MATHEMATICS 322
ASSIGNMENT 4
Due: September 30, 2015

01° Let J be the interval:

$$J = [0, \pi]$$

Let α and β be complex valued functions defined on J:

$$\alpha(\phi), \quad \beta(\phi) \qquad (\phi \in J)$$

for which:

$$\alpha(0) = 0 = \alpha(\pi), \quad \beta(0) = 0 = \beta(\pi)$$

Find all complex valued functions f defined on $J \times \mathbf{R}$:

$$f(t, \phi) \qquad ((t, \phi) \in J \times \mathbf{R})$$

for which:

(•) $f(0, \phi) = \alpha(\phi), \quad f_t(0, \phi) = \beta(\phi) \qquad (\phi \in J)$

(•) $f(t, 0) = 0 = f(t, \pi) \qquad (t \in \mathbf{R})$

(•) $f_{tt}(t, \phi) = f_{\phi\phi}(t, \phi) \qquad ((t, \phi) \in J \times \mathbf{R})$

To do so, apply the Fourier Sine Series:

$$f(t, \phi) = \sum_{k=1}^{\infty} c_k(t) sin(k\phi) \qquad ((t, \phi) \in J \times \mathbf{R})$$

where:

$$c_k(t) = \frac{2}{\pi} \int_0^{\pi} f(t, \phi) sin(k\phi) d\phi$$

Note that:

$$c_k(0) = \frac{2}{\pi} \int_0^{\pi} \alpha(\phi) sin(k\phi) d\phi$$

and:

$$c_k^{\circ}(0) = \frac{2}{\pi} \int_0^{\pi} \beta(\phi) sin(k\phi) d\phi$$

02° Recall the following relations governing Bessel's Functions:

$$e^{ixsin(\theta)} = \sum_{n=-\infty}^{+\infty} J_n(x)e^{in\theta} \qquad (x \in \mathbf{R},\ -\pi \leq \theta \leq \pi)$$

where:

$$J_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{ixsin(\theta)}e^{-in\theta}d\theta \qquad (n \in \mathbf{Z},\ x \in \mathbf{R})$$

Prove that:

$$J_n(u+v) = \sum_{m=-\infty}^{+\infty} J_m(u)J_{n-m}(v) \quad (n \in \mathbf{Z},\ u \in \mathbf{R},\ v \in \mathbf{R})$$