Lucas Imperfect Information Model

- The Lucas model was the first of the modern, microfoundations models of aggregate supply and macroeconomics
  - It built directly on the Friedman-Phelps analysis of the Phillips curve that we have studied
  - It led to the "Lucas supply curve" in which $y_t = \bar{y} + b\left(p - p^e\right)$
  - Similar supply curves can (and will) be derived from other sets of assumptions
- Key assumption: Lucas model assumes perfect price flexibility, but imperfect information
  - Romer presents in terms of "yeoman farmer" production rather than having explicit firms
    - This is not an important assumption
    - Can derive similar model with firms and wages
- Key assumption: Market structure
  - Economy is composed of a large number of goods *i*
  - Each good is produced by a large number of firms (rather than being a monopoly as in the imperfect competition model)
- Key assumption: There are two kinds of random shocks
  - Aggregate shocks that affect AD curve (*M*)
  - Good-specific shocks that affect demand for individual good *i* (z_i) but average out to zero across all goods

Household behavior

- $U_i = C_i - \frac{1}{\gamma} L_i^{\gamma}$
- Production function: $Y_i = L_i$
- Budget constraint: $PC_i = P_i Y_i$
  - This is critical: C_i is an index of consumption by a representative producer of good *i*, not consumption of good *i*
  - Household buys *all* goods, but sells only good *i*
- Putting together: $U_i = \frac{P_i}{P} Y_i - \frac{1}{\gamma} Y_i^{\gamma}$
- Price-taking households, so they take both *P* and P_i as given
  - Note the difference introduced by the competition assumption rather than the "polygopoly" model
- Utility maximization:

- $\frac{dU_i}{dY_i} = \frac{P_i}{P} - Y_i^{\gamma-1} = 0$
- $Y_i = \left(\frac{P_i}{P}\right)^{\frac{1}{\gamma-1}}$
- In log terms: $y_i = \frac{1}{\gamma-1}\left(p_i - p\right)$
- This is the supply curve for good *i*

Aggregate and single-good demand

- Demand for good *i* is given by $y_i = y + z_i - \eta\left(p_i - p\right)$
- Aggregate demand: $y = m - p$
- $y_i = m - p + z_i - \eta\left(p_i - p\right)$
  - This is exactly the same demand curve as in the polygopoly model, except we now have a random, good-specific shock z_i

Aggregation and equilibrium

- Assume that $p = \overline{p}$ and $y = \overline{y}$ are the log-average price and quantity indexes
- What matters to producers is their relative price: $r_i \equiv p_i - p$
  - If they can observe both p_i and p, then the model can be solved directly
  - Setting demand = supply for each market:

    $$\frac{1}{\gamma-1}\left(p_i - p\right) = m - p + z_i - \eta\left(p_i - p\right)$$

    $$p_i - p = \frac{m - p + z_i}{\eta + 1/\left(\gamma - 1\right)}$$

  - Since p is the average of all the p_i values, when we average across all markets
    - $p_i = p$
      $z_i = 0$
  - So solution is $p = m$ and money is neutral
    - Aggregate output is $y = 0$ ($Y = 1$)
    - Note that this is fully efficient in comparison to polygolopy model because there is no monopoly behavior here
- Lucas assumes that agents have imperfect information
  - They know the price of the good they sell
  - They don't know all the prices of the goods that they buy
  - This is probably fairly realistic because most people are more specialized in the goods they sell than the ones they buy

- Signal-extraction problem
  - Agents observe p_i but not p
  - Must attempt to infer p and r_i from p_i
  - Known variable $p_i = r_i + p$ (signal plus noise)
- Under imperfect information, supply curve becomes $y_i = \frac{1}{\gamma - 1} E(r_i \mid p_i)$
- Shocks
  - Assume that the aggregate shock m is normally distributed with mean zero and variance V_m
  - "Local" shocks z_i are normal with mean zero and variance V_z
  - From agent's perspective, we consider p and r_i as unobserved random functions of m and z_i
- Optimal signal-extraction rule: $E(r_i \mid p_i) = \frac{V_r}{V_r + V_p}\left[p_i - E(p_i)\right]$, where V_r and V_p are the variances of r_i and p from the agent's viewpoint
  - $\frac{V_r}{V_r + V_p}$ is "signal-to-noise" ratio: how much of what we hear is the "signal" r as opposed to the noise V_p?
  - Note what happens to $E(r_i \mid p_i)$ when V_p becomes large or small
  - No aggregate-price variation means ratio = 1 and all shocks are assumed to be local
    - Supply curve is elastic because agents assume shocks are relative and respond strongly
  - Infinite aggregate-price variation (or zero local-price variation) means ratio = 0 and all shocks are assumed to the aggregate
    - Supply curve is inelastic because agents assume shocks are aggregate and don't respond
  - Values of V_r and V_p are endogenous, depending on how r and p respond to shocks
- Lucas supply curve
  - Plug signal-extraction rule back into supply curve

    $$y_i = \frac{1}{\gamma - 1}\frac{V_r}{V_r + V_p}\left[p_i - E(p_i)\right]$$
  - Averaging across all i: $y = \frac{1}{\gamma - 1}\frac{V_r}{V_r + V_p}\left[p - E(p)\right] \equiv b\left[p - E(p)\right]$
  - Note similarity to modern Phillips curve relating $u - u_n$ to $\pi - \pi^e$
  - Using methods you need not learn, we can show that $\frac{V_r}{V_r + V_p} = \frac{V_z}{V_z + V_m}$ in the case that $\eta \to 1$

  - Show graph of SRAS and AD, with SRAS passing through $y = 0$ and $p = E(p)$
  - Note cross-country implications:
    - Countries with high V_m will have inelastic AS curves relative to countries with low V_m
    - This is the basis for the empirical work in Lucas's paper of the week (and also the Ball, Mankiw, and Romer paper for next week)
- Equilibrium
  - AS: $y = b\left[p - E(p)\right]$
  - AD: $y = m - p$
  - Solving together yields

    $$bp - bE(p) = m - p$$
    $$(1+b)p = m + bE(p)$$
    $$p = \frac{1}{1+b}m + \frac{b}{1+b}E(p)$$
    $$y = \frac{b}{1+b}m + \frac{-b}{1+b}E(p)$$
- What are expectations of p?
  - "Rational expectations" hypothesis says that agents form expectations in a way that is consistent with model
  - They don't know the shocks m and z_i, but they do know the model and the coefficients of the model
  - $$E(p) = \frac{1}{1+b}E(m) + \frac{b}{1+b}E\left[E(p)\right] = \frac{1}{1+b}E(m) + \frac{b}{1+b}E(p)$$
    $$E(p) - \frac{b}{1+b}E(p) = \frac{1}{1+b}E(m)$$
    $$\left[1 - \frac{b}{1+b}\right]E(p) = \frac{1}{1+b}E(m)$$
    $$\left[\frac{1+b-b}{1+b}\right]E(p) = \frac{1}{1+b}E(m)$$
    $$\frac{1}{1+b}E(p) = \frac{1}{1+b}E(m)$$
    $$E(p) = E(m)$$
  - Thus,

    $$y = \frac{b}{1+b}\left[m - E(m)\right]$$
    $$p = \frac{1}{1+b}m + \frac{b}{1+b}E(m)$$
  - Show setting of $E(p)$ and thus SRAS based on AD curve corresponding to $E(m)$

Policy ineffectiveness proposition

- The output equation shows that ***only unanticipated changes in aggregate demand affect real output***.
- If *m* and *E*(*m*) go up by the same amount, then money is neutral:
    - Effect on *y* is zero
    - Effect on *p* is one
- This undermines the case for countercyclical AD policy, because if people correctly recognize the need for policy as quickly as policymaker does, then they will "fix the economy" on their own and any policy change will affect only *p*
- This was highly controversial conclusion of Lucas model in early 1970s, rejected by Keynesians, leading to "new Keynesian" models with good microfoundations but with positive role for macroeconomic stabilization policy
- Empirical evidence was initially positive (Barro), but later most evidence demonstrated that even policy changes that were correctly anticipatable had real effects