Investment Models: Introduction

Capital stock vs. capital services

- Input to production = services of capital
  - In principle, firm can rent capital goods or buy them
  - Macroeconomy as a whole must "buy" them by forgoing consumption of output and using it for investment
- The interesting aspects of capital/investment theory relate to the stock-flow dynamics
  - How does investment in new capital respond to the demand for capital services?

Capital services as an input

- Suppose that r_K is the nominal market price at which a firm can rent one unit of capital (i.e., buy one unit of capital services)
  - Firm that sells its output at P will set MPK = r_K/P just as firm sets MPL = W/P
  - Demand function for capital services is downward sloping MPK curve
  - Given r_K/P in the market, firm's optimal K^* is determined by MPK curve

Capital as an asset: User cost of capital

- What determines the rental rate?
  - If firm owns its capital, then we usually call the rental rate the "user cost" of capital
  - It is the rate at which the firm would rent capital to itself, which is the opportunity cost of renting the capital out to another user at the rental rate
- Rental rate must compensate owner of capital for two costs:
  - Opportunity cost of owning a bond
  - Depreciation (net of capital gain) on the capital
- Three choices available to wealth-holding firm (assume no inflation in goods prices so $i = r$)
  - **Own a bond worth equivalent of one unit of capital**
    - Gross return next period = $p_K(1+r)$
  - **Own capital and rent it out**
    - Get rental fee of r_K
    - Get capital gain or loss of $\dot{p}_K$ if market price of capital goods changes
    - Have $(1-\delta)p_K$ of capital left
    - Gross return is $r_K + \dot{p}_K + (1-\delta)p_K$
  - **Own capital and use it in production**
    - Get increased output worth $p \times$ MPK

    - Get capital gain or loss of $\dot{p}_K$ if market price of capital goods changes
    - Have $(1-\delta)p_K$ of capital left
    - Gross return is $p \times \text{MPK} + \dot{p}_K + (1-\delta)p_K$
- In equilibrium, all three choices must yield the same return
  - Between bond-holding and capital renting-out:

    $$p_K(1+r) = r_K + \dot{p}_K + (1-\delta)p_K$$

    $$r_K = p_K\left(r + \delta - \frac{\dot{p}_K}{p_K}\right)$$

    - This is the formula for the user-cost of capital that is Romer's equation (9.4)
  - Between using and renting-out capital:

    $$r_K + \dot{p}_K + (1-\delta)p_K = p \times \text{MPK} + \dot{p}_K + (1-\delta)p_K$$

    $$\text{MPK} = \frac{r_K}{p}$$

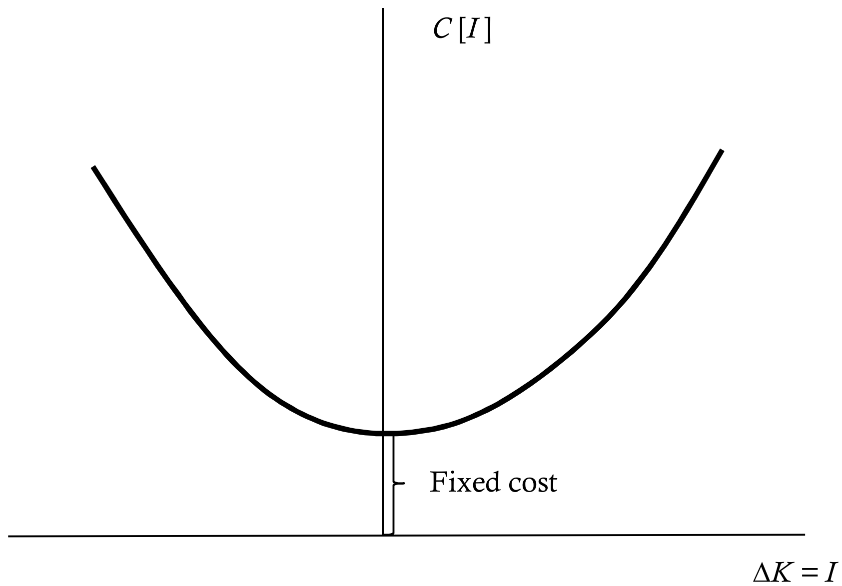
    - This is our marginal product condition for profit maximization above
- If firms adjusted to their optimal capital stock instantaneously, then these two conditions would always hold exactly.
  - In continuous time, this would mean that a change in the interest rate, for example, would cause a jump in MPK, which would mean a jump in K
  - A jump in K (upward) means infinite investment at that instant
  - Since investment comes out of output, this means either infinite output or negative infinite consumption at that instant, neither of which can happen
- Firms do not adjust capital instantly
  - Example: Increase in demand for dorms causes Reed to invest in new dorms
  - How quickly does this happen?
  - Why is adjustment delayed? **Adjustment costs**

Investment with Adjustment Costs

Modeling adjustment costs

- Firms probably incur two kinds of adjustment costs in changing their capital stocks
  - Fixed costs associated with initiating an investment project and supervising it
    - These costs may be independent of the amount of investment being undertaken
    - Fixed adjustment costs cause firms to make "lumpy" investments: adding a whole new dorm rather than one room at a time

- Variable costs that may increase at an increasing rate as investment rises
    - These costs limit the speed of adjustment
    - More costly to build 10 dorms than one at a time?
    - More costly to try to build a dorm in six months than to stretch it over a year
    - Increasing marginal cost of adjustment cause firms to smooth investment: not adjust instantaneously but spread out adjustment over time



- Our model incorporates the variable cost, but not the fixed cost
    - This is serious shortcoming, but there are models in the literature that also include fixed costs
    - The rising variable costs prevent immediate adjustment of capital, but make investment very smooth rather than lumpy

Structure of model

- We will stick to the continuous-time version of the model and skip Romer's discrete-time analysis
- Our model is at the industry level, with *N* firms in the industry
    - Each firm is small enough that it is a price-taker and neglects the effects of its own decisions on the industry as a whole
- The representative firm has a capital stock of $\kappa(t)$ at time *t*, so the total industry stock is $K(t) \equiv N\kappa(t)$
- "Operating profit" (revenue minus labor cost) at the level of the individual firm is proportional to its capital stock.

  - Operating profit per unit of capital is $\pi\left[K(t)\right]$, which is a negative function of industry capital $\pi'\left[K(t)\right] < 0$
  - More capacity in the industry means lower output prices and less profit per unit of capital for each firm in the industry
  - The firm's operating profit is thus $\pi\left[K(t)\right]\kappa(t)$
- No depreciation so $\dot{\kappa}(t) = I(t)$
- Real adjustment costs $C\left[\dot{\kappa}(t)\right]$ have the following properties:

  $C(0) = 0$
  $C'(0) = 0$
  $C''(\dot{\kappa}) > 0$
- One function that satisfies these properties is $C = \frac{1}{2}a\dot{\kappa}^2$
  - Romer does the model in general functional terms, but we will simplify and assume that costs are quadratic
- Price of capital goods is one (same as consumer goods)

Maximizing long-run net worth

- At time *t*, the firm gets **net cash flow** (not profits, though Romer calls it this) of

  $$\pi\left[K(t)\right]\kappa(t) - I(t) - C\left[I(t)\right] = \pi\left[K(t)\right]\kappa(t) - I(t) - \frac{1}{2}aI(t)^2$$
  - "Profits" usually deduct only depreciation cost of capital
  - "Net cash flow" is the actual inflow and outflow of (real) dollars to the firm
- Firm choose investment path to maximize lifetime present value of net cash flow:

  $$\max_{I(t)} \int_{t=0}^{\infty} e^{-rt}\left[\pi\left[K(t)\right]\kappa(t) - I(t) - \frac{1}{2}aI(t)^2\right]dt$$
  - This maximization is subject to a dynamic constraint because $I(t) \equiv \dot{\kappa}(t)$
  - We call κ the "state variable" in the model and *I* the "control variable"
  - The dynamic analog to the Lagrangian is the Hamiltonian
    - Maximizing the Hamiltonian is equivalent to maximizing the integral expression subject to the constraint that $I(t) \equiv \dot{\kappa}(t)$
  - $H\left[\kappa(t), I(t)\right] = \pi\left[K(t)\right]\kappa(t) - I(t) - \frac{1}{2}aI(t)^2 + q(t)I(t)$
    - $q(t)$ is called the "costate variable" and plays the role of the Lagrange multiplier

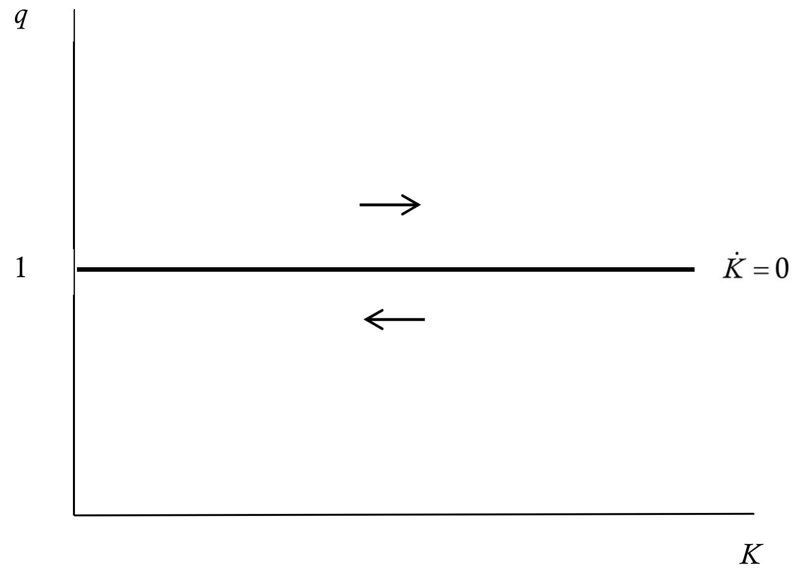
        - It is the "shadow price of installed capital"—the additional value to the firm of having another unit invested without having to incur the cost
    - Maximization of the Hamiltonian involves three conditions:
        - $\frac{\partial H}{\partial I(t)} = 0, \forall t$
            - In this model, $\frac{\partial H}{\partial I(t)} = -1 - C'\left[I(t)\right] + q(t) = -1 - aI(t) + q(t) = 0,$ or $q(t) = 1 + C'\left[I(t)\right] = 1 + aI(t)$
            - This gives us a relationship between q and I: $C'\left[I(t)\right] = aI(t) = q(t) - 1$, or, inverting the C' function, $I(t) = C'^{-1}\left[q(t) - 1\right] = \frac{1}{a}\left[q(t) - 1\right]$
            - Investment is a function of q
                - When $q = 1$, $q - 1 = 0$, $C'^{-1} = 0$, and $I = 0$ because $C' = 0$ only when $I = 0$
                - When $q > 1$, $C'^{-1} > 0$ and $I > 0$
            - This is the essence of "neoclassical investment theory"
                - This theory can be expressed in several different ways: using q or using MPK = r_K/p
                - The bottom line is that investment depends on the return to capital projects (MPK or π) and the cost of capital (r_K)
                - As we will see in a few minutes, q incorporates both of these
        - $\frac{\partial H}{\partial \kappa(t)} = rq(t) - \dot{q}(t)$
            - In this model, $\frac{\partial H}{\partial \kappa(t)} = \pi\left[K(t)\right] = rq(t) - \dot{q}(t),$ or $\dot{q}(t) = rq(t) - \pi\left[K(t)\right]$
            - This equation has an interpretation
                - π = MPK
                - q = real price of installed capital
                - $rq - \dot{q}$ = forgone interest minus capital gain, which is user cost of capital
        - Transversality condition: $\lim_{t \to \infty} e^{-rt} q(t)\kappa(t) = 0$

            - Transversality condition assures that present value of firm's capital stock at future date t does not grow without bound

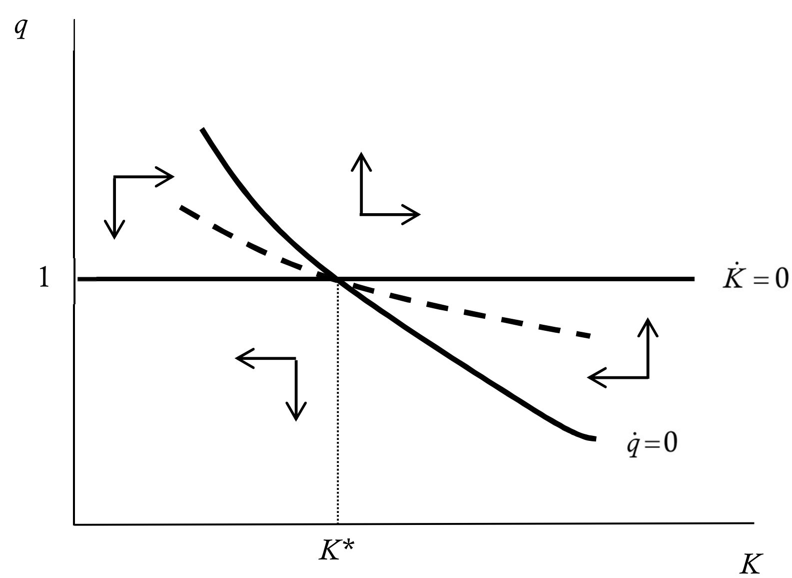
- Interpretation of q
    - We can show that $q(0) = \int_{t=0}^{\infty} e^{-rt} \pi\left[K(t)\right] dt$, which is the present value of the future operating profits from a unit of capital
    - q depends on two things:
        - π represents the MPK
        - r represents the user cost of capital (remember that there is no δ)
        - The q theory is formally similar to the theory that MPK = r_K/p but incorporating costs of adjustment
    - q is the ratio of the value of installed capital to the value of uninstalled capital (which is always one in our model)
        - $q = 1$ means that an installed unit of capital has the same value as one "on the shelf"
        - $q > 1$ means that it installed capital is more expensive than on-the-shelf capital goods
        - Would a firm wanting capital find it cheaper to invest or to buy another firm that has already invested?
            - In equilibrium, the price of shares of existing firms must make these two alternatives equally costly
        - Can think of q as value of one unit of capital on the stock market, rather than on the shelf
            - If there are no adjustment costs (or if firms have had time to adjust their capital stocks fully) then $q = 1$
            - If capital has become more desirable and firms have not yet fully increased their stocks to reflect that, then $q > 1$

Dynamics of the q model

- By aggregation, $K(t) = N\kappa(t)$, so $\dot{K}(t) = N\dot{\kappa}(t) = NI(t)$ (assuming N is fixed)
    - But from above, this means $\dot{K}(t) = \frac{N}{a}\left[q(t) - 1\right] \equiv f\left[q(t)\right]$
        - $f(1) = 0,$ $f' > 0$
        - $q = 1 \Rightarrow \dot{K} = 0$
        - $q > 1 \Rightarrow \dot{K} > 0$
        - $q < 1 \Rightarrow \dot{K} < 0$



- From the Euler equation above,
  $\dot{q}(t) = rq(t) - \pi[K(t)]$
  - $\dot{q}(t) = 0 \Rightarrow q(t) = \dfrac{\pi[K(t)]}{r}$, which is downward-sloping in K because $\pi' < 0$



- Dynamics of model: saddle-path convergence to equilibrium
  - Given the capital stock at time 0, q will be at level determined by saddle path
  - Over time, K returns to K^* and q returns to its long-run value of 1

- Increase in industry profitability (increase in demand) increases π function and shifts $\dot{q}=0$ curve upward
    - In short run, *q* jumps (increase in stock value), then returns back down as actual capital stock catches up to increase in desired
    - Value of company is still higher, but *q* is value of company *per unit of capital*, so the denominator increases as *K* goes up
- Decrease in *r* has same effect as increase in π
- Nothing shifts $\dot{K}=0$ curve, right?
    - How about a tax or tax credit on investment?
    - Investment tax credit (proportional, but doesn't apply to adjustment costs) shifts the $\dot{K}=0$ curve downward to $1-\theta$ (where θ is the rate of credit)

Variations and Empirical Considerations

Irreversible investment

- Influential book by Dixit and Pindyck: *Investment under Uncertainty*
    - First couple of chapters are worth reading
- Three key elements in their model:
    - Return to capital in period $t+s$ is uncertain, but uncertainty decreases as *s* gets smaller because we get better information
        - Reed will know more about future residence-hall demand next year than it does now
    - Investment is (at least partially) irreversible
        - If Reed builds a dorm, it can't get all of its costs back by "unbuilding" it
    - Investment can be postponed
        - Reed can build a dorm either now or in a year
- When these three elements are present, there is an ***option value*** of waiting rather than committing to an investment project immediately.
    - Investing now requires forfeiting the option of not investing
    - This option has value under the conditions above
    - Thus firms may choose not to invest up to the full optimal K^* of the neoclassical model.
    - Degree of uncertainty can have an important impact on investment, with firms choosing to postpone investment when prospects are uncertain and new information is likely to arrive soon.

Financing investment

- Firms have three basic methods of acquiring the funds they need for investment:
    - Retained earnings (internal funding)
    - Borrowing (issuing bonds)
    - Issuing new equity/ownership shares
- We have so far assumed that the method of finance does not matter. This is justified under special conditions by the ***Modigliani-Miller Theorem***.
- Firms that incur high debt to finance their investments are using ***leverage***
- Consider the Coursebook example of three firms with different leveraging.
    - Each has $1000 in capital
    - Shares cost $1 each
    - Interest rate = 10%
    - Each has good and bad years with equal probability:
        - Profit in good years = $150 (15% ROR)
        - Profit in bad years = $50 (5% ROR)

	LL	**HL**	**RHL**
Shares	1000	500	1
Debt	$0	$500	$999
Debt service	$0	$50	$99.90
Profits in good year	$150	$100	$50.10
Profits in bad year	$50	$0	$–49.90
Profits/share: good	$0.15	$0.20	$50.10
Profits/share: bad	$0.05	$0.00	$–49.90
Profits/share: ave	$0.10	$0.10	$0.10

- Notice that regardless of how firm finances its investment, the average (expected) rate of return per share is 10%.
- Thus, regardless of the firm's leveraging strategy, it has the same expected gain per share and should undertake the project if the expected return is higher than the interest rate (cost of capital)
- However, these firms look like they have very different ***risk***
    - The risk that matters in financial markets is "***non-diversifiable risk***"
    - Consider how an owner of shares in HL can diversify the additional risk relative to LL ($0.20 or $0.00 vs. $0.15 or $0.05)
    - If I have $10 to invest,
        - I could buy 10 shares of LL (10% of company) and get $1.50 or $0.50
        - Alternatively, I could buy 5 shares of HL (10% of company) and $5 of bonds (perhaps HL bonds). In this case, I get $0.50 in

interest on the $10 in bonds in either state of the world, plus either $1.00 (5 × $0.20) or $0.00 in profit on my 5 shares. Together, these two assets give me the same $1.50 or $0.50 that I get with 10 shares of LL.

- Moreover, by choosing HL rather than LL, the firm conveniently supplies exactly the $5 worth of bonds to the market that I want to hold.

  - This is the M-M Theorem: The method of finance does not matter. The underlying risk of the firm's operation (if this firm represents the entire macroeconomy) is non-diversifiable, but the added risk associated with leveraging is diversifiable by simply diluting the highly leveraged firm's shares in your portfolio.

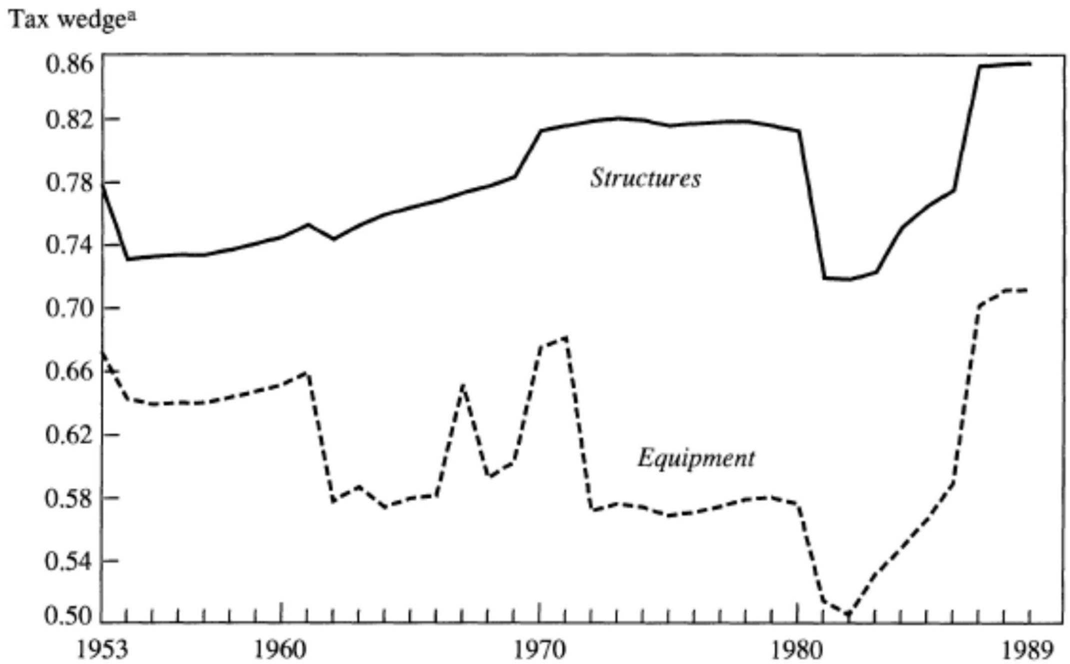
- Other examples of diversification:
  - Should auto companies invest their funds in oil companies so that they can get the profits from oil production when high oil prices depress auto sales?
    - No. People investing in auto companies can perform this diversification on their own. There is no need for the auto companies to do it for them.
- Of course, all of these examples ignore information asymmetries and the possibility of bankruptcy, fraud, etc.

Empirical evidence

- Empirical analysis of investment is frustrating!
  - Universally accepted core model says that investment should depend fundamentally on comparing the return on capital to the cost of capital (interest rate)
  - But regressions of investment on the q, the cost of capital, or the interest rate do not show a strong (or any) relationship
  - We all "know" that this effect is there, but we can't find it in the data
  - For example, the Reed performing arts building was built as a direct result of the low interest rate
- Why can't we find a strong relationship between investment and the cost of capital?
  - Perhaps because we cannot really measure the expected rate of return (MRP) on capital, and that where most of the variation occurs.
  - Perhaps because of the long lags we would expect in the relationship: a change in q or the interest rate in 2014 would lead to investment expenditures lasting several years, and probably peaking one or two years in the future.
  - Perhaps because we cannot measure "marginal q" or the real interest rate accurately

   - Perhaps due to endogeneity: changes in investment demand change interest rates so causality runs in both directions
- Cummins, Hassett, and Hubbard (1994) look for effects due to tax reforms
   - When tax reform occurs, the cost of capital changes discretely at a specific date, often by more than the changes induced by interest rates
   - This is a "natural experiment" with unambiguous causality
   - 13 significant changes in corporate tax code between 1962 and 1988, all of which would have an effect on tax-adjusted user cost of capital (or *q*)

Figure 1. After-Tax Cost of One Dollar of Investment: Equipment and Structures, 1953–89

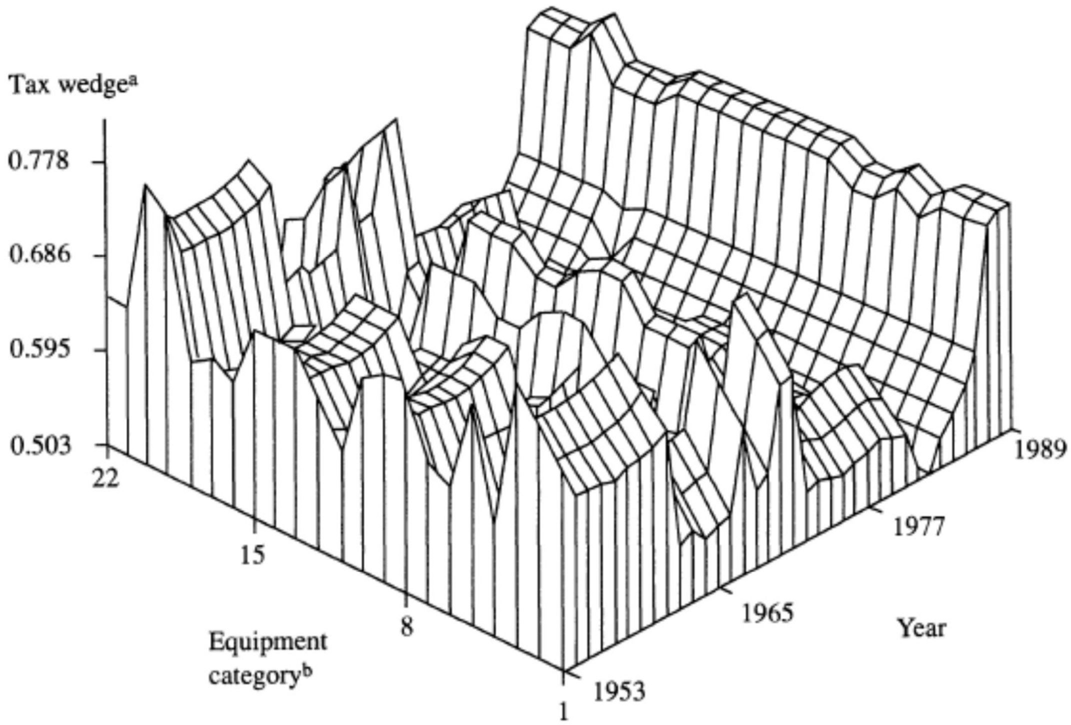


Source: Authors' calculations from Auerbach (1982, 1983a), Pechman (1987), and U.S. Senate Committee on the Budget (1986, 1992).

a. The tax wedge is calculated from Γ, which is the sum of the present value of tax savings from depreciation allowances and the investment tax credit. Higher values for the tax wedge (1 − Γ) correspond to higher after-tax costs of investing.

   - Some of these changes applied differently to different categories of capital

Figure 2. After-Tax Cost of One Dollar of Equipment Investment, 1953–89



Source: Authors' calculations based upon data from the Bureau of Economic Analysis.
a. The tax wedge is calculated from Γ, which is the sum of the present value of tax savings from depreciation allowances and the investment tax credit. Higher values for $(1-\Gamma)$ correspond to higher after-tax costs of investing.
b. See table 2 for BEA classifications.

- Summary table of results from Coursebook (p. 15-38):

Table 2. Estimated effects of Q in tax-reform and other years.

	Number of years	Average of Q coefficients	Range of Q coefficients	Average $\lvert t \rvert$ on Q coefficients
No tax reform	13	0.056	–0.119 to 0.138	0.77
Minor tax reform	9	0.555	0.446 to 0.742	5.14
Major tax reform	4	0.639	0.470 to 0.874	5.33

Source: Cummins, Hassett, and Hubbard (1994), Table 5.

- Effect of q is much stronger in tax-reform years than in no-reform years
- Changes in tax parameters seem to induce changes in investment spending through their effect on tax-adjusted q

- Fazzari, Hubbard, and Peterson (1988)

- Cost of external finance > cost of internal finance, especially for smaller firms
- High cash flow may be a strong determinant of investment if firms prefer to finance investment internally rather than borrowing
- How to finance internally? Don't pay dividends
- FHP divide firms into 3 classes based on their propensity to pay dividends
- Cost of capital or q has stronger effect (and cash flow a weaker effect) among dividend-paying firms, which tended to be larger and older

Table 3. Regression coefficients on Q and cash flow, 1970–84 sample.

	Class 1	Class 2	Class 3
Q	0.0008 (0.0004)	0.0046 (0.0009)	0.0020 (0.0003)
CF/K	0.461 (0.027)	0.363 (0.039)	0.230 (0.010)
$\bar{R}^2$	0.46	0.28	0.19

Source: Fazzari, Hubbard, and Petersen (1988), Table 4.

- This suggests that financing constraints might be important for many firms in the economy
- This is also a source of monetary-policy effectiveness:
    - When credit conditions are tight, banks lend less, which constrains smaller firms (who don't have access to direct credit markets such as bonds and commercial paper), who cut investment