Designing a Halfpipe for Advanced Snurfers

One of the 2011 COMAP problems was optimizing the design of a halfpipe for professional snowboarders. We used a variety of methods and models to answer this problem. This presentation is a summary of how we used the 96 hours to determine, through mathematical models, the optimal shape of a halfpipe.

2011 COMAP Team:

-Alexander Macavoy
-Jadon Herron
-Rachel Burton

Contents

- What is COMAP?
- Our Approach
  - Two Dimensional Energy Model
  - Two Dimensional Force Model
  - Three Dimensional Force Model
- Reality Check
- Solutions and Conclusion

What is COMAP?

- COMAP (Consortium On Mathematics And its Applications)
- 96 hour long worldwide mathematics marathon
- End result is a carefully worded technical article
- How can you place?
  - Unsuccessful Participant
  - Successful Participant
  - Honorable Mention
  - Meritorious
  - Finalist
  - Outstanding

What kind of people do math for 96 hours?

Alexander Macavoy
Major: Mathematics
Minor: Geology, German
Current location: Tübingen, Germany
Why COMAP? It was mostly curiosity and what better way to celebrate 21 birthday?

Rachel Burton
Major: Mathematics
Minor: Physics, Biology, Chemistry
Why COMAP? I missed Physics and I was persuaded by Alex.

Jadon Herron
Major: Civil Engineering
Why COMAP? I have always like competition and I really enjoyed my physics and engineering classes.

Before the Contest Began…

- Assigned roles and discuss schedule
  - Alexander Macavoy-Computer Programmer
  - Jadon Herron-Math Guy
  - Rachel Burton-Writer/Taskmaster

- Practice writing abstracts based on previous year's problems

- Reading through previous Outstanding papers

2011 COMAP Questions

Question A –Continuous Problem

-Determine the shape of a "halfpipe" snowboard course to maximize the production of "vertical air" by a skilled snowboarder.

Question B-Discrete Problem

-Determine the minimum number of VHF radio spectrum repeaters necessary to accommodate 1,000 simultaneous users in a 40-mile radius.

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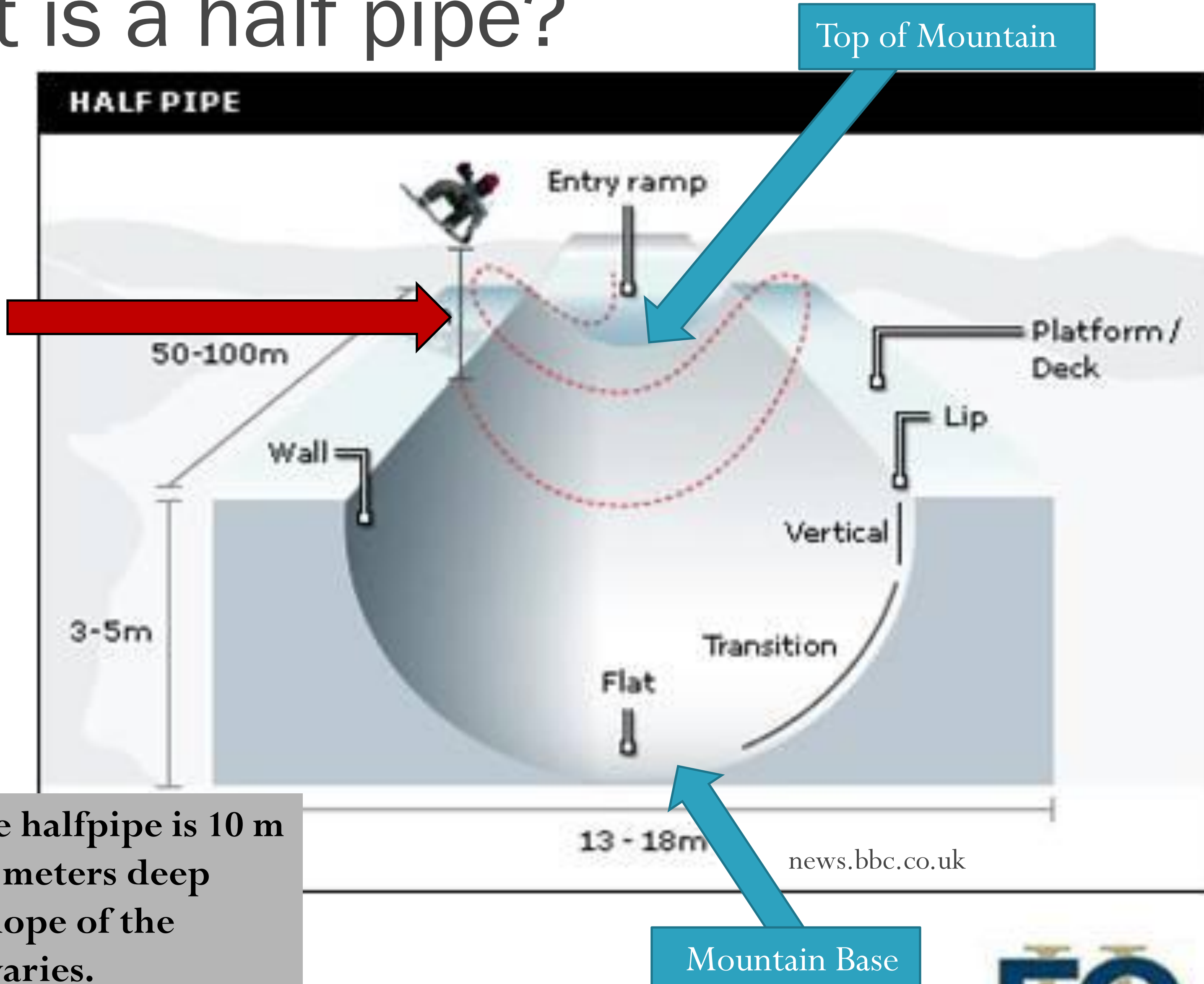
Our Approach

- Brainstorms! Brainstorm! Brainstorm!
- Three Dimensional Models
- Shaun White YouTube videos!
- Snowboard lingo
  - What is does a halfpipe look like?
  - What is 'vertical air'

What is a half pipe?



The average halfpipe is 10 m wide and 7 meters deep while the slope of the mountain varies.

Our Three Models

1. **Energy Model-**Conservation of Energy
   - Does not allow for any air resistance, friction, or centripetal acceleration
   - Cross section of the halfpipe

2. **Two dimensional Force Model-**sum of the y,z forces
   - Allows for the addition of work, air resistance, friction, and centripetal acceleration
   - Cross section of the halfpipe

3. **Three dimensional Force Model-**sum of the x,y,z forces
   - Allows for the addition of work, air resistance, friction, centripetal acceleration
   - Three dimensional!

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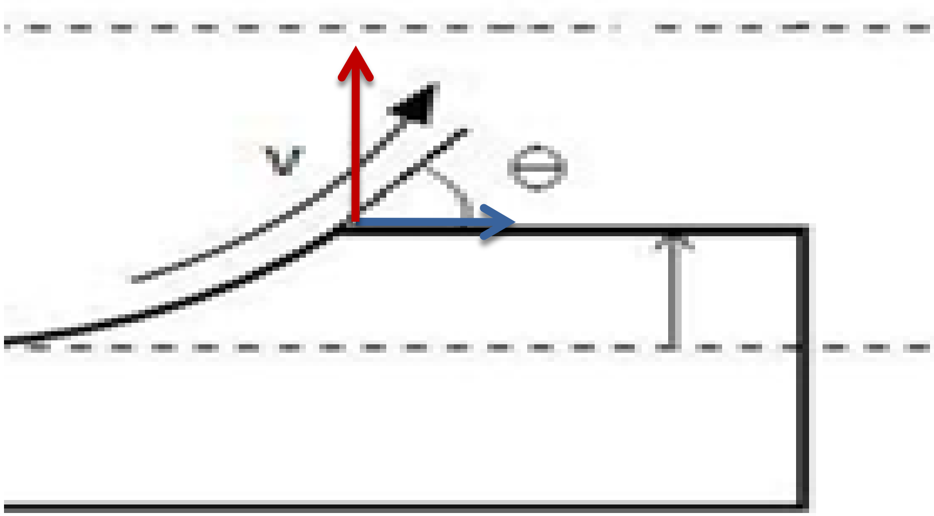
Two Dimensional Energy Model



$$KE_o + PE_o = KE + PE$$

$$v_y = \sin\theta\left(\sqrt{(2g(h_i - h_f) + 2W)}\right)$$

$$v_x = \cos\theta\left(\sqrt{(2g(h_i - h_f) + 2W)}\right)$$

- Developed a model representing a two dimensional or "basic" halfpipe.
- Theoretical model to help describe a simplistic view at what vertical air and factors like work would mean.
- There were no computer simulations for this model

We want to maximize this velocity for optimal vertical air

The force model approach

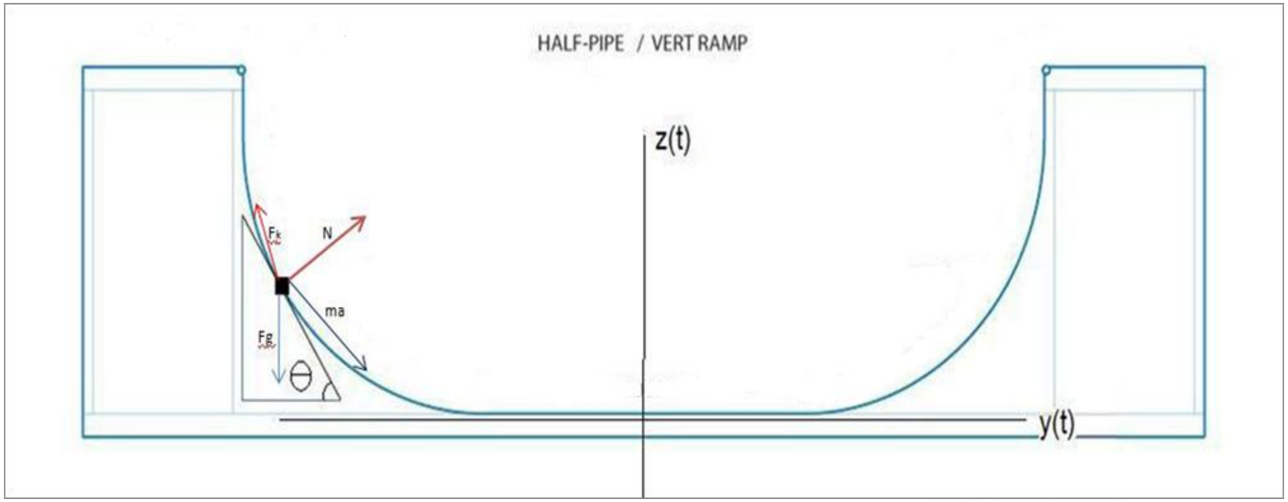
- Given a function of the shape of the halfpipe model the path of the snowboarder
- Test different shapes of the halfpipe using the same force equations
- Analyze the data to determine the best shape

Two dimensional Force Model

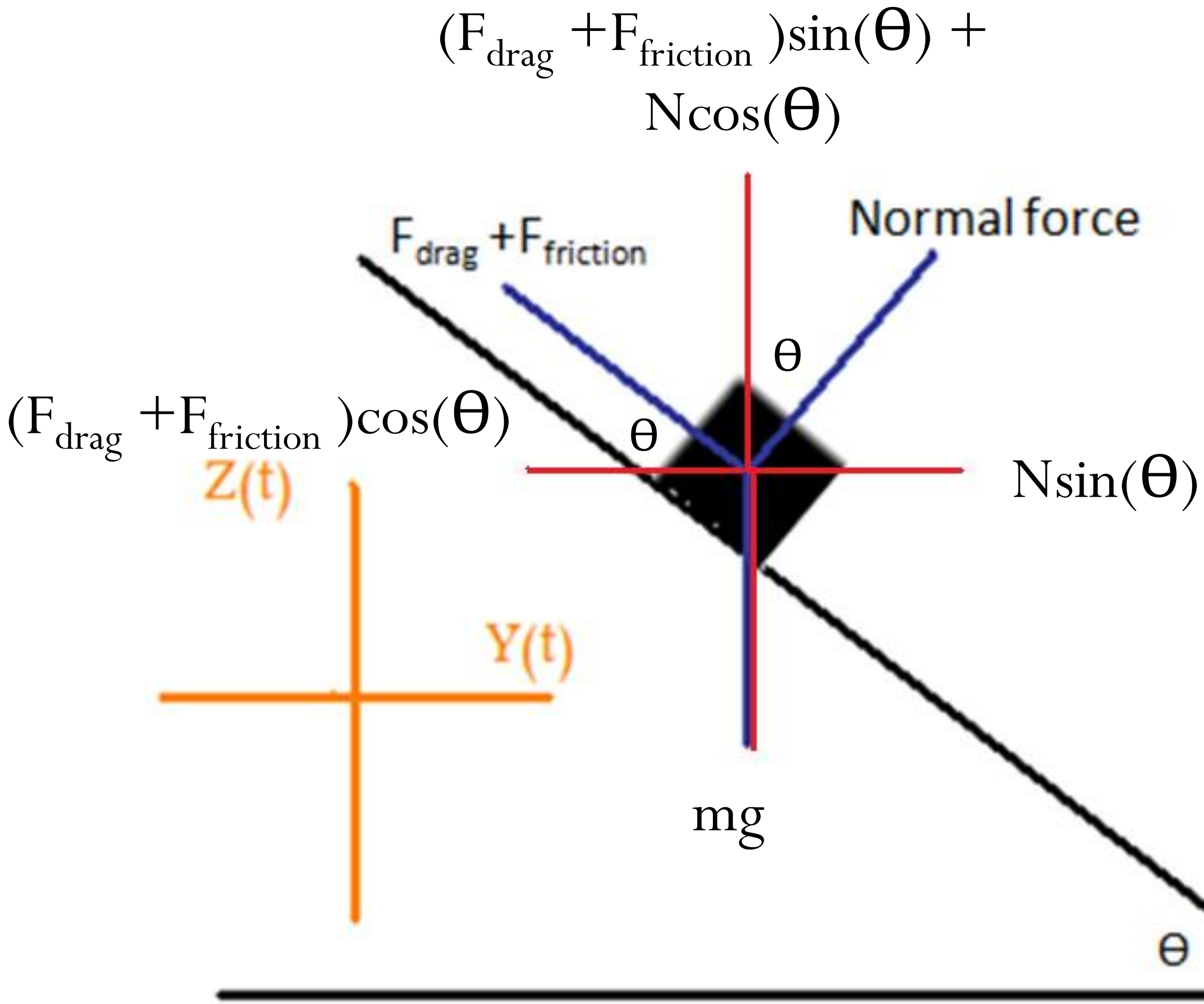
Using Newton's Second Law of Motion
F=ma

For the 2D model the forces are summed in the two directions giving two second order differential equations.

- $F_z = ma_z$
- $F_y = ma_y$



Componentizing the Forces

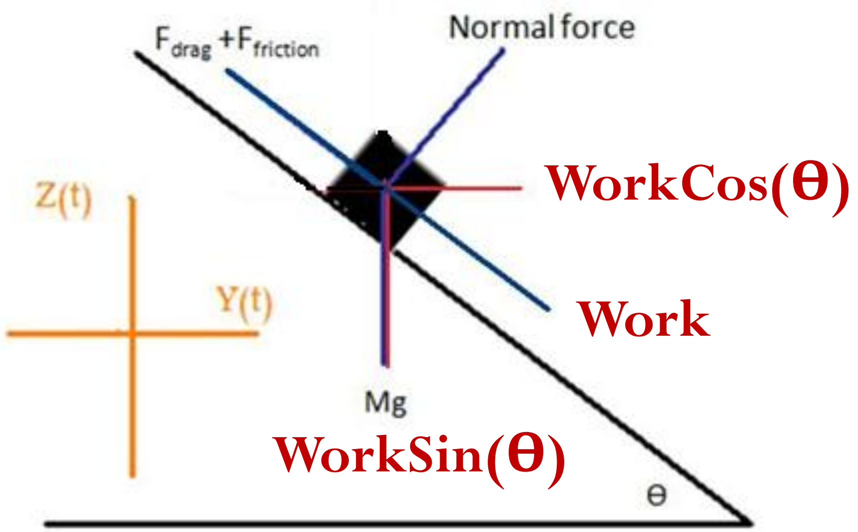


$\theta=\theta(z,y)$

The Addition of Work

•There is work being added to the system from the snowboarder

•To model this force we added a constant force in the direction of travel of the snowboarder



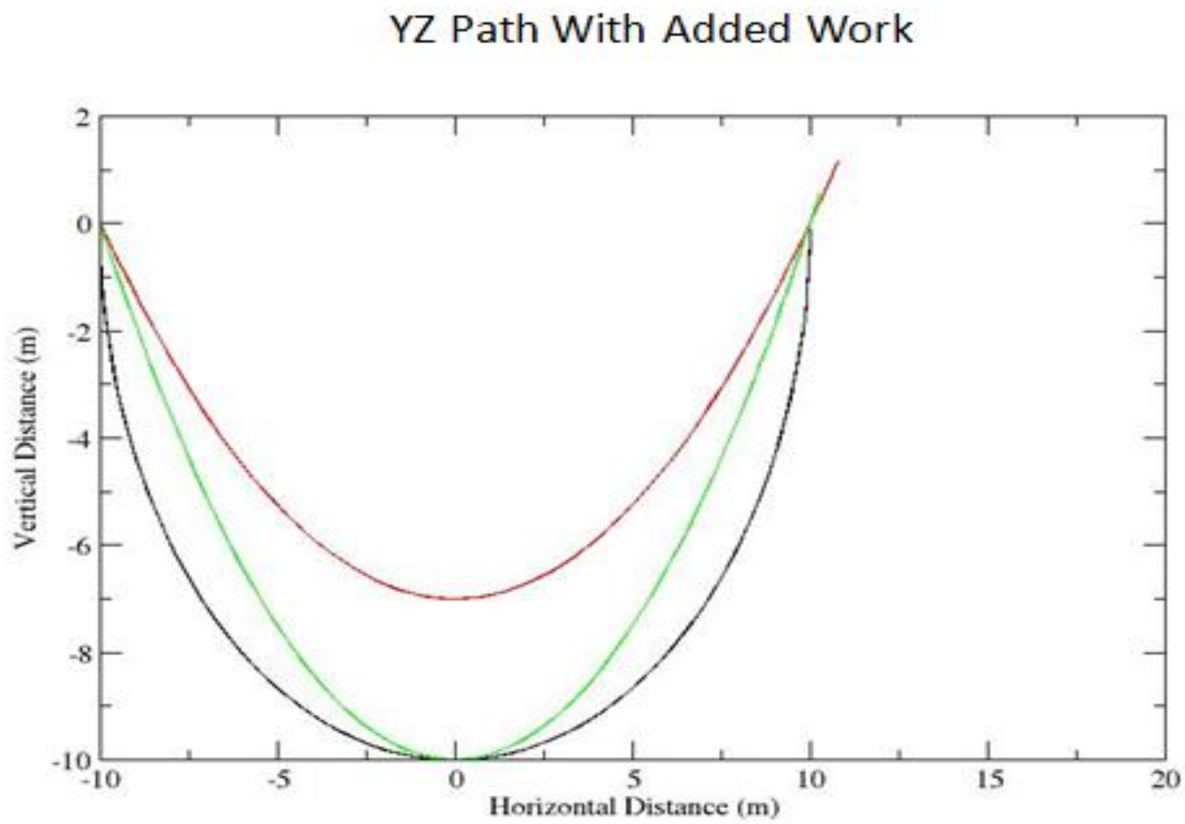
Two Dimensional Model

1. $dv_y/dt = N\sin(\theta) - (F_{drag} + F_{friction})\cos(\theta) + (work)\cos(\theta)$
2. $dy/dt = v_y$
3. $dv_z/dt = -g + N\cos(\theta) + (F_{drag} + F_{friction})\sin(\theta) - (work)\sin(\theta)$
4. $dz/dt = v_z$

- $N = g\cos(\theta) - \frac{(v_y^2 + v_z^2)}{\rho}$
- $F_{drag} = \frac{.5\, \rho_{air}\, a c_d\, (v_y^2 + v_z^2)}{m}$
- $F_{friction} = \mu N$
- $\rho(z,y)$ = radius of curvature $= \frac{(1+(dz/dy)^2)^{3/2}}{d^2 z/dy^2}$
- Work = constant force
- $\Theta = \arctan(dz/dy)$

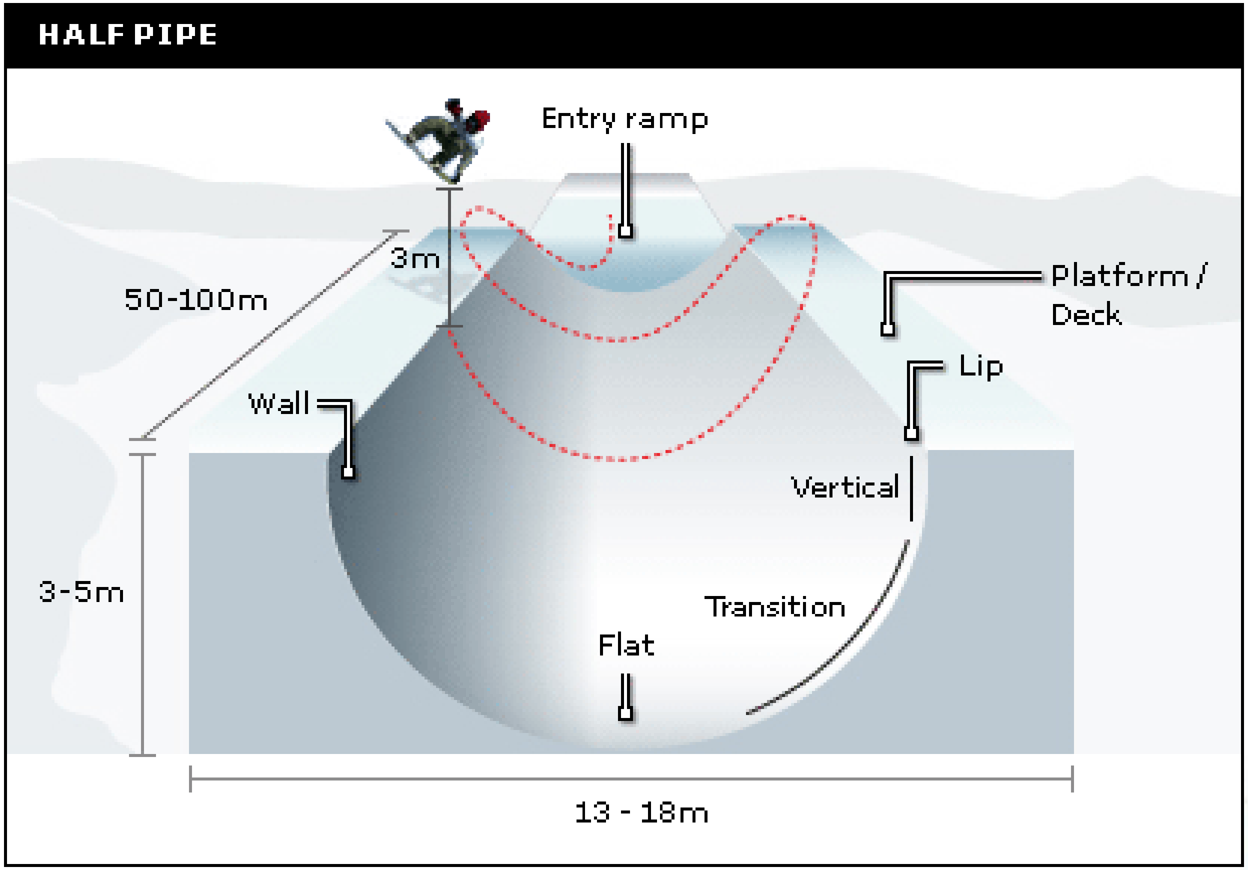
Computer Simulation

•Next we used the fourth order Runge-Kutta approximation simulated with c++ to solve the four differential equations

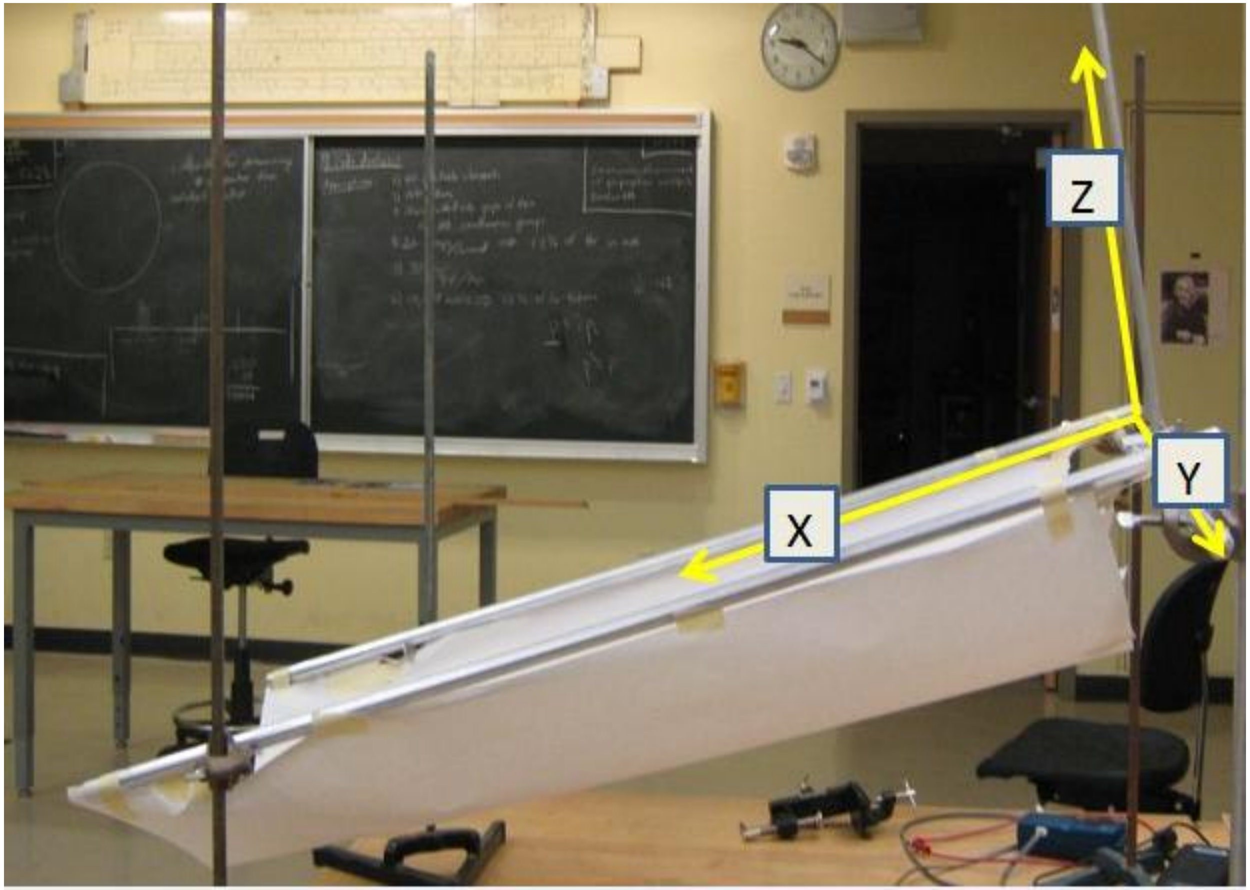


Three dimensional Force Model

- The third dimension was added



To find the equation for three dimensions we shifted the x, y, z axis



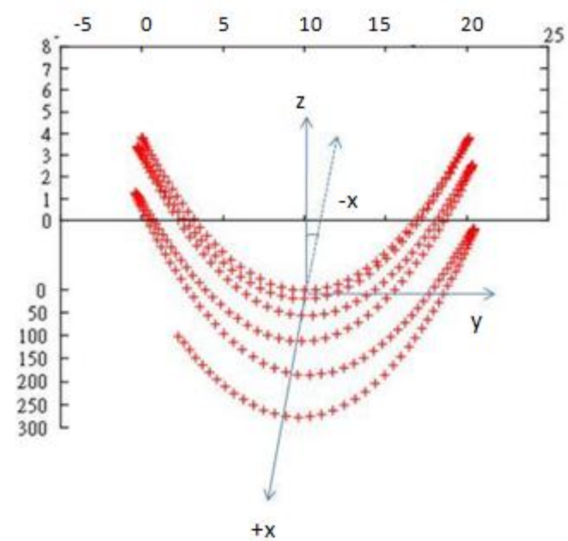
Three Dimensional Model

1. $dv_x /dt = g \sin(\phi)\, ma - g \cos(\phi)\, \mu - F_{drag\ x}$
2. $dv_y /dt = N \sin(\Theta) - (F_{drag\ yz} + F_{friction}) \cos(\Theta) + (work) \cos(\Theta)$
3. $dv_z /dt = -g \cos(\phi) + N \cos(\Theta) + (F_{drag\ yz} + F_{friction}) \sin(\Theta) - (work) \sin(\Theta)$
4. $dx/dt = v_x$
5. $dy/dt = v_y$
6. $dz/dt = v_z$

The big change from 2D

• $N = g \cos(\phi) \cos(\Theta) - \dfrac{(v_y^2 + v_z^2)}{\rho}$

• Φ is a set angle and is the slope of the halfpipe on the hill



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Computer Program Fidelity

- Testing the 2D model
  1. Set the frictional and work force to zero
  2. Set the work equal to zero

- Testing the 3D model
  1. Set ϕ, work, and the frictional forces equal to zero
  2. Set ϕ equal to zero
  3. Running the full 3D model and comparing the velocities with the 2D model

So...do these models work?

-Is the physics sound?

-Does the math match in the program like it does on paper?

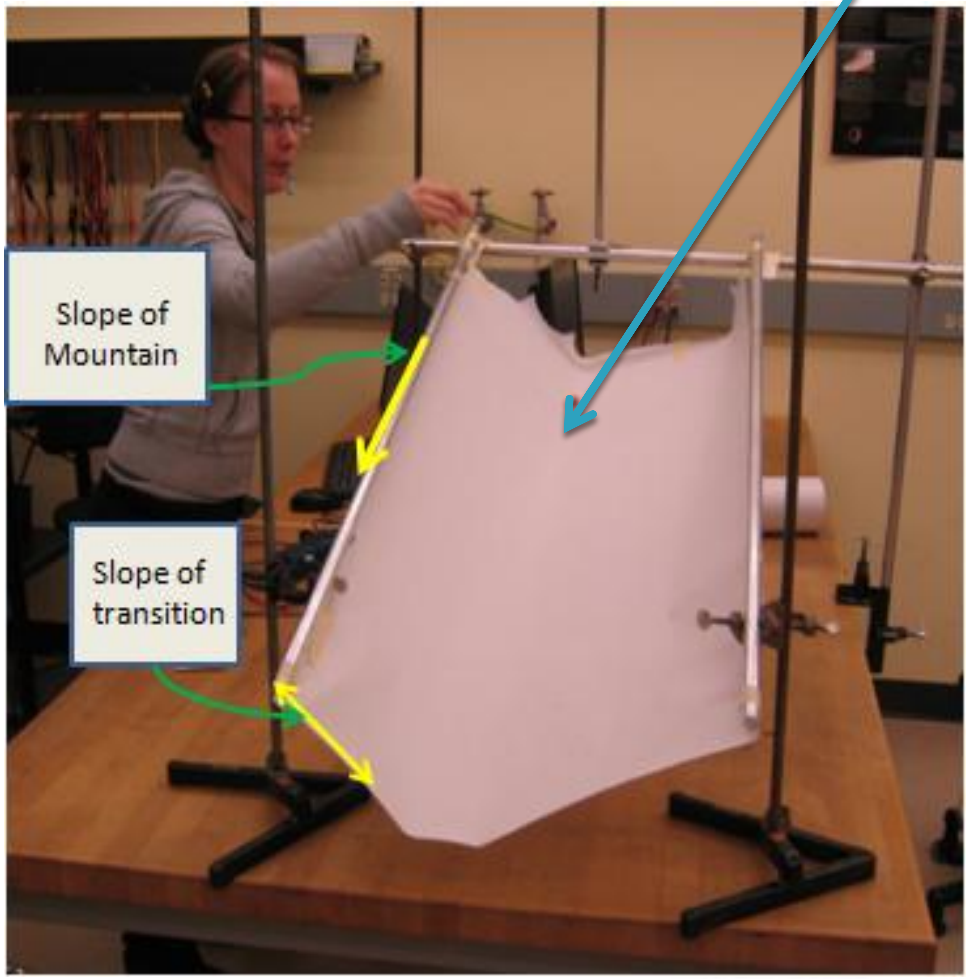
-Did we answer the exact question?

-Does the paper reflect the hard work?

Testing Methods

"Snowboarder" =Marble

- •Physical Models:
  glue guns, butcher paper, paper clips, weight stands ect.
- •Computer Modeling:
  4th order Runge Kutta in C++
- •Free Body Diagrams
- •Shaun White videos (available on YouTube)
- •Suggested field testing



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Comparing 3 Halfpipe Functions and 3 Mountain Slopes

Parabola 1: $z(y)=0.1y^2-10$

cycle	Φ	V_y (m/s)	V_z(m/s)	V_x (m/s)
1	25	0	0	14.857514
2	25	0	0	29.20664
3	25	0	0	43.49515
1	35	0.472654	0.94674	21.10855
2	35	0.646961	1.297154	42.41051
1	45	1.279261	2.561209	26.495354
2	45	1.639812	3.306971	55.532605

Best Halfpipe function

Best Practical Mountain Slope

Parabola 2: $z(y)=0.07y^2-7$

cycle	Φ	V_y (m/s)	V_z(m/s)	V_x (m/s)
1	25	1.277128	1.802008	13.51889
[illegible]	25	1.860634	2.626752	27.67469
3	25	2.389199	3.345365	41.7776
1	35	1.709591	2.413389	19.2354
2	35	2.353462	3.34072	40.26938
1	45	2.148841	3.034146	24.92837
2	45	2.98102	4.231989	53.61694

circle: $z(y)=\sqrt{100-y^2}$

cycle	Φ	V_y (m/s)	V_z(m/s)	V_x (m/s)
1	25	0	0	14.634575
2	25	0	0	28.76926
3	25	0	0	42.636571
1	35	0.06168	1.013371	20.796595
2	35	0.07151	1.25022	41.799215
1	45	0.128915	2.572859	26.495354

Optimal Vertical air

Our Solution

By comparing the data from three different functions and three different angles we were able to determine the best halfpipe for both an average snowboarder and an advanced snowboarder.

- Practical halfpipe:

$$z(y) = 0.07y^2 - 7$$

Slope of 25 °

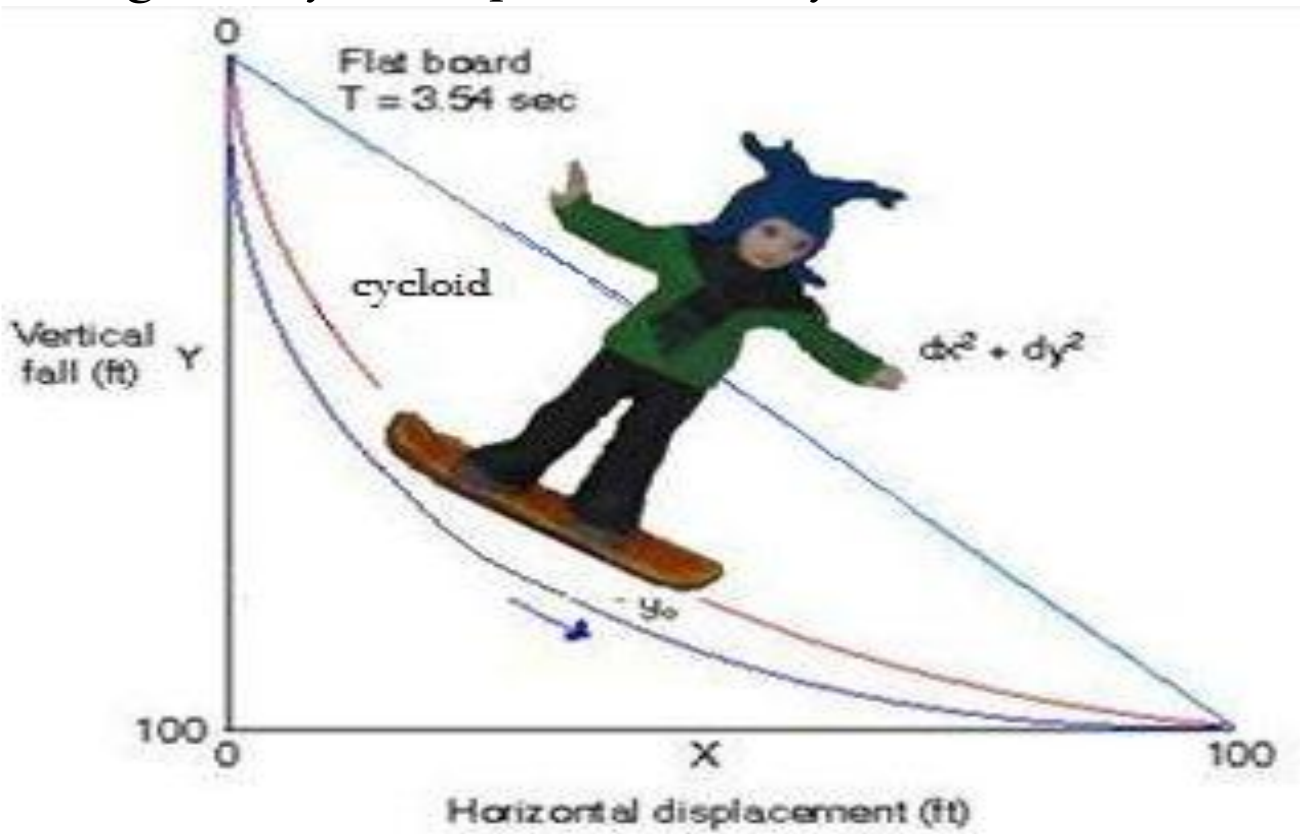
- Advanced halfpipe:

$$z(y) = 0.07y^2 - 7$$

Slope of 45°

If we only had more time…

A possible future model could be a half-pipe down the slope given by the equation of a cycloid.



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Special Thanks

- **Dr. Tovar-** Thank you for all those C++ programs you assigned, for those long term papers that took up the entire dead week, for spending the extra time to coach us, and for persuading us to try.
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- **Reed College-**For giving us the opportunity to present here.
- **EOU Math Club**-getting us here and supporting us!
- **Shaun White** for posting all your videos on YouTube and providing necessary information!

Just in Case…