MATHEMATICS 322
ASSIGNMENT 9
Due: November 11, 2015

01$^\bullet$ Consider the following Autonomous First Order ODE on $\mathbf{R}^3$:

$$(\circ)\qquad \begin{array}{rcl} x^\circ & = & -\sigma x + \sigma y \\ y^\circ & = & rx - y - xz \\ z^\circ & = & -bz + xy \end{array}$$

where b, r, and s are positive numbers. We assume that $b + 1 < \sigma$. Show that the integral curves for $(\circ)$ are complete, that is, that they are defined for all time (past and future). To that end, let (a, b, c) be an initial condition for which:

$$(a, b, c) \neq (0, 0, 0)$$

and let γ be the maximum integral curve for $(\circ)$ passing through (a, b, c) at time 0:

$$\gamma(t) = (x(t), y(t), z(t)), \quad \gamma(0) = (a, b, c)$$

Let δ be the function defined as follows:

$$\delta(t) = x(t)^2 + y(t)^2 + z(t)^2$$

Show that there is a positive number λ such that:

$$|\delta^\circ(t)| \leq \lambda\delta(t)$$

Then show that:

$$max\{\delta(-t), \delta(t)\} \leq exp(\lambda t) \qquad (0 \leq t)$$

Finally, show that γ would be future bounded if future incomplete and would be past bounded if past incomplete, in either case a contradiction. To see the contradiction, review article 12° (*Escape to the Boundary*) in Chapter 1 of our "text."