Economics 314 Spring 2019
Daily Problem #37 April 15

The fundamental underlying equation of dynamic programming is the Bellman equation. We can write this equation as

$$V_i(0) = \lim_{\Delta t \to 0} V_i(\Delta t) \equiv \lim_{\Delta t \to 0} \left\{ \int_{t=0}^{\Delta t} u(t \mid \text{state at } 0 = i)\, dt + e^{-\rho \Delta t} E\left[V(\Delta t \mid \text{state at } 0 = i) \right] \right\},$$

Where $V_i(0)$ is expected lifetime utility from date (0) of someone who is currently in state i. (Remember that there are three states in the Shapiro-Stiglitz model: E, S, and U.)

Our utility function is $U = \int_0^\infty e^{-\rho t} u(t)\, dt$, where

$$u(t) = \begin{cases} w(t) - \bar{e}, & \text{if state is E,} \\ w(t), & \text{if state is S, and} \\ 0, & \text{if state is U.} \end{cases}$$

1. What is the intuition and interpretation of the $u(t)$ function?

2. For someone in state E at time 0, the probability of still being in state E t time units later is e^{-bt} and the probability of having moved into state U is $\left(1 - e^{bt}\right)$. Explain intuitively why the Bellman equation for state E at time 0 can be written:

$$V_E(0) = \lim_{\Delta t \to 0} \left\{ \int_{t=0}^{\Delta t} e^{-\rho t} \left[e^{-bt}(w - \bar{e}) + \left(1 - e^{-bt}\right)(0) \right] dt + e^{-\rho \Delta t} \left[e^{-b\Delta t} V_E(\Delta t) + \left(1 - e^{-b\Delta t}\right) V_U(\Delta t) \right] \right\}.$$