MATHEMATICS 321
ASSIGNMENT 3
Due: September 23, 2015

01• Let $I \equiv [\,0,1\,]$ be the closed unit interval in $\mathbf{R}$. Let F be a continuous mapping carrying I to itself. Show that there must be at least one number x in I such that $F(x) = x$.

02• Let X_1 and X_2 be metric spaces and let F be a mapping carrying X_1 to X_2. Let Γ be the graph of F, that is, let Γ be the subset of $X_1 \times X_2$ defined as follows:

$$\Gamma = \{(x_1, x_2) \in X_1 \times X_2 \,:\, x_2 = F(x_1)\}$$

Show that if F is continuous then X_1 and Γ are homeomorphic.

03• Let X be a metric space, with metric d. One says that X is *connected* iff, for any subsets U and V of X, if U and V are open, if $U \cap V = \emptyset$, and if $U \cup V = X$ then $U = \emptyset$ or $V = \emptyset$. For instance, $\mathbf{R}^2$ (with the conventional metric) is connected. See the fourth problem in the first assignment. Again, let X be a metric space, with metric d. Let Y be a subset of X. Of course, both Y and $clo(Y)$ are themselves metric spaces, as one may restrict d to $Y \times Y$ and $clo(Y) \times clo(Y)$, respectively. Prove that if Y is connected then $clo(Y)$ is connected. Show by example that $clo(Y)$ may be connected while Y is not.

04• Let C be a circle in the Euclidean plane for which the radius is 1. Let P_1 be an equilateral triangle in the plane circumscribed about C and let C_1 be the circle in the plane circumscribed about P_1. Let P_2 be a square in the plane circumscribed about C_1 and let C_2 be the circle in the plane circumscribed about P_2. Let P_3 be a regular pentagon in the plane circumscribed about C_2 and let C_3 be the circle in the plane circumscribed about P_3. In general, for each positive integer j, let P_{j+1} be a regular $(j+2)$-gon in the plane circumscribed about C_j and let C_{j+1} be the circle in the plane circumscribed about P_{j+1}. Let X be the subset of $\mathbf{R}$ composed of the radii of the various circles. Find the supremum of X. See the figure.