**MATHEMATICS 321**
ASSIGNMENT 1
Due: September 9, 2015

01$^\bullet$ Let $X$ be a metric space, with metric $d$. Let $U$ and $V$ be subsets of $X$. Show that if $U$ and $V$ are open then $U \cap V$ is open. To that end, argue from first principles. That is, show that if $int(U) = U$ and $int(V) = V$ then $int(U \cap V) = U \cap V$.

02$^\bullet$ For the 5-adic metric space $\mathbf{Z}$, show that $N_{0.01}(0)$ consists of all $k$ in $\mathbf{Z}$ for which $|k|$ is divisible by $5^3$.

03$^\bullet$ Let $\bar{\mathbf{M}}_r(\mathbf{R})$ be the uniform metric space on $\mathbf{R}$, consisting of all bounded real valued functions defined on $\mathbf{R}$. Let $\bar{\mathbf{M}}_r^+(\mathbf{R})$ be the subset of $\bar{\mathbf{M}}_r(\mathbf{R})$ consisting of all functions $f$ in $\bar{\mathbf{M}}_r(\mathbf{R})$ such that, for each $t$ in $\mathbf{R}$, $0 < f(t)$. Show that $\bar{\mathbf{M}}_r^+(\mathbf{R})$ is neither open nor closed in $\bar{\mathbf{M}}_r(\mathbf{R})$.

04$^\bullet$ Let $U$ and $V$ be open subsets of $\mathbf{R}^2$ for which:

$$U \cap V = \emptyset \quad \text{and} \quad \mathbf{R}^2 = U \cup V$$

Show that either $U = \emptyset$ or $V = \emptyset$.

05$^\bullet$ Let $\rho$ be a number in the circle group $\mathbf{T}$. One says that $\rho$ is a *root of unity* iff there is some integer $k$ such that $\rho^k = 1$. Let $\sigma$ be a number in $\mathbf{T}$ which is not (!) a root of unity. Let $A$ be the subset of $\mathbf{C}$ consisting of all numbers of the form:

$$r\sigma^k \qquad (0 \leq r \leq 1, \;\; k \in \mathbf{Z})$$

where $r$ runs through the closed unit interval $[\,0, 1\,]$ in $\mathbf{R}$ and where $k$ runs through $\mathbf{Z}$. Describe the closure of $A$. Of course, we intend that $\mathbf{C}$ be regarded as a metric space, in the usual sense.