MATHEMATICS 331
ASSIGNMENT 7
Due: March 12, 2015

01° Let $\mathbf{V}$ be a finite dimensional linear space and let L_1 and L_2 be linear mappings carrying $\mathbf{V}$ to itself. Show that if $L_1 \cdot L_2 = I$ then $L_2 \cdot L_1 = I$. Of course, I is the identity mapping carrying $\mathbf{V}$ to itself.

02° Let $\mathcal{B}'$ and $\mathcal{B}''$ be bases for $\mathbf{V}$:

$$\mathcal{B}' : B'_1, B'_2, \ldots, B'_n; \quad \mathcal{B}'' : B''_1, B''_2, \ldots, B''_n$$

Let L be the linear mapping carrying $\mathbf{V}$ to itself, defined by the following relations:

$$L(B'_j) = B''_j$$

where j is any index $(1 \leq j \leq n)$. Show that $det(L) \neq 0$.

03° Let L be a linear mapping carrying $\mathbf{V}$ to itself, which meets the following condition:

$$L \cdot L = L$$

Show the characteristic values for L can only be 0 or 1.

04° Imagine that the following square arrays (aka matrices):

$$\begin{pmatrix} 6 & 3 \\ 3 & 2 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 1 & 1 \\ 0 & 6 & 3 \\ 0 & 3 & 2 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

derive from linear mappings. Calculate the determinants and the characteristic values. Which among them are invertible?

05° Let L', L, and L'' be linear mappings carrying $\mathbf{V}$ to itself. In particular, let L be invertible. Show that if:

$$L'' = L \cdot L' \cdot L^{-1}$$

then L' and L'' have the same characteristic values.