# General Equilibrium with Imperfect Competition

- In order to examine price-setting behavior, we need a model in which firms make a non-trivial choice about prices
  - Perfect competition cannot support this because firms all *must* set the market-equilibrium price
  - So we cannot do the microfoundations of price-setting and sticky prices without a macro general-equilibrium model of imperfect competition
- Model is like short-run of monopolistic competition model in Econ 201 (but no fixed cost and no entry to drive long-run profit to zero)
  - Many firms in economy (infinitely many)
  - Each firm produces a variant of the product that is distinct from all other firms
    - Can think of location variation: each firm is located along the interval [0, 1] and they are dense on that line
  - Each firm's product is a close substitute for those of other firms, but not a perfect substitute
    - The elasticity of each firm's demand is $\eta$
    - $1 < \eta < \infty$
    - Higher $\eta$ means closer to perfect competition
  - "Polygopoly"
- Production function: $Y_i = L_i$
  - No diminishing returns (none necessary)
  - No capital or other fixed inputs
  - Choose units so that one unit of output = product of one unit of labor input
- Utility function: $U = C - \frac{1}{\gamma} L^{\gamma}$
  - $\gamma > 1$ to make the marginal disutility of work increasing in work effort
  - Note the additivity of the utility function
    - Cross partials are zero
    - $C$ does not affect $MU_L$ and $L$ does not affect $MU_C$
    - This makes analysis much simpler by allowing us to analyze optimal consumption and optimal labor effort separately
- No investment, government, or foreign sector to all output is consumed: $Y = C$

## Behavior of single household in isolation

- Suppose that the economy consisted of just one household consuming only its one variety of the consumption good

  - We rule this out in the model by assuming that people desire a variety of variants of consumption goods
  - We use it here because it is a useful benchmark to assess the efficiency of the market economy
  - Household's only choice is how much to work $\bar{L}$, which determines its output and consumption at $\bar{L}$ as well
- Given the production function and the absence of non-consumption uses for output, $C = L$, so $U = \bar{L} - \frac{1}{\gamma}\bar{L}^{\gamma}$
  - $\frac{\partial U}{\partial \bar{L}} = 1 - \bar{L}^{\gamma-1} = 0$
  - Optimal $\bar{L} = 1$ and $Y = C = 1$ as well

## Behavior of household with infinite variety of goods

- We now must think carefully about what we mean by $C$
  - There are many goods and they are not identical, so there is no single $C$ that should unambiguously be in the utility function
  - A tractable approach to this problem is the construct an aggregate ***index*** of the household's consumption of all of the goods (a kind of average)
    - A simple arithmetic average or sum would be a logical choice, but doesn't turn out to have convenient properties
  - We use the Dixit-Stiglitz aggregate index to calculate an average of consumption across all of the goods
  - With finite number of goods $N$ the Dixit-Stiglitz index would be

$$C = \left( \sum_{i=1}^{N} C_i^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

    - Note "CRTS" property of index: doubling all $C_i$ doubles $C$
  - We want the number of goods to approach $\infty$
    - Only as $N \to \infty$ does each firm become a vanishingly small part of the overall market
    - Instead of indexing firms/goods by $i = 1, 2, \ldots, N$ we index them by $i \in [0, 1]$
  - With a "continuum of goods," the Dixit-Stiglitz index becomes

$$C = \left( \int_{i=0}^{1} C_i^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}$$ (Note omission of $di$ in Romer's equation (6.39))

- We analyze household decision in two steps:

- Given total nominal expenditures $S$, how does the household decide which of the products to buy? In other words, what is the demand for $C_i$ given the overall level of $S$ (and $C$) chosen?
- What determines the overall level of consumption and spending $C$ and $S$?
  - There is no saving here, so these will be determined by the amount of income earned, which depends on labor supplied
  - Basically, $C = Y = L$, so this will depend on labor supply

## Demand for $C_i$ taking S as given

- Lagrangian: $\mathcal{L} = \left( \int_{i=0}^{1} C_i^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} + \lambda \left( S - \int_{i=0}^{1} P_i C_i di \right)$
  - First expression is objective function to be maximized
  - λ is the Lagrange multiplier, a variable whose value is to be determined
  - The second parenthetical expression is the constraint $S = \int_{i=0}^{1} P_i C_i di$, that total nominal spending is the sum of spending on all goods
    - This is expressed in a difference form that equals zero when the constraint is satisfied
- Maximizing $\mathcal{L}$ with respect to the choice variables and λ is equivalent to maximizing the utility function subject to the constraint
  - $\frac{\partial \mathcal{L}}{\partial \lambda} = S - \int_{i=0}^{1} P_i C_i di = 0$ assures that the constraint is satisfied
  - $\frac{\partial \mathcal{L}}{\partial C_i} = \frac{\eta}{\eta-1} \left( \int_{j=0}^{1} C_j^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}-1} \frac{\eta-1}{\eta} C_i^{\frac{\eta-1}{\eta}-1} - \lambda P_i = 0, \ \forall i \in [0,1]$

$$C_i^{-\frac{1}{\eta}} = \frac{\lambda}{\left( \int_{j=0}^{1} C_j^{\frac{\eta-1}{\eta}} dj \right)^{\frac{1}{\eta-1}}} P_i$$

$$C_i = \frac{\lambda^{-\eta}}{\left( \int_{j=0}^{1} C_j^{\frac{\eta-1}{\eta}} dj \right)^{-\frac{\eta}{\eta-1}}} P_i^{-\eta} = C\lambda^{-\eta} P_i^{-\eta} = AP_i^{-\eta}$$

- Because λ is not yet known, we must solve the model further to determine $A$ and thus the consumer's demand function for $C_i$

- Plugging $C_i = AP_i^{-\eta}$ from FOC $\frac{\partial \mathcal{L}}{\partial C_i} = 0$ into the FOC $\frac{\partial \mathcal{L}}{\partial \lambda} = 0$, or $S = \int_{j=0}^{1} P_j C_j dj$ :

$$\int_{j=0}^{1} P_j A P_j^{-\eta} dj = S$$
$$A \int_{j=0}^{1} P_j^{1-\eta} dj = S$$
$$A = \frac{S}{\int_{j=0}^{1} P_j^{1-\eta} dj}.$$

- Following the analysis of Romer's equation (6.46), we can plug this value of $A$ into the consumption equation to get $C_i = \frac{S}{\int_{j=0}^{1} P_j^{1-\eta} dj} P_i^{-\eta}$ , and then substitute this into the aggregation equation for $C$ to get

$$C = \left[ \int_{i=0}^{1} \left( \frac{S P_i^{-\eta}}{\int_{j=0}^{1} P_j^{1-\eta} dj} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} = \frac{S}{\int_{j=0}^{1} P_j^{1-\eta} dj} \left( \int_{i=0}^{1} \left( P_i^{-\eta} \right)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}$$
$$= \frac{S}{\int_{j=0}^{1} P_j^{1-\eta} dj} \left( \int_{i=0}^{1} P_i^{1-\eta} di \right)^{\frac{\eta}{\eta-1}} = S \left( \int_{i=0}^{1} P_i^{1-\eta} di \right)^{\frac{\eta}{\eta-1}-1} = S \left( \int_{i=0}^{1} P_i^{1-\eta} di \right)^{\frac{1}{\eta-1}}$$
$$= \frac{S}{\left( \int_{i=0}^{1} P_i^{1-\eta} di \right)^{\frac{1}{1-\eta}}} \equiv \frac{S}{P}.$$

- We define $P$ to be the price index defined by the denominator of this expression, which is natural because it means that $S = PC$.
  - This price index is not unlike the CPI in that it "averages" prices across goods, but it does so in a power metric rather than linearly.
- Given this solution for $A$, $C_i = \left( \frac{P_i}{P} \right)^{-\eta} C$
  - Consumer demand for product $i$ is average consumer demand across products times the relative price of product $i$ taken to the power $-\eta$
  - $-\eta$ is the price elasticity of demand for product $i$

## Supply of labor

- Again we take advantage of the separability of the utility function to analyze the decision of how much to work in isolation from the consumption decision
- For the household, consumption is equal to its income (there is no saving) of $\frac{WL + R}{P}$, with $R$ being its share of firms' profits.

  - Profits were zero in our growth and RBC models because firms were perfectly competitive.
  - There is no capital income here because there is no capital.
- Household chooses labor effort to maximize utility:
  - $\frac{W}{P} - L^{\gamma-1} = 0$
  - $L = \left(\frac{W}{P}\right)^{\frac{1}{\gamma-1}}$
  - Note that this will correspond to optimal (individual) labor supply of $L = 1$ only if $W/P = 1$

## Firms' behavior

- Following our aggregate index for $C$, let $Y = \left(\int_{i=0}^{1} Y_i^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$ be an index of aggregate output, "averaged" across goods
- Since $C_i = Y_i$ (and thus $C = Y$), the demand for good $i$ is $Y_i = \left(\frac{P_i}{P}\right)^{-\eta} Y$
- Firm $i$'s real profit is $\frac{R_i}{P} = \frac{P_i Y_i - W L_i}{P} = \left(\frac{P_i}{P}\right) Y_i - \left(\frac{W}{P}\right) L_i$
  - There are three choice variables of the firm here: $P_i/P$, $Y_i$, and $L_i$
  - The three variables are constrained by
    - The production function $Y_i = L_i$ and
    - The demand equation $Y_i = \left(\frac{P_i}{P}\right)^{-\eta} Y$
  - Substituting for $Y_i$ and $L_i$ yields $\frac{R_i}{P} = \left(\frac{P_i}{P}\right)^{1-\eta} Y - \left(\frac{W}{P}\right)\left(\frac{P_i}{P}\right)^{-\eta} Y$, in which the only choice variable is $P_i/P$
- Differentiating with respect to $P_i/P$ and setting equal to zero:

$$\frac{\partial(R_i / P)}{\partial(P_i / P)} = (1-\eta)\left(\frac{P_i}{P}\right)^{-\eta} Y + \eta\left(\frac{W}{P}\right)\left(\frac{P_i}{P}\right)^{-\eta-1} Y = 0$$

Dividing by $\left(\frac{P_i}{P}\right)^{-\eta} Y$,

$$\left(1-\eta\right) = -\eta\left(\frac{W}{P}\right)\left(\frac{P_i}{P}\right)^{-1} \text{ or}$$

$$\frac{P_i}{P} = \frac{\eta}{\eta-1}\left(\frac{W}{P}\right).$$

- This is a key equation showing the monopoly firm's pricing policy
- The relative price of firm *i* is a markup on marginal cost $W/P$
- The markup ratio is $\eta/(\eta - 1) > 1$
- It is a standard result from monopoly theory that the markup ratio depends on the elasticity of demand in this way
- As the economy becomes more competitive, $\eta \to \infty$ and the markup ratio $\to 1$
  - Perfectly competitive firms price at marginal cost with no markup (and earn zero profits)
  - As $\eta \to 1$ from above, the firm's optimal price $\to \infty$

## Equilibrium in the aggregate economy

- Suppose that all firms and households behave according to the equations derived above and that markets clear
  - $\frac{P_i}{P} = \frac{\eta}{\eta-1}\left(\frac{W}{P}\right)$
  - $Y_i = \left(\frac{P_i}{P}\right)^{-\eta} Y$
  - $L_i = \left(\frac{W}{P}\right)^{\frac{1}{\gamma-1}}$
  - $Y_i = L_i$
- Only one piece is missing to determine the equilibrium levels of all the variables of the model: **aggregate demand**
  - What is *Y*?
  - $Y = C = \frac{S}{P}$, but what are *S* and *P*?
  - We need a theory of aggregate demand to couple with our supply model
  - Quantity theory: $PY = MV$ is simpler than *IS/LM* framework, but generates a downward-sloping AD curve
    - We make it even simpler by defining the units of the money stock in such a way that $V = 1$, so $PY = S = M$ .
    - We can think of *M* as the money stock with a velocity constrained to one
    - Or we can think of *M* as a general variable reflecting the level of aggregate demand

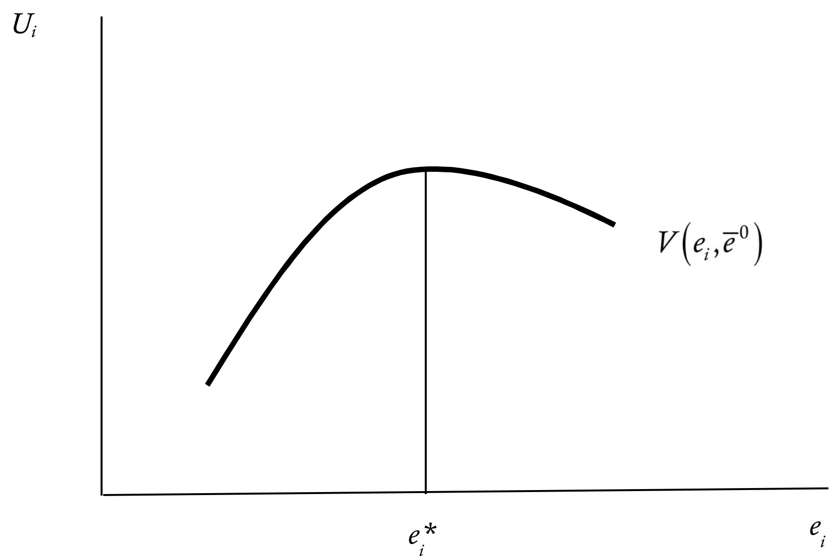
    - In either case, we take it to be exogenous and subject to shocks and to manipulation by monetary (and perhaps fiscal) policy
  - $Y = \frac{M}{P}$ is our theory of aggregate demand
- Plugging this in, $Y = L = \left(\frac{W}{P}\right)^{\frac{1}{\gamma-1}}$, or $\frac{W}{P} = Y^{\gamma-1} = \left(\frac{M}{P}\right)^{\gamma-1}$
- Plugging this into the pricing equation, $\frac{P_i}{P} = \frac{\eta}{\eta-1} Y^{\gamma-1}$ or $Y = \left(\frac{\eta-1}{\eta}\right)^{\frac{1}{\gamma-1}} \left(\frac{P_i}{P}\right)^{\frac{1}{\gamma-1}}$
- All firms in the model are symmetric (they all have the same η), so they must all choose the same price $P_i$
  - If all $P_i$ are the same, then the aggregation of the prices $P = P_i$ and $\frac{P_i}{P} = 1$
- Thus, the equilibrium level of output in the model is $Y = \left(\frac{\eta-1}{\eta}\right)^{\frac{1}{\gamma-1}} < 1$
- Equilibrium price is $P = \frac{M}{Y} = \frac{M}{\left(\frac{\eta-1}{\eta}\right)^{\frac{1}{\gamma-1}}}$

## Properties of equilibrium in the model

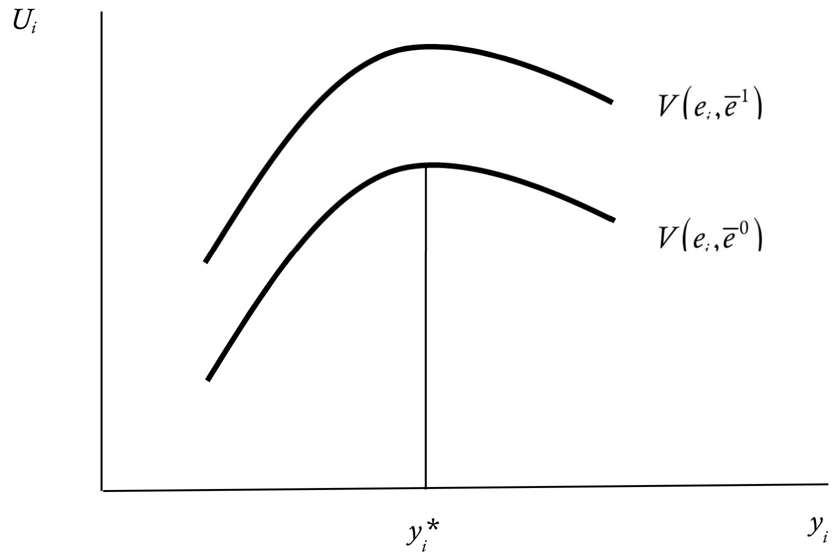
- Equilibrium is inefficient
  - We showed that households that internalize all aspects of production and consumption would choose $Y = \bar{L} = 1$
  - The decentralized equilibrium has $Y < 1$
  - Monopolies produce too little due to contrived scarcity
  - This leads to the conclusion that booms can be good and recessions particularly bad
    - If equilibrium is efficient, then deviations both above and below the natural level of output are bad
- Money is neutral
  - Change in $M$ leads to proportional change in $P$ and $W$ with no change in $Y$
  - In order to introduce a role for AD-based monetary of fiscal stabilization policy we must add price stickiness; imperfect competition alone is not enough
- There is a positive AD externality of each firm's production
  - If one firm were to produce more (sacrificing profit), it would raise $Y$ a little bit, which would add to the demand for other firms and make everyone better off

# Coordination Failures

- Our analysis of coordination failures is based on the general framework of Cooper and John (1988)
  - It shows how strategic complementarity can lead the economy to be "stuck" at a suboptimal equilibrium
- Let each of many individuals in the model have utility function $U_i = V\left(e_i, \bar{e}\right)$
  - $Y$ is a variable that measures a decision/activity of each individual
  - $e_i$ is the $i$th individual's choice of $e$
  - $\bar{e}$ is the choice of the average person
  - Assume that $V_{11}\left(e_i, \bar{e}\right) \equiv \frac{\partial^2 V}{\partial e_i^2} < 0$, which is necessary second-order condition for unique maximum choice of $e_i$
  - Diagram shows $V$ function of $e_i$ for some chosen value of $\bar{e}^0$




  - (We will apply as $P_i$ and $P$)
- **Spillovers**
  - If $V_2\left(e_i, \bar{e}\right) \equiv \frac{\partial V}{\partial \bar{e}} > 0$, then we have positive spillovers (externalities) in that other people choosing a higher $y$ improves person $i$'s utility
  - Negative spillovers: $V_2\left(e_i, \bar{e}\right) < 0$

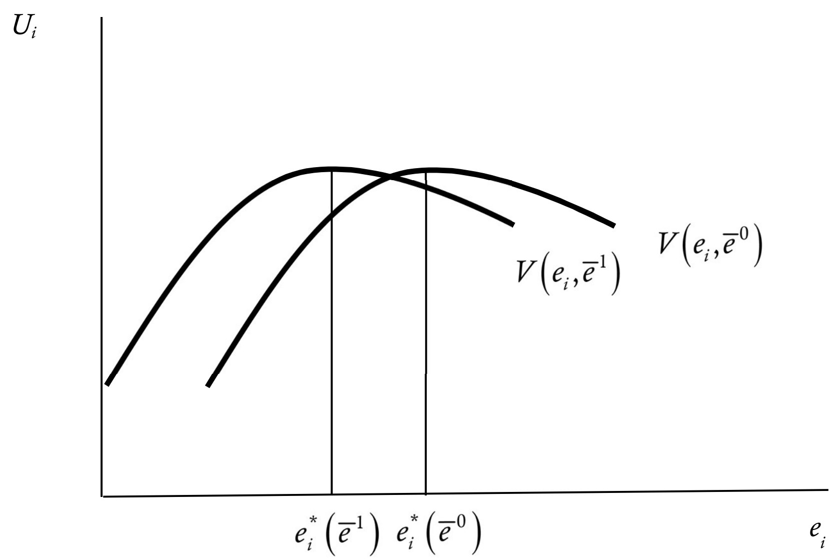



- Positive spillover means that an increase in average $\bar{e}$ from $\bar{e}^0$ to $\bar{e}^1$ moves utility function of individual upward

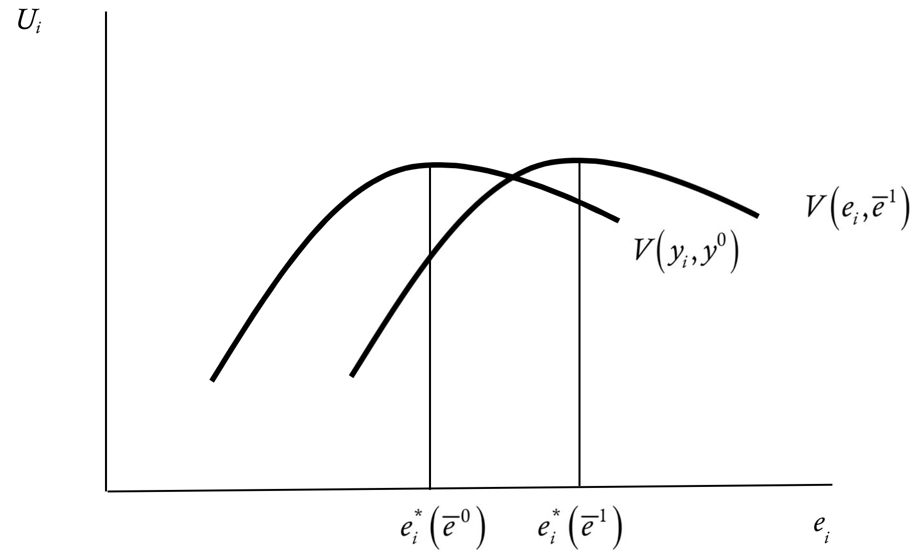
- **Maximization of utility**
  - Individual $i$ chooses $y_i$ to maximize $U_i$
    - $\frac{\partial U_i}{\partial e_i} = V_1(e_i, \bar{e}) = 0$
    - Solving this FOC for $e_i$ yields a reaction function $e_i^* = e_i^*(\bar{e})$ telling what level of $e_i$ the individual chooses if everyone else chooses $\bar{e}$
  - How does $\bar{e}$ affect $e_i^*$? What is sign of $\frac{\partial e_i^*}{\partial \bar{e}}$?
    - $e_i^*(\bar{e})$ is defined by the FOC $V_1(e_i^*, \bar{e}) = 0$
    - To find the effect of $\bar{e}$ on this condition, take the derivative with respect to $\bar{e}$:
      - $V_{11}(e_i^*, \bar{e})\frac{\partial e_i^*}{\partial \bar{e}} + V_{12}(e_i^*, \bar{e}) = 0$
      - $\frac{\partial e_i^*}{\partial \bar{e}} = -\frac{V_{12}(e_i^*, \bar{e})}{V_{11}(e_i^*, \bar{e})}$.
    - We know that $V_{11} < 0$ is necessary second-order condition for an interior maximum
      - (Utility function must have a peak, not a trough, at $y^*$)

- Therefore $\operatorname{sgn}\left(\frac{\partial e_i^*}{\partial \bar{e}}\right) = \operatorname{sgn}\left(V_{12}\left(e_i^*, \bar{e}\right)\right)$

- If $V_{12}\left(e_i^*, \bar{e}\right) < 0$, then other people increasing their $e$ makes each individual want to reduce $e_i$
    - This is called ***strategic substitutability***
    - This is the "usual case" in economics: if everyone else buys something, it raises its price and causes me to buy less
    - This kind of effect leads to stable interior, rather than extreme corner, solutions for most economic equilibria

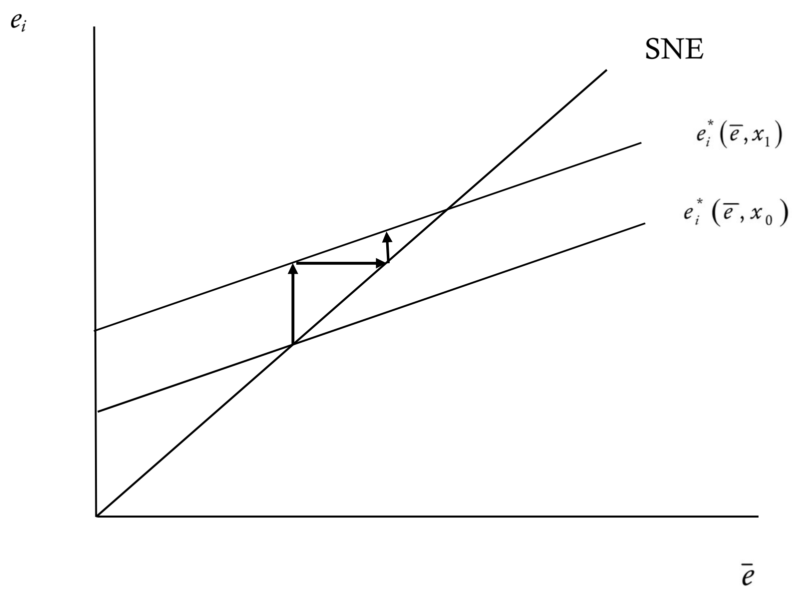



- If $V_{12}\left(e_i^*, \bar{e}\right) > 0$, then other people increasing $e$ makes the individual want to increase her own $e_i$
    - This is ***strategic complementarity***
    - This can lead to self-reinforcing movements toward the extremes because each individual increasing $e_i$ leads to an increase in $e$, which makes each individual increase $e_i$ further
    - This sounds like a ***multiplier*** process akin to Keynes's multiplier, and it is



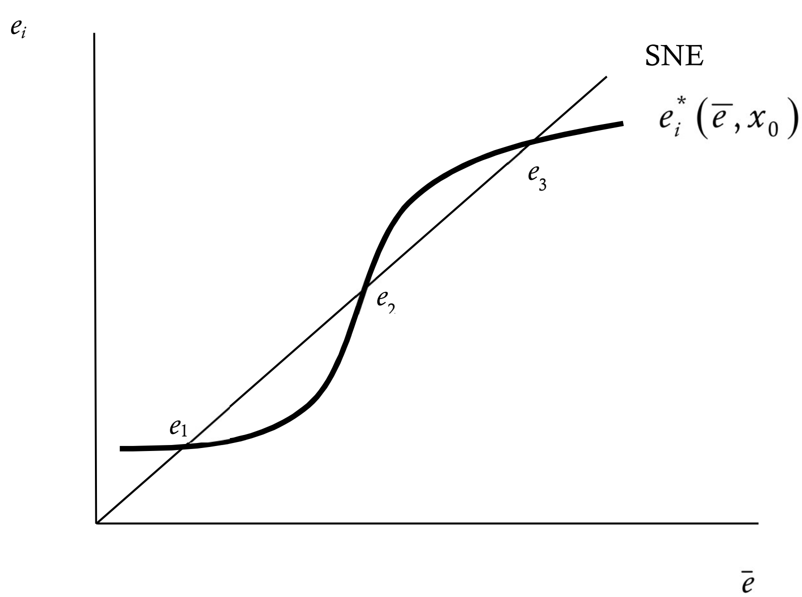

- Intermediate case is strategic independence: $V_{12}(e_i^*, \bar{e}) = 0$
- Simple example:
  - If your roommate has a TV, you are less likely to buy one (strategic substitutability)
  - If your roommate watches TV, you are more likely to watch (strategic complementarity)
- Note that strategic complementarity/substitutability can occur with or without spillovers
  - Strategic interaction moves the left/right position of the maximum
  - Spillovers move the utility function up or down

- We can graph $e_i$ as a function of $\bar{e}$
  - Strategic substitutability $\Rightarrow$ downward sloping
  - Strategic complementarity $\Rightarrow$ upward sloping
  - Strategic independence $\Rightarrow$ horizontal
- Symmetric Nash equilibrium: if everyone is the same, all will choose the same $e$ and $e_i = \bar{e}$
  - This is 45-degree SNE line in graph
- Show multiplier effect in simple strategic complementarity
  - Exogenous shift that raises $e_i^*(\bar{e})$ curve leads to increase in $e$ that is larger than original shift, just like Keynesian multiplier
  - Suppose that there is a variable $x$ that affects MU of $e$ so that an increase in $x$ from $x_0$ to $x_1$ leads to an increase in desired $e_i$



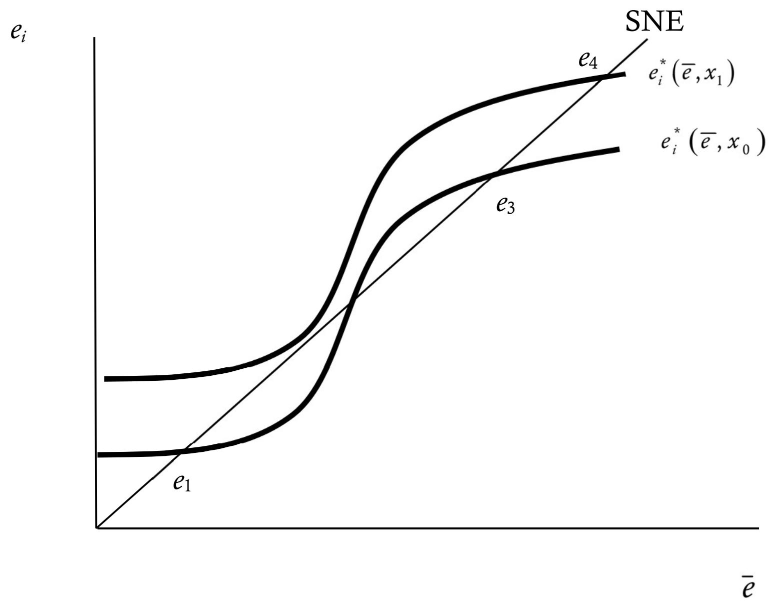

- Equilibrium increase in $y$ is larger than original change
- What if $\frac{\partial e_i^*}{\partial \bar{e}} > 1$?
  - That would lead to instability and infinite $e$ if reaction function is linear
  - But an interesting situation occurs if there are regions where the slope > 1

- Multiple equilibria
  - Consider the reaction function below




- There are three equilibria here:

    - $e_1$ and $e_3$ are stable; $e_2$ is unstable
  - Equilibria are Pareto ranked: more $y_i$ is better, so $e_3$ is preferred to others
  - Bus system example
  - Poverty traps




- Multiplier + multiple equilibria allows for large effects of small policy changes:
  - Suppose that economy is stuck at low equilibrium $e_1$
  - Small change in policy $x_0$ to $x_1$ shifts each agent's response function upward as shown
  - Natural dynamics of system moves economy to $e_4$
  - After economy is at $e_4$ (or close), can switch policy back to $x_0$ and economy will converge to $e_3$

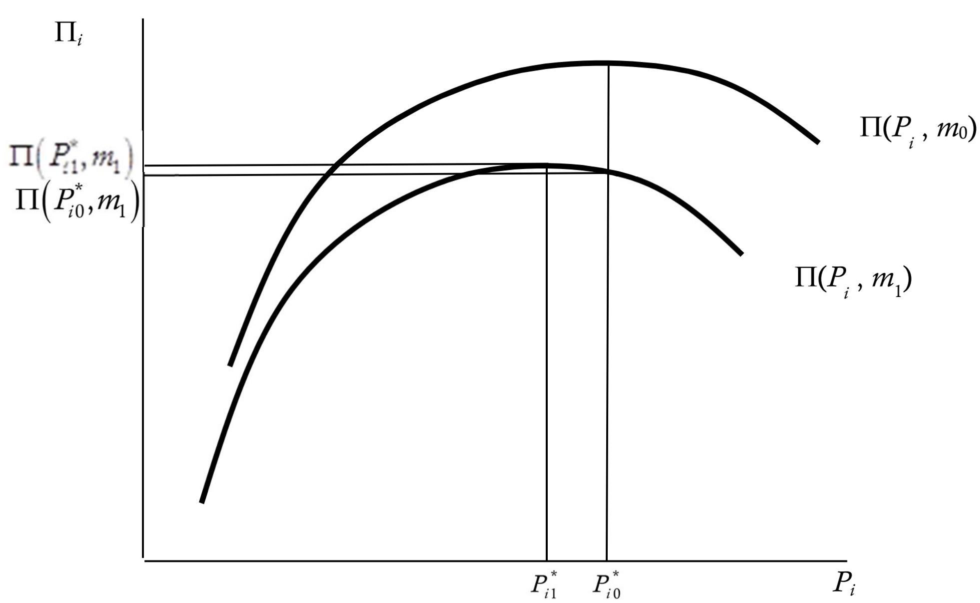
# Nominal and Real Rigidities

## Distinction between nominal and real rigidities

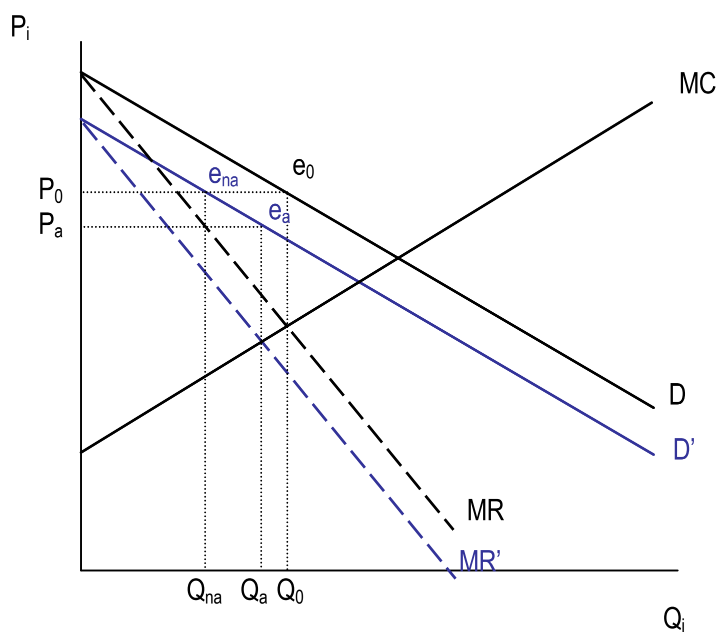
- Nominal rigidities are considerations that cause firms to keep price fixed in nominal (dollar) terms
    - Menu costs are an example
    - Nominal rigidities result in stickiness in $P_i$
- Real rigidities are considerations that cause firms to keep price fixed in relative terms
    - Real rigidities result in stickiness in $\frac{P_i}{P}$.

## Profits and price stickiness

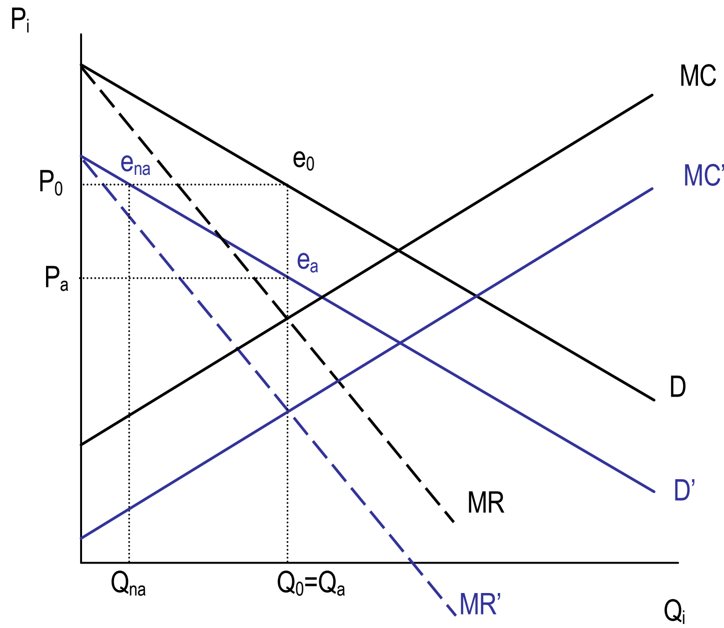
- An insight due to a number of authors (Parkin, Akerlof and Yellen, and Mankiw) is that the lost profit due to a small deviation from the optimal price may be second-order small
- Romer's Figure 6.11 illustrates this.
    - Let $P_{i0}^*$ be the firm's initial price, which is optimal when aggregate demand is high (at $m_0$)
    - $P_{i1}^*$ is the firm's optimal price when aggregate demand is low (at $m_1$)
    - Because of flatness of profit function near maximum, a small menu cost $z$ may still induce the firm to keep price fixed: price stickiness is optimal if $z > \Pi\left(P_{i1}^*, m_1\right) - \Pi\left(P_{i0}^*, m_1\right)$, which is very small



- How does firm lose profit by not adjusting? Romer's Figure 6.10 illustrates monopolist's production problem
  - Figure below illustrates the effects of a decline in AD, assuming that other firms do not adjust their prices.
  - If prices in the rest of the economy do not adjust, then our firm's decline in demand is smaller (because our relative price is still competitive) and marginal cost does not adjust (because no change in input prices)
  - Non-adjustment: Price at $P_0$ and production at $Q_{na}$
  - Adjustment of price: Price at $P_a$ and production at $Q_a$
  - Lost profit from non-adjustment is triangle bounded by MR′, MC, and $Q_{na}$




- How is cost of non-adjustment different if all other prices in the economy do adjust?
  - Now demand falls by more because other firms' prices are lower (and our good is a substitute for theirs) and MC falls in proportion to demand (because input prices have fallen by same amount as demand)
  - Cost of non-adjustment is now large triangle bounded by MR′, MC′, and $Q_{na}$

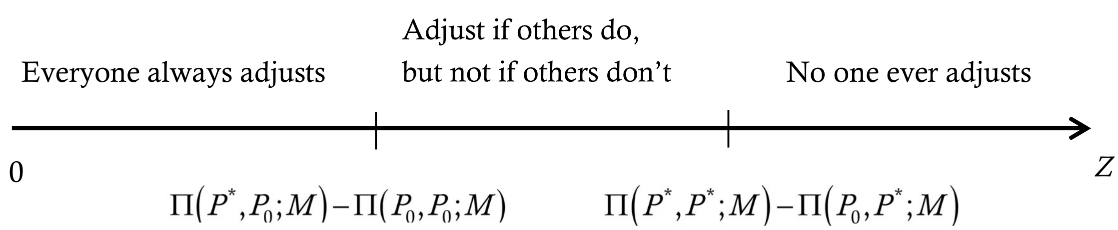



- Moral of the story: It is much more costly for our firm *not* to adjust its price if everyone else *does* adjust its price
  - We will see this soon as "real rigidities" reinforcing nominal rigidities
  - It's also a case of "strategic complementarity," which can lead to multiple equilibria and coordination failures

# Coordination Failures in Price Setting

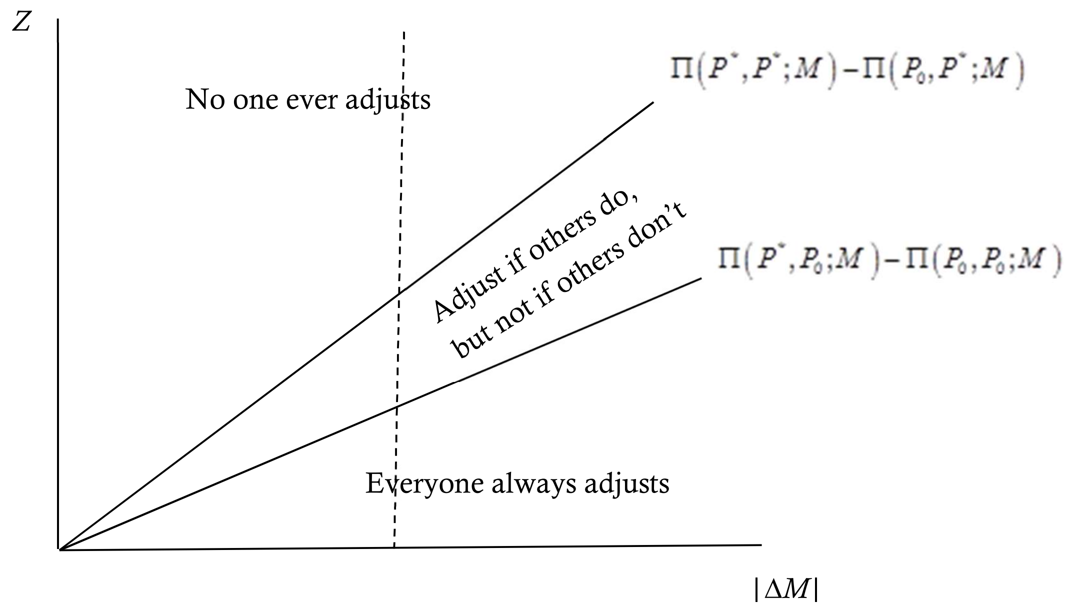
## Real and nominal rigidities

- We have seen that firms are more likely to adjust their price is other firms do because the lost profits from non-adjustment are greater if other firms adjust
- If other firms do not adjust, then our firm will choose not to adjust if $\Pi\left(P^*, P_0; M\right) - \Pi\left(P_0, P_0; M\right) < Z$ , where $Z$ is the menu cost
- If other firms do adjust, then out firm will not adjust if $\Pi\left(P^*, P^*; M\right) - \Pi\left(P_0, P^*; M\right) < Z$
- Because $\Pi\left(P^*, P_0; M\right) - \Pi\left(P_0, P_0; M\right) < \Pi\left(P^*, P^*; M\right) - \Pi\left(P_0, P^*; M\right)$, it is possible that for some range of changes in $M$, $\Pi\left(P^*, P_0; M\right) - \Pi\left(P_0, P_0; M\right) < Z < \Pi\left(P^*, P^*; M\right) - \Pi\left(P_0, P^*; M\right)$




  - If, given $Z$, $M$ is in a range that satisfies this condition, then each firm adjusts only if other firms adjust, and keeps price fixed if other firms keep prices fixed
  - An increase in $\Delta M$ shifts both of the threshold points in the graph above to the right, so all of the ranges move to the right.
    - Big $|\Delta M|$ means everyone adjusts unless $Z$ is really large
    - Small $|\Delta M|$ means that firms are unlikely to adjust
    - An intermediate range of $|\Delta M|$ may put firms into the middle range, with multiple equilibria
  - In this case, there are two equilibria: all firms keep prices fixed and all firms adjust prices
  - The diagram below illustrates the tradeoff in two dimensions, with the magnitude of the AD shock on the horizontal axis and the magnitude of menu costs on the vertical axis.
    - The dashed line represents the one-dimensional graph above, turned to be vertical at one particular level of $|\Delta M|$.
    - For very high menu costs or very small AD changes, we are in the upper-left section and no one adjusts prices.

- For very large AD changes or very small menu costs, we are in the lower-right section and everyone adjusts.
- In the intermediate section of the graph, there are multiple equilibria: full adjustment or full non-adjustment is possible.




- The price-adjustment equilibrium is Pareto-superior, but without external coordination no firm has incentive to be the first to adjust price
- We could have a sub-optimal equilibrium in which all firms choose price stickiness even though all would be better off if they all adjusted prices
- This is a coordination failure: $50 bills are being left on the sidewalk because of the strategic complementarity that leads to a sub-optimal equilibrium
  - No one has an incentive, on her own, to change behavior

- Why should we care???
  - If firm's optimal behavior is to keep prices constant, doesn't that mean that it is optimal for prices to stay constant?
  - Not if there are externalities from price stickiness:
    - With strategic complementarities ***and*** positive spillovers, it is possible to have multiple equilibria in which the **social cost** of price stickiness is high but the **private cost** is low
    - This would be a coordination failure in which price stickiness leads to a bad outcome that no one has a strong incentive to fix
- Given what we think we know about the parameters of the model ($\eta$ and $\nu$), how likely is it that $Z$ is large enough to cause price stickiness for meaningfully large changes in $M$?

## Romer's quantitative examples

- Profit maximizing pricing equation is $\frac{P_i}{P} = \left(\frac{\eta}{\eta - 1}\right)\left(\frac{W}{P}\right)$ or $\frac{P_i}{P} = \frac{\eta}{\eta - 1} Y^{\gamma - 1}$
    - We will find it convenient to write this equation in log form as $p_i^* - p = c + \phi y$, where $c \equiv \ln\left(\frac{\eta}{\eta - 1}\right)$ and $\phi = \gamma - 1 > 0$
    - Greater real rigidity corresponds to a smaller value of $\phi$, which is the reciprocal of the elasticity of labor supply
    - Greater real rigidity happens when optimal relative-price response to AD shock is small
- Firm's profit is $\Pi(P_i, P) = Y_i\left(\frac{P_i}{P}\right) - \left(\frac{W}{P}\right)Y_i$, recognizing that $Y_i = L_i$
    - Recall that in equilibrium, $Y_i = \left(\frac{P_i}{P}\right)Y$ and $\frac{W}{P} = Y^{\frac{1}{\nu}}$, where $\nu \equiv \frac{1}{\gamma - 1}$ is the elasticity of labor supply, and $Y = \frac{M}{P}$, where $M$ is our exogenous aggregate-demand variable
    - Substituting, $\Pi(P_i, P; M) = \frac{M}{P}\left(\frac{P_i}{P}\right)^{1-\eta} - \left(\frac{M}{P}\right)^{\frac{1+\nu}{\nu}}\left(\frac{P_i}{P}\right)^{-\eta}$
- **Flex-price equilibrium**
    - If firm sets profit-maximizing price with no price stickiness, the resulting equilibrium (with all firms setting same price) is $\frac{M}{P} = \left(\frac{\eta - 1}{\eta}\right)^{\nu}$, which reflects our earlier equilibrium condition
- **Fixed-price equilibrium**
    - Suppose that other firms keep price at pre-existing $P_0$
    - Our firm's profit if it keeps price fixed is $\Pi_{FIX} = \Pi(P_0, P_0; M) = \frac{M}{P_0} - \left(\frac{M}{P_0}\right)^{\frac{\nu - 1}{\nu}}$ because $\frac{P_i}{P} = \frac{P_0}{P_0} = 1$
    - If our firm changes price, then it sets $\frac{P_i^*}{P_0} = \left(\frac{P_i}{P}\right)^* = \frac{\eta}{\eta - 1}\left(\frac{M}{P}\right)^{\frac{1}{\nu}}$

- Profit is $\Pi_{ADJ} = \Pi\left(P^*, P_0; M\right) = \frac{M}{P_0}\left(\frac{P^*}{P_0}\right)^{1-\eta} - \left(\frac{M}{P_0}\right)^{\frac{\nu+1}{\nu}}\left(\frac{P^*}{P_0}\right)^{-\eta}$ with (from the original profit-maximizing condition)

$$\left(\frac{P^*}{P_0}\right) = \left(\frac{P_i}{P}\right)^* = \frac{\eta}{\eta-1}\left(\frac{W}{P}\right) = \frac{\eta}{\eta-1}\left(\frac{M}{P}\right)^{\frac{1}{\nu}} = \frac{\eta}{\eta-1}\left(\frac{M}{P_0}\right)^{\frac{1}{\nu}}$$

- Thus,

$$\Pi\left(P^*, P_0; M\right) = \frac{M}{P_0}\left[\frac{\eta}{\eta-1}\left(\frac{M}{P_0}\right)^{\frac{1}{\nu}}\right]^{1-\eta} - \left(\frac{M}{P_0}\right)^{\frac{\nu+1}{\nu}}\left[\frac{\eta}{\eta-1}\left(\frac{M}{P_0}\right)^{\frac{1}{\nu}}\right]^{-\eta}$$

$$= \frac{1}{\eta-1}\left(\frac{\eta}{\eta-1}\right)^{-\eta}\left(\frac{M}{P_0}\right)^{\frac{1+\nu-\eta}{\nu}}$$

- **Will firm adjust price?**
  - Firm will adjust if menu cost $Z < \Pi\left(P^*, P_0; M\right) - \Pi\left(P_0, P_0; M\right)$
  - Romer calibrates $\eta = 5$ (markup = 25%) and $\nu = 0.1$
  - Under this calibration, $Z$ must be 25% of total revenue to justify price stickiness in response to 3% change in $M$
  - It is very unlikely that menu costs are 25% of revenue, so it doesn't seem like we have a good microeconomic rationale for price stickiness from this model
  - Show Table 1 from Ball and Romer paper
    - This model predicts significant price stickiness only with unreasonably low $\eta$ or unreasonably high $\nu$
    - With low $\nu$ and a clearing labor market, as in the RBC model, the real wage moves a lot with small changes in employment
    - The $\phi$ parameter in $p_i^* - p = c + \phi y$ is $1/\nu$ and inversely measures real rigidity, so small $\nu$ means little real rigidity
    - This changes costs a lot and makes price stickiness much more costly
- **Models with greater real rigidity**
  - Many possible ways of making marginal revenue and/or marginal cost less responsive to changes in demand
  - **Customer-markets model**
    - Each customer has a particular "home" market at which he shops until something happens that triggers him to search
      - Once search is triggered, he surveys markets and chooses the one with the lowest price as him new home market
    - Price information is imperfect and asymmetric

    - Because he shops at the home market, he observes any price changes immediately
    - He does not observe any other market's prices unless he decides to search
  - Increase in price will be observed by all home customers, many of whom may decide to search and thus will be lost as customers
    - Increase in price leads to very elastic response: many customers reduce purchases
  - Decrease in price will keep home customers, but they are already shopping there; but only a few searching customers will be attracted
    - Decrease in price leads to very inelastic response: not many new customers attracted
  - Demand curve has a kink at the current price
    - This leads to discontinuity in the MR curve
    - Small changes in MC do not change optimal price because they stay within the gap
    - Small changes in demand (horizontal) also do not change optimal price
  - Ball and Romer implement this model as a limiting case of differentiable demand curve with $\rho = -\left.\frac{\partial^2 \ln Q_i^d}{\partial\left[\ln\left(P_i / P\right)\right]^2}\right|_{\frac{P_i}{P}=1}$ = elasticity of price elasticity of demand with respect to relative price
    - $\rho = 0 \Rightarrow$ constant elasticity of demand (as in our base model)
    - $\rho \rightarrow \infty \Rightarrow$ kinked demand with discontinuous elasticity at $\frac{P_i}{P} = 1$
  - Table 2 from Ball and Romer shows that a large $\rho$ and realistic $\eta$ can lead to situation where
    - Private cost of nominal rigidity is small enough that firms will keep prices fixed for meaningful changes in $M$
    - Social cost of rigidity is much higher than private cost, so this price rigidity can lead to a serious coordination failure
- **Romer's second quantitative example: "Real-wage function"**
  - Suppose that labor market does not clear and real wages are relatively inelastic with respect to output/employment:
    - Real-wage function: $\frac{W}{P} = AY^{\beta}$
    - Rest of analysis is the same except with $AY^{\beta}$ replacing $Y^{1/\nu}$

    - If $\beta = 0.1$, this is equivalent to a $\nu$ of 10
        - Labor-supply elasticity of 10 is very unrealistic
        - But empirical evidence shows that real wage does not move much over the cycle, so 10 might be appropriate in non-market-clearing model
- With $\beta = 0.1$ and realistic markup, can have small private cost and large social cost of stickiness