MATHEMATICS 212
ASSIGNMENT 10
Due: April 27, 2015

01° Let μ be the differential 1-form:

$$\mu = (y+z)dx + (x+z)dy + (x+y)dz$$

on $\mathbf{R}^3$. Verify that $d\mu = 0$. Find all differential 0-forms λ:

$$\lambda = \lambda(x, y, z)$$

on $\mathbf{R}^3$ such that:

$$d\lambda = \mu$$

To that end, introduce the *homotopy* H carrying $\mathbf{I} \times \mathbf{R}^3$ to $\mathbf{R}^3$, defined as follows:

$$H(t, x, y, z) = (tx, ty, tz) \qquad (0 \leq t \leq 1)$$

Compute:

$$\lambda = K(H^*(\mu))$$

as described in the lectures. Then find all differential 0-forms ω on $\mathbf{R}^3$ for which $d\omega = 0$. That done, you are finished. Why?

02° Let μ be the differential 2-form:

$$\mu = 3xy^2zdydz - y^3zdzdx + x^2y^2dxdy$$

on $\mathbf{R}^3$. Verify that $d\mu = 0$. Find all differential 1-forms λ:

$$\lambda = f(x, y, z)dx + g(x, y, z)dy + h(x, y, z)dz$$

on $\mathbf{R}^3$ such that:

$$d\lambda = \mu$$

To that end, introduce the *homotopy* H carrying $\mathbf{I} \times \mathbf{R}^3$ to $\mathbf{R}^3$, defined as follows:

$$H(t, x, y, z) = (tx, ty, tz) \qquad (0 \leq t \leq 1)$$

Compute:

$$\lambda = K(H^*(\mu))$$

as described in the lectures. Then find all differential 1-forms ω on $\mathbf{R}^3$ for which $d\omega = 0$. That done, you are finished. Why?

03° Let Ω be the positive octant in $\mathbf{R}^3$:

$$0 < x,\ 0 < y,\ 0 < z$$

Let μ be the differential 1-form on Ω, defined as follows:

$$\mu = x\frac{1 - log(r)}{r^3}dx + y\frac{1 - log(r)}{r^3}dy + z\frac{1 - log(r)}{r^3}dz$$

As usual, $r^2 = x^2 + y^2 + z^2$. Verify that $d\mu = 0$. Find all differential 0-forms λ on Ω such that:

$$d\lambda = \mu$$

You might want to apply the homotopy H carrying $I \times \Omega$ to Ω, defined as follows:

$$H(t, x, y, z) = (1 - t)(1, 1, 1) + t(x, y, z) \qquad (0 \leq t \leq 1,\ (x, y, z) \in \Omega)$$

Compute:

$$\lambda = K(H^*(\mu))$$

as described in the lectures. Then find all differential 0-forms ω on $\mathbf{R}^3$ for which $d\omega = 0$. That done, you are finished. Why?

04° Let μ be the differential 2-form on $\mathbf{R}^3$, defined as follows:

$$\mu = (xy - xz)dydz + (yz - yx)dzdx + (zx - zy)dxdy$$

Verify that $d\mu = 0$. Find all differential 1-forms λ:

$$\lambda = f(x, y, z)dx + g(x, y, z)dy + h(x, y, z)dz$$

on $\mathbf{R}^3$ such that:

$$d\lambda = \mu$$

Naturally, you will want to apply the homotopy H carrying $I \times \mathbf{R}^3$ to $\mathbf{R}^3$, defined as follows:

$$H(t, x, y, z) = (tx, ty, tz) \qquad (0 \leq t \leq 1,\ (x, y, z) \in \mathbf{R}^3)$$

Compute:

$$\lambda = K(H^*(\mu))$$

as described in the lectures. Then find all differential 1-forms ω on $\mathbf{R}^3$ for which $d\omega = 0$. That done, you are finished. Why?