Alternating Integrals

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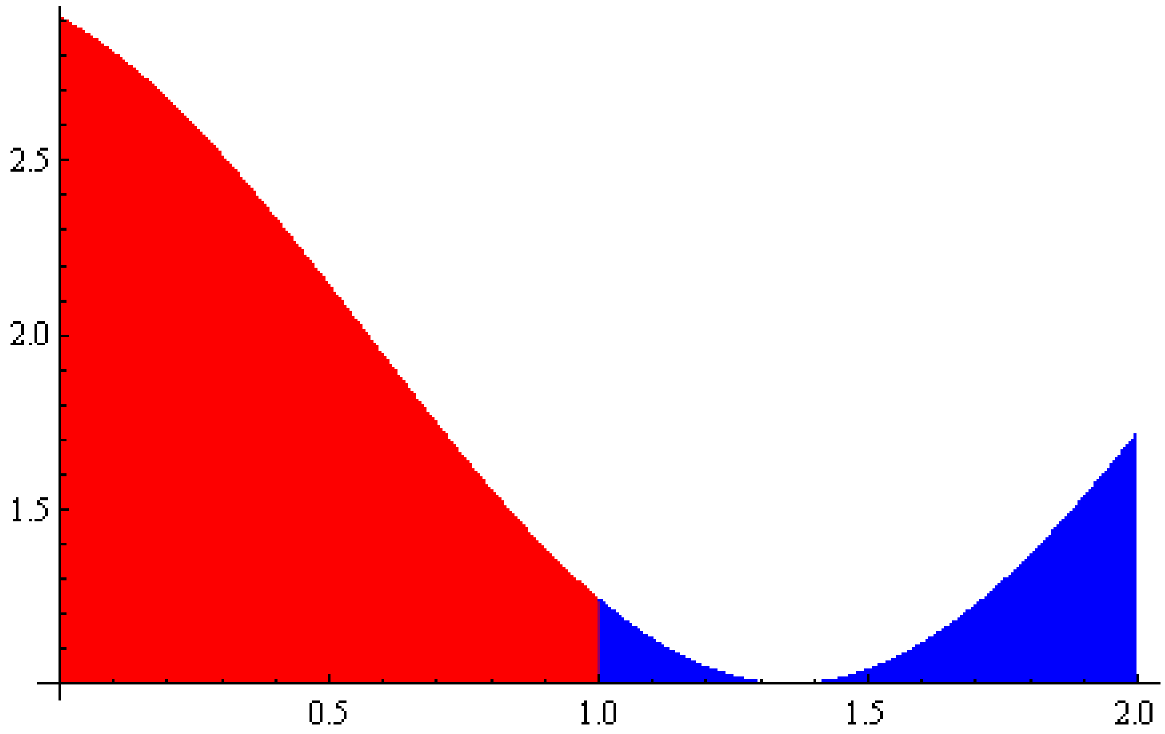
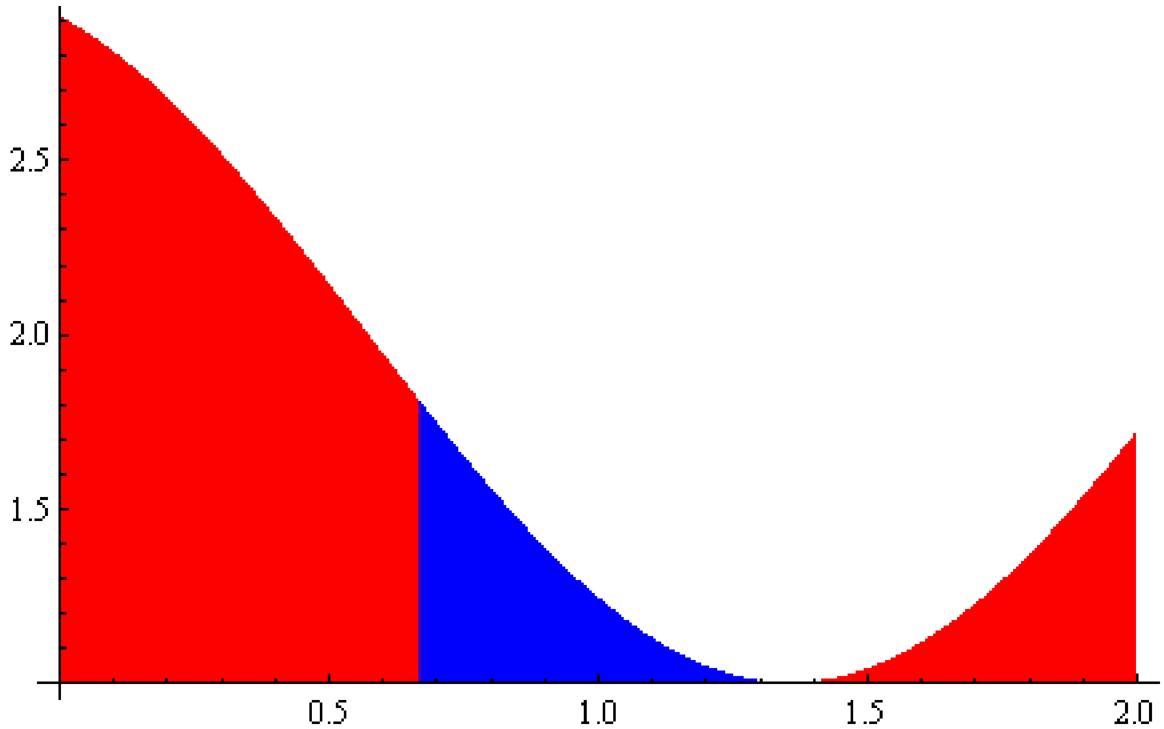
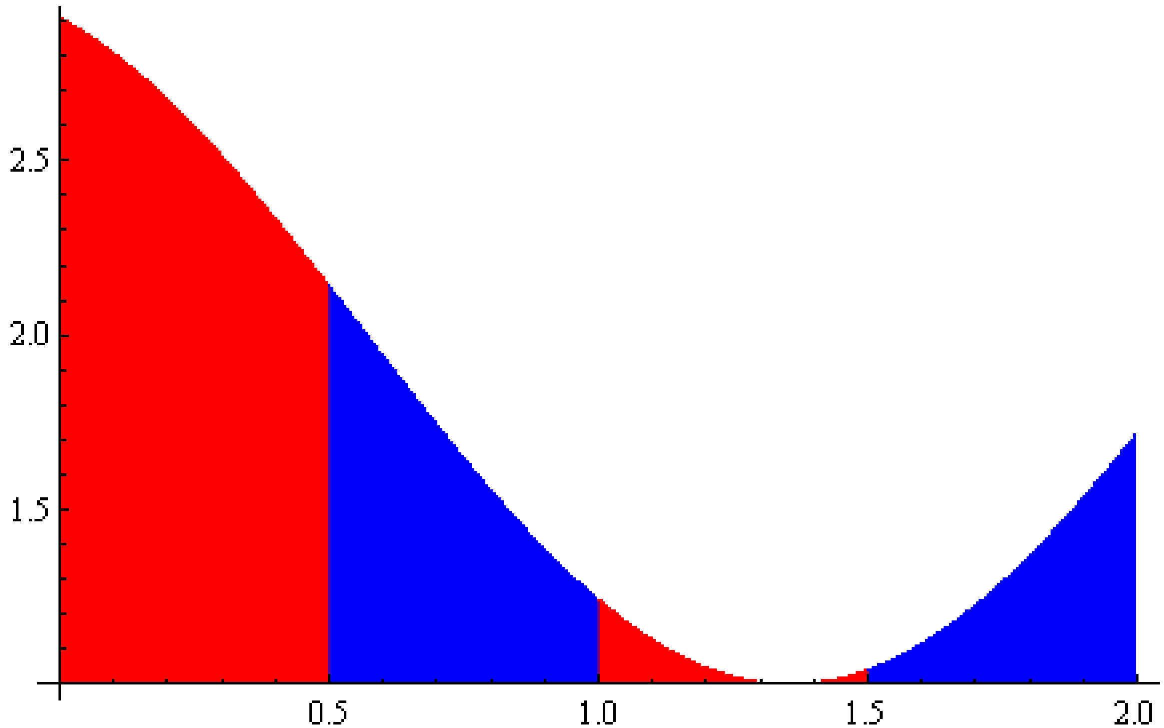
2 Intervals



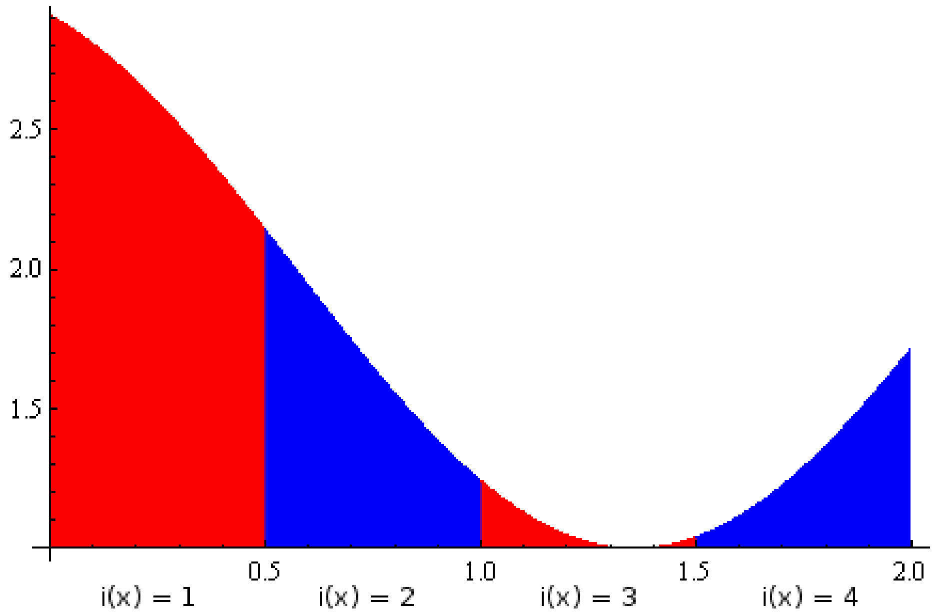
3 Intervals



4 intervals



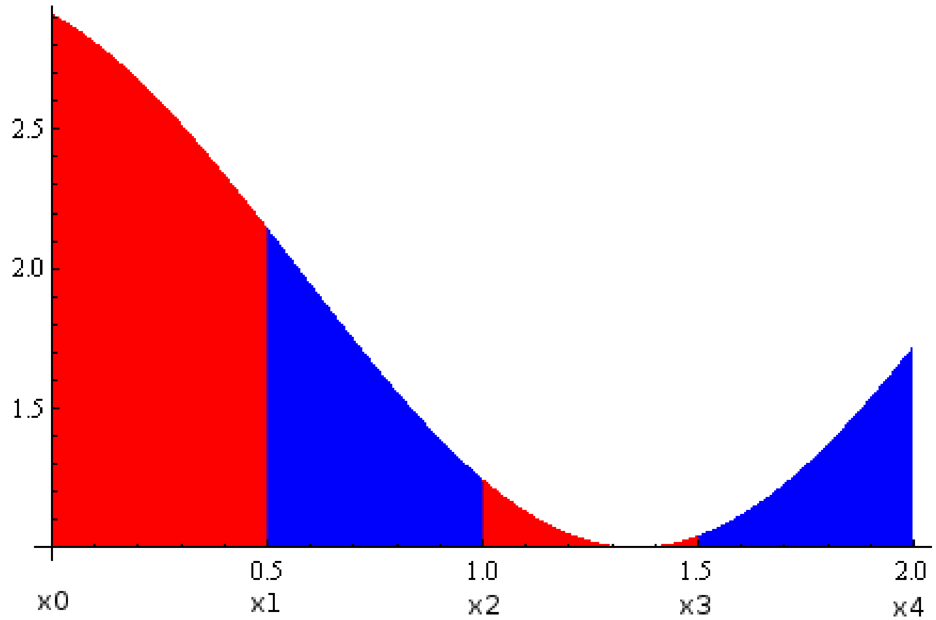
A Formula!



$$s_f(n)\Big|_a^b = \int_a^b (-1)^{i(x)} f(x)\,dx$$

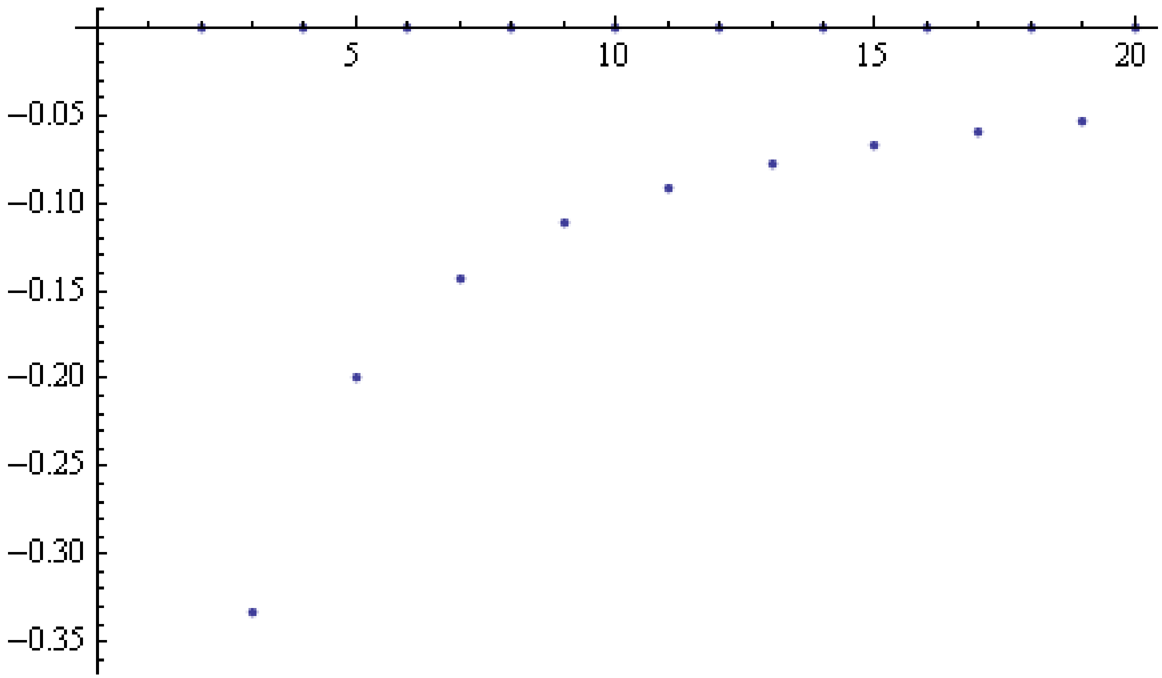
where $i(x) = \left\lfloor \frac{n(x-a)}{b-a} \right\rfloor + 1$ . . .

What I actually ended up using looks like this.

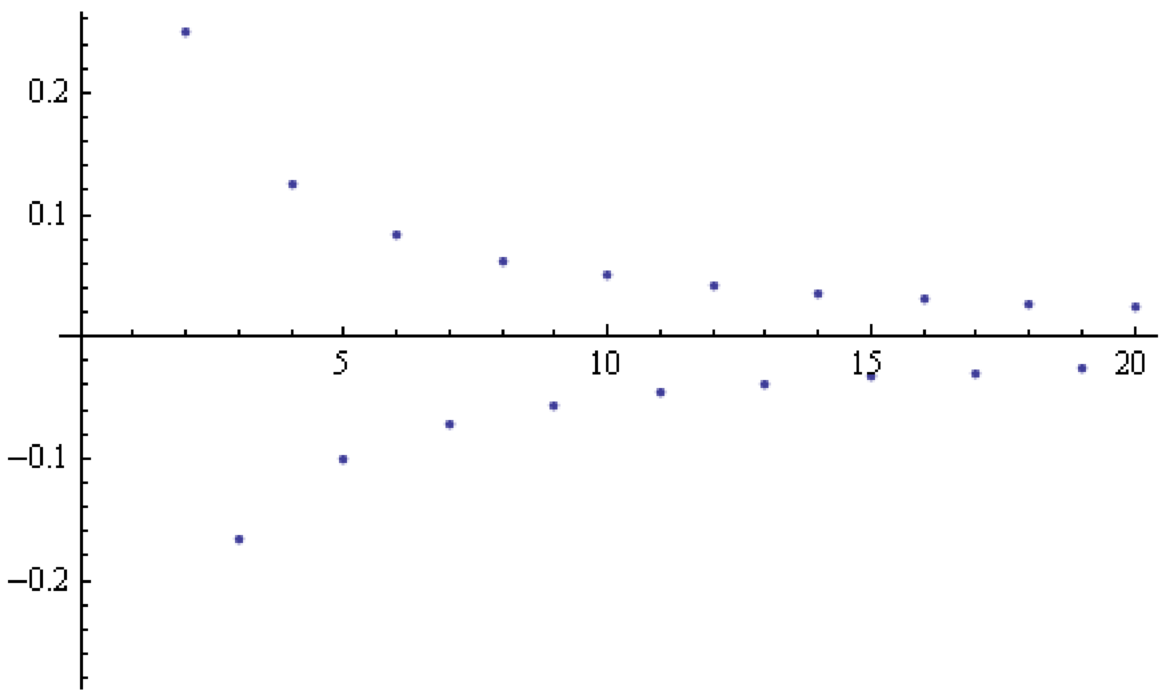


$$s_f(n)\Big|_a^b = \sum_{i=1}^{n}\left((-1)^i \int_{x_{i-1}}^{x_i} f(x)\,dx\right)$$

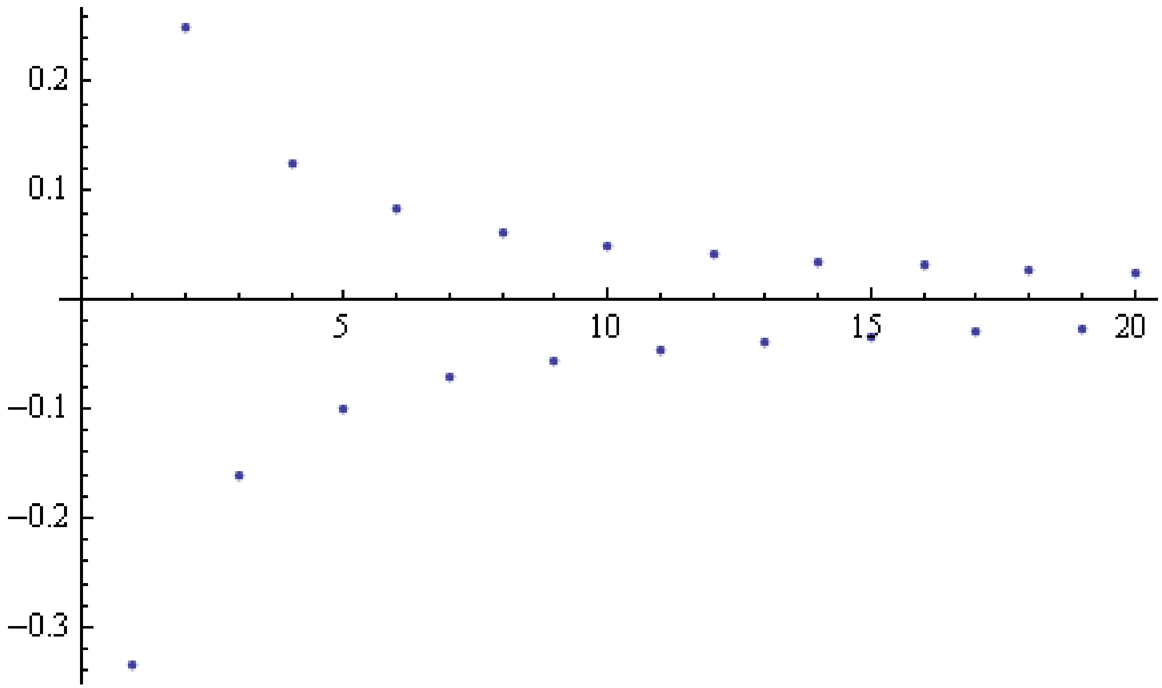
$$f(x) = 1$$



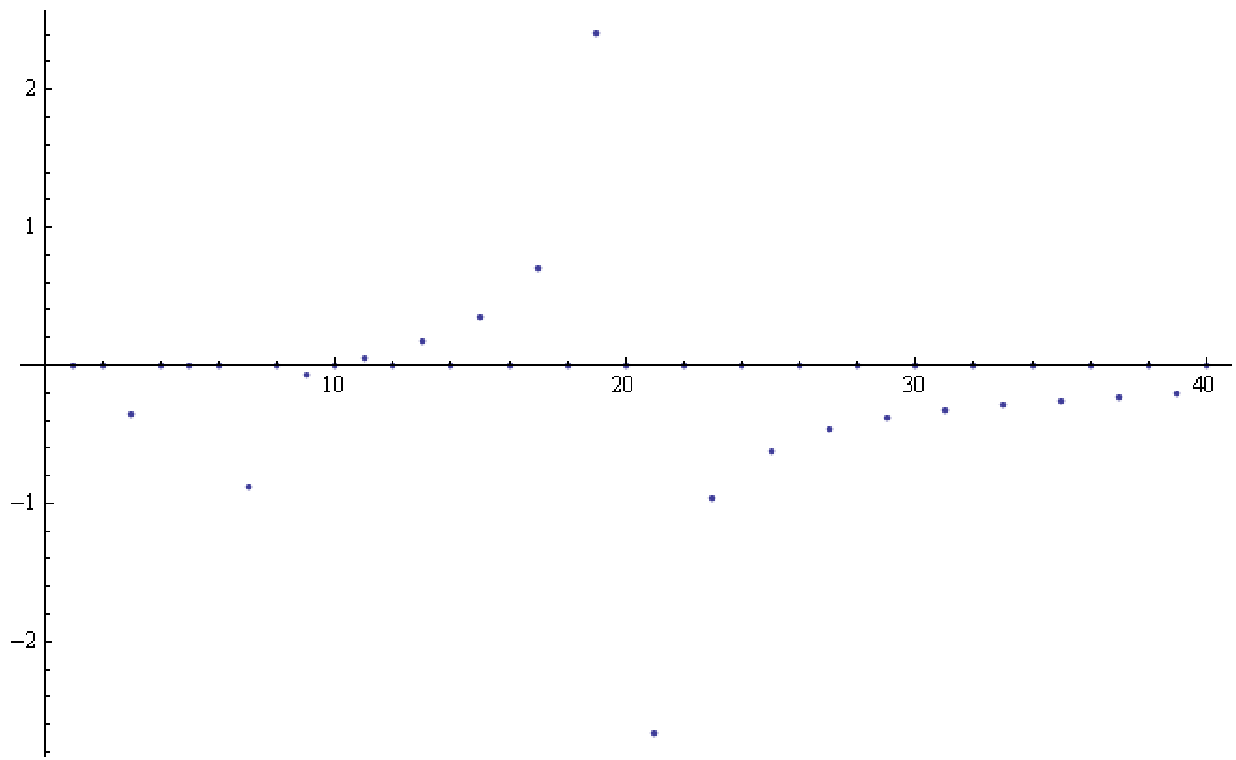
$$f(x) = x$$



$$f(x) = x^2$$



$$s_f(n)\Big|_0^{2\pi} \text{ for } f(x) = \cos(20x)$$



Basic Formulas

Recall:

$$s_f(n)\Big|_a^b = \sum_{i=1}^{n} \left((-1)^i \int_{x_{i-1}}^{x_i} f(x)\,dx \right)$$

Addition and subtraction:

$$s_{f\pm g}(n)\Big|_a^b = s_f(n)\Big|_a^b \pm s_g(n)\Big|_a^b$$

Constant multiplier:

$$s_{cf}(n)\Big|_a^b = cs_f(n)\Big|_a^b$$

Monomials

Recall:

$$s_f(n)\Big|_a^b = \sum_{i=1}^{n}\left((-1)^i \int_{x_{i-1}}^{x_i} f(x)\,dx\right)$$

$$f(x) = x^m$$

$$s_f(n)\Big|_0^b = b^{m+1} s_f(n)\Big|_0^1$$

Monomials

m	$s_f(n)\Big\vert_0^1$ for even n	$s_f(n)\Big\vert_0^1$ for odd n	$s_f(n)\Big\vert_0^1$ in general
0	0	$-\frac{1}{n}$	
1	$\mathbf{\frac{1}{2n}}$	$-\frac{1}{2n}$	$(-1)^n\left(\frac{1}{2n}\right)$
2	$\mathbf{\frac{1}{2n}}$	$\frac{1-3n^2}{6n^3}$	
3	$-\frac{1-2n^2}{4n^3}$	$\frac{1-2n^2}{4n^3}$	$(-1)^n\left(\frac{-1+2n^2}{4n^3}\right)$
4	$-\frac{1-n^2}{2n^3}$	$-\frac{2-5n^2+5n^4}{10n^5}$	
5	$\frac{3-5n^2+3n^4}{6n^5}$	$-\frac{3-5n^2+3n^4}{6n^5}$	$(-1)^n\left(\frac{3-5n^2+3n^4}{6n^5}\right)$

All $O\left(\frac{1}{n}\right)$...

Theorem.

$$\lim_{n\to\infty} s_f(n)\Big|_a^b = 0$$

Where to go from here?

Thank you!

Special Thanks to: Chris Hallstrom