Threshold Dynamical Systems

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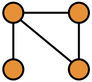
What is a Threshold Dynamical System?

- Implementation of dynamical systems on graphs.



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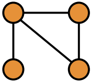

- Similar to cellular automata.

Specifics

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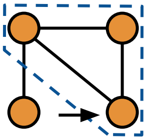
- Associated with each vertex is also a *threshold*.
- The state of a vertex is governed by a *threshold function*.

Threshold Functions

- The 1-neighborhood of a vertex v is all vertices connected to v by one edge or fewer.



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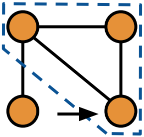
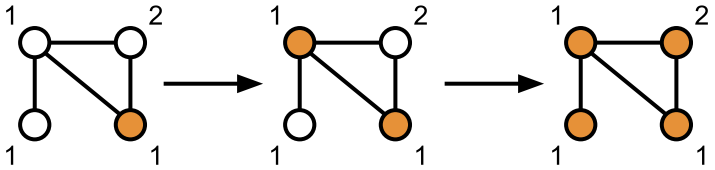


- Let k be the threshold of vertex v; then v will turn on if and only if the number of on vertices in v's 1-neighborhood exceeds k.

Example: Simultaneous Update



Use of TDS

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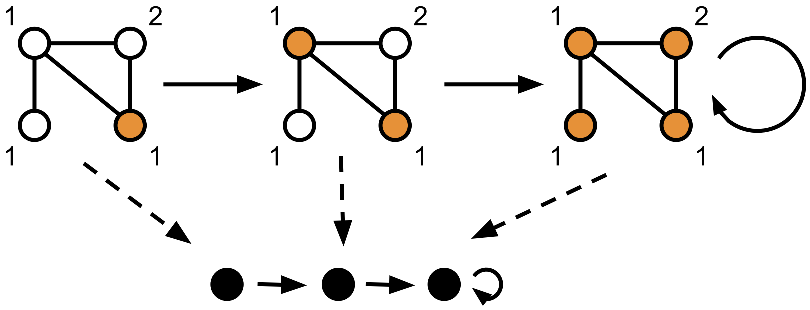
- Social contagions: rumors, smoking
- Biological contagions: malaria, acquired viral load

State Space



What Do We Want To Know?

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- States space informs us of overall population ‘infected’

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- This can represent changing immunity.
- Threshold of v is increased when it is turned on, or its threshold decreased when it is turned off, or both. Thresholds are bounded.

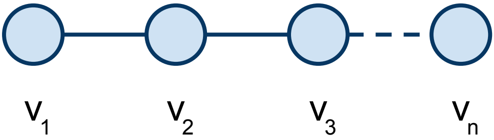
No Limit Cycles

It is already known that non-evolving threshold systems do not have limit cycles. We extended this result to evolving thresholds. This depends on the boundedness of the thresholds.

Fixed Points

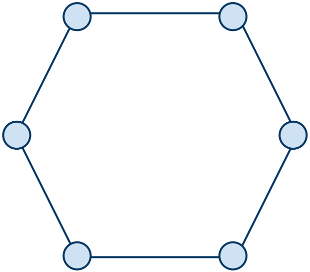
The number of fixed points in the state space remains the same among increasing, decreasing, and mixed evolving thresholds and regardless of update order. Notice that once a point in state space is fixed, it remains fixed in all these different systems.

Path



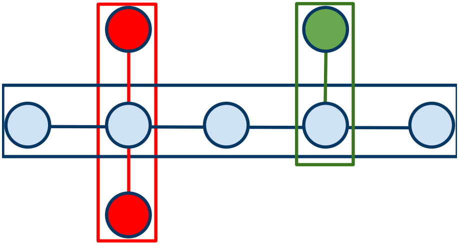
Number of fixed points on a path of length n is $\text{Fib}(3n-1)$. This is the same for increasing, decreasing, and mixed evolving threshold systems. This was computed through recursion.

Circle



Number of fixed points on a circle with n vertices is $2+\mathrm{Luc}(3n-1)$.
This was computed through the matrix transfer method.

Tree



Because the number of vertices does not determine a unique tree, we did not attempt to find a formula based on n. Instead, we found an algorithm to compute the number of fixed points in state space by considering the tree as an intersection of paths.

Further Research

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- Other types of evolving threshold functions.
- Graph classes that more accurately represent social networks.
- Stability of limit sets.

Thanks

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