MATHEMATICS 322
ASSIGNMENT 2
Due: September 16, 2015

Wronskian

01$^\bullet$ Let h_1 and h_2 be solutions of the Second Order Linear ODE:

$$(\circ) \qquad f^{\circ\circ}(t) + p_1(t)f^{\circ}(t) + p_0(t)f(t) = 0 \qquad (t \in J)$$

Show that the wronskian:

$$w(t) = det \begin{pmatrix} h_1(t) & h_2(t) \\ h_1^{\circ}(t) & h_2^{\circ}(t) \end{pmatrix}$$

for h_1 and h_2 satisfies the First Order Linear ODE:

$$w^{\circ}(t) + p_1(t)w(t) = 0$$

Show that either: (1) for each t in J, $w(t) = 0$; or: (2) for each t in J, $w(t) \neq 0$.

Variation of Constants

02$^\bullet$ Find the (unique) solution f of the following (Inhomogeneous) Second Order Linear ODE:

$$(\circ) \qquad f^{\circ\circ}(t) + 4t^{-1}f^{\circ}(t) + 2t^{-2}f(t) = 1 \qquad (0 < t)$$

such that:

$$f(1) = 1, \;\; f^{\circ}(1) = 0$$

To that end, note that the functions:

$$h_1(t) = t^{-2}, \quad h_2(t) = t^{-1} \qquad (0 < t)$$

compose a basis of solutions for $(\circ)$.

Power Series'

03$^\bullet$ Consider the Second Order Linear ODE of Airy:

$$(\circ) \qquad f^{\circ\circ}(t) + tf(t) = 0 \qquad (t \in \mathbf{R})$$

Find a basis of solutions for $(\circ)$ in the form of power series':

$$f(t) = c_0 + c_1t + c_2t^2 + c_3t^3 + \;\cdots\; + \; c_jt^j + \;\cdots$$