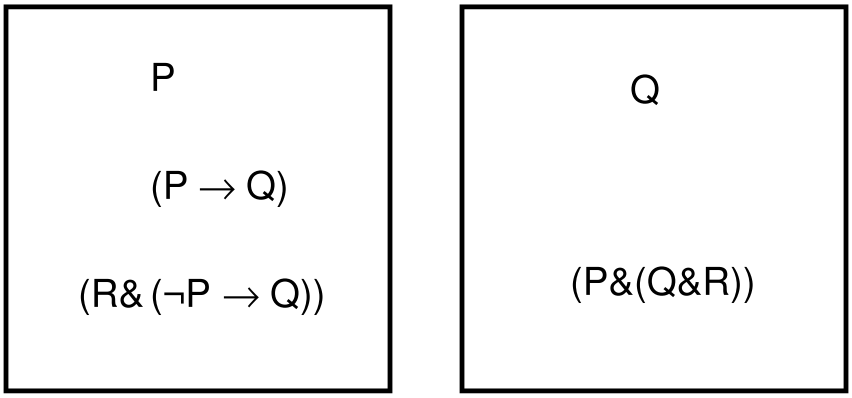
Further, it says that all facts that there are, including facts about composite objects, are really facts about mereological atoms. What appears on the surface of language and thought to be a single thing, is actually composed of many atoms, and indeed actually *is* those atoms. Not that it is each of those atoms, nor is it *one* of those atoms, either; it is identical with those atoms taken together.

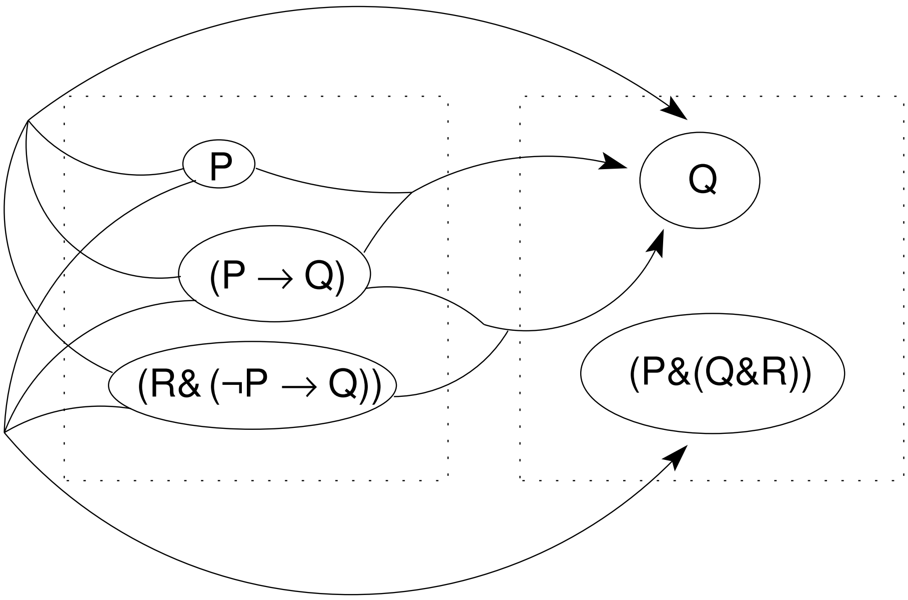
I now turn to the Sub-realist picture of language. The key notion for our us is the notion of an expression referring once, or twice, or some other fixed number of times. Singular terms will be terms that refer only once, while plural terms may refer multiple times.

Now it is important not to be misled into thinking that I really mean something temporal, when I say a plural expression refers multiple times.

What I mean to be counting are not events, but *holdings* of the relation of reference.

Here is a helpful analogy. Look at the sentences in the boxes, and ask yourself how many times the relation of entailment runs from sentences in the left box to sentences in the right box. That is, how many different instances are there in which sentences in the left box entail Q and in which entail $P \wedge (Q \wedge R)$?



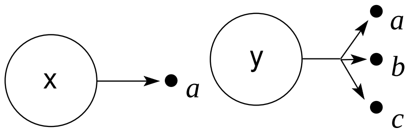


The second diagram uses branching arrows to indicate the bearings of the entailment relation, from left box to right box. Since the top two sentences in the left box together entail Q, there is an arrow from them to Q. Not two arrows; not an arrow from each of them to Q, as neither of them alone entails Q. Looking at the right box, notice that Q is born the entailment relation three times, while $P \wedge (Q \wedge R)$ is born it only once. This talk of bearing the relation some number of *times*, you see, has nothing essentially to do with time; it is a logical notion, having to do with the number of *instances* of something.

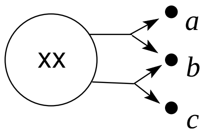
The semantics will use this notion extensively. The semantic difference between singular terms and plural terms will be that singular terms refer only once, while plural terms bear the reference relation many times over.

Sub-realist semantic values relative to an assignment relation:

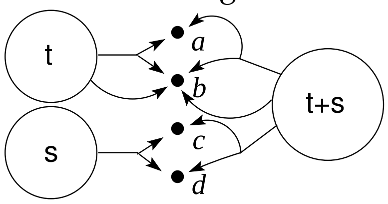
Singular terms t — t refers once, to some atom or some atoms collectively



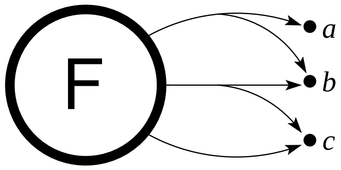
Plural terms t — t refers multiple times (or just once), each time to some atom or some atoms collectively



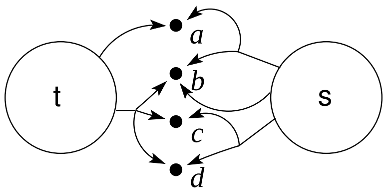
Lists t + s — if t refers to an atom or some atoms, so does t + s; if s refers to an atom or some atoms, so does t + s; and t + s refers to nothing else



Predicates F — for a one-place predicate F, F refers multiple times, each time to some atom or some atoms collectively



t = s — satisfied just in case every atom among some atoms referred to by t is among some atoms referred to by s, and vice-versa



Atomic wffs $\mathsf{F}(\mathsf{t})$	if t is singular, $\mathsf{F}(\mathsf{t})$ is satisfied just in case the atom or atoms collectively referred to by t are collectively referred to by F; if t is plural, consider the atoms (call them "t-ATOMS") such that: every atom that is one of some atoms that t refers to is one of t-ATOMS, and no other atoms are among t-ATOMS. $\mathsf{F}(\mathsf{t})$ is satisfied just in case F refers to, collectively, t-ATOMS.
$\exists(\mathsf{x},\mathsf{t})\phi(\mathsf{x})$	satisfied just in case there is some atom or are some atoms collectively referred to by t such that $\phi(\mathsf{x})$ is satisfied on the assignment relation just like the current one except that it assigns x to them.

The Sub-realist can claim to make good on the two promises of the thesis that composition is identity. First, the Sub-realist claims that his ontology is minimal. The only single things you find in it are atoms. There are no additional composite objects; there are just ways of referring to atoms that superficially appear to be reference to single composite objects. These terms don't refer to single atoms, of course: the name 'John', for example does not refer to a single atom; it refers to 10^{28} atoms, collectively. But it is singular in an important sense: it only refers to them; it refers to no other thing or things. The Sub-realist goes on to give semantics for all expressions, including plural terms, quantifiers and predicates, using only atoms, referred to in various sophisticated ways.

Second, the Sub-realist can really make good on the claim to explain mereological notions. The sentences that express the formalized theory of Atomistic Classical mereology become validities of his semantics. On the handout is a formalization of Atomistic Mereology, and I leave it as an exercise to prove that these sentences are all semantically valid—true in all models—with the Sub-realist semantics. I like to think of this as vindicating the claim that Mereology can be derived from definitions and logic, something like the way Frege wanted to derive arithmetic from definitions and logic—only we require that it be the logic of plurals, not mere first-order logic.

4.2 Realist

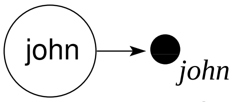
Now the Sub-realist might be said to "reduce" composite things to atoms, though not perhaps in the strictest sense of the word "reduce". The second defense I will consider is certainly not reductivist; it holds that what appears to be a single thing composed of many atoms is really a single thing. But it is *also, really* many things. On this view, there simply is no single absolute number metaphysically associated with an object or with some objects; typical objects will be one thing, and also two things, and also n things, for many different n. I call it a "Realist" defense, for it emphatically holds that in the finally analysis, every composite thing really is a single thing, a single thing that is identical with many things. The Sub-realist, on the other hand, holds that each apparently single thing is really a unique larger number of things. The Sub-realist says that you will not find, in the final analysis, any single composite things. Only atoms, taken singly, collectively, and multiply, but still only atoms, appear in the Sub-realist semantics.

The Realist says that when we refer to John, we refer to this one thing, John. Suppose that John is composed of 10^{28} mereological atoms. Then John is identical with those 10^{28} atoms, and when we refer to John, we refer to those atoms collectively. We also refer to Johnleft and Johnright, collectively. These are just many ways of describing the same fact. The Sub-realist, on the other hand, only holds that the name 'John' refers to the atoms collectively. Speaking strictly, there is no single thing that the name refers to. Still speaking strictly, the name is nonetheless "singular" in an important respect. It refers only once: to those atoms, collectively, and to nothing else.

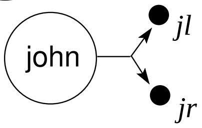
Thus the Realist would accept that all three of the following pictures are equally correct ways of picturing the same fact, that 'John' refers to John.

Three ways of looking at the semantics of 'john':

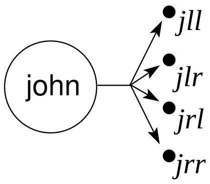
'john' refers to John



'john' refers to Johnleft and Johnright (collectively)



'john' refers to Johnleftleft, Johnleftright, Johnrightleft, and Johnrightright (collectively)



The Sub-realist would only accept, in his strict and philosophical mode, a picture that cannot be drawn here: one with 10^{28} arrowheads, but a single shaft emanating from the name.

The difference between the Realist and the Sub-realist is subtle. The Sub-realist is committed to Mereological Atomism: everything is composed of atoms, but atoms have no parts besides themselves. The Realist is not committed Atomism, but his view is compatible with Atomism. On the assumption of Atomism, they will agree on many things.

Let me try to bring out the issue one other way, by looking again at the diagnosis of the error in reasoning in the argument from Yi. Turn back to Yi's argument.

The Realist will say that the plural expression 'Johnleft and Johnright' refers twice. It refers to two things. It refers to Johnleft, and it also refers to Johnright. It does not refer to John. The expression 'John' refers once. It refers to one thing. Sentence (3c) is true, while sentence (3b) is false. (3c) is true because the things referred to by the expression 'Johnleft and Johnright' are, together, identical with the thing referred to by the expression 'John'. (3b) is false because the things referred to by the first expression are not each identical with the thing referred to by the second expression.

The Sub-realist will not exactly reject this story, but will want to say that it is underwritten by a deeper analysis of the situation, in which we only speak of reference, reference of various kinds, to atoms.

The Realist can give a much-simplified semantics for the formal language. For example, singular terms each refer once, to a single thing, and plural terms refer multiple times, each time to one thing. The clause for identity wffs is simple: $t = s$ is satisfied just in case the thing or things referred to by t, taken together, are identical with the thing or things referred to by s, taken together.

It is less clear the extent to which the Realist makes good on the two virtues of Composition as Identity. The Realist claims that ontological commitment to each of some things is, by virtue of identity facts, commitment to something that is identical with all of those things taken together. One might argue that this really is a further commitment; the Sub-realist, for example, does not appear to be committed to such a thing. Further, it is not clear that the Realist can give as complete an explanation of mereological notions as the Sub-realist. For the Sub-realist, mereological talk ultimately reduces to quantification, by way of his reductive semantics for mereological talk. The Realist has a different perspective. He can define the part-whole relation in terms of identity, but its behavior does not automatically flow from his semantics; it must be taken as primitive in the meta-language.