Economics 314 — Spring 2019
Project #7 Assignment — Due: 9am, Wednesday, April 10

Partner assignments

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Problems

1. Work Romer's Problem 6.11.

- Notice that $\phi < 1$ is the normal case and $\phi > 1$ is a case of "negative real rigidity." This may help you interpret the results.
- Show on your graph the values of the gains from price adjustment at $f = 0$ and $f = 1$ in each case.
- As an addition to part (c): For a given value of Z, find the range of values of m' (if any) for which multiple equilibria may exist (i.e., for which either full price adjustment or non-adjustment is an equilibrium).

2. Work the following variant of Romer's Problem 6.13 on wage indexation.

The economy is composed of a large number of firms distinguished by subscript *i*. Firm *i* has a production function given by $Y_i = SL_i^{\alpha}$, where S is a supply shock and $0 < \alpha \leq 1$. The log of the firm's production is $y_i = s + \alpha \ell_i$, where the lower-case letters refer to the logs of the respective capital letter variables. Aggregation takes the form of averaging the logs of variables, so aggregate output is $y = s + \alpha \ell$. (This form of aggregation makes the aggregate variable equal to the geometric mean of the individual values rather than the sum or the arithmetic mean.) Aggregate demand is given by $y = m - p$. The logs of the supply shock and the aggregate-demand shock (s and m) are random variables with mean of zero and variances of V_s and V_m, respectively. Firms are price-takers in product markets.

a. What is the marginal product of labor? Price-taking firms will set the marginal product of labor equal to the real wage. Can you derive Romer's equation $p_i = w_i + (1-\alpha)\ell_i - s$ from this condition? Why or why not? Does his assumption that we can simply set the constant equal to zero seem reasonable to you? (Making this assumption does not change any of the subsequent results, so we will use it below.)

b. Suppose that labor supply is inelastic at the level of one unit per person (and per firm), so $L^s = 1$ and $\ell^s = 0$. Let aggregate labor demand be given (in log terms) by the mean across firms of Romer's price equation: $\ell^d = \frac{1}{1-\alpha}(p - w + s)$. Assume initially that the wage is perfectly flexible, in other words, that it adjusts to make aggregate labor supply equal to labor demand. Show that when $s = 0$ and $m = 0$, the equilibrium values of the logs of the endogenous variables are $w = p = y = \ell = 0$.

c. Now suppose that nominal wages are set in contracts that last one period. At the beginning of the period, firms and workers determine a rule for setting the nominal wage. The rule may set a specific nominal wage for the period, or it may allow the nominal wage to vary depending on how high or low prices turn out to be: an *indexed* contract. Given the nominal wage, the quantity of employment is then determined by firms' labor demand. (As part of the contracting process, workers agree to work as much as firms want them to.) We will consider three types of contracts: non-indexed contracts that fix the nominal wage, fully-indexed contracts that adjust the nominal wage fully in proportion to changes in prices (effectively fixing the real wage), and partially indexed contracts that adjust the nominal wage partially to changes in prices.

If they were going to set a fixed nominal wage for the period, they would presumably set the log-wage to zero, since that is the level that clears the market when there are no shocks (i.e., when the shocks are at their expected values—zero). Find expressions for p, y, and ℓ as functions of the shocks m and s and the parameter α when contracts are not indexed and the nominal wage is fixed at its expected equilibrium value of zero. Use these expressions to analyze how p, y, and ℓ are affected by the two shocks and explain the intuitive economics of what is happening for each shock.

d. Now suppose that contracts are fully indexed, so that $w = p$. This means that whenever the price level deviates from its expected value of zero, nominal wages are adjusted in a corresponding way to keep the (log) *real* wage $w - p$ at *its* expected value of zero. Find the corresponding expressions for p, y, and ℓ as functions of the shocks m and s and the parameter α in this case and, as above, use them to analyze, both mathematically and intuitively, how the shocks affect the three main endogenous variables. Compare these results to those of part c and explain why they are different.

e. Now consider the intermediate case with partial indexation. Suppose that firms and workers agree on an indexing rule stipulating that $w = \theta p$. We assume that $0 \leq \theta \leq 1$. If $\theta > 0$, this rule moves the wage away from zero whenever a money or supply shock pushes prices away from zero. Find expressions for p, y, and ℓ as functions of the shocks m and s and the parameters θ and α. How does the sensitivity of each of the variables to m and to s

change as the indexation parameter θ increases? Is that consistent with your results from above?

f. In evaluating the desirability of alternative policy rules, we often look for rules that minimize the effects of shocks. We implement this idea by seeking parameter values that minimize the variation of employment or output. Here, the "policy" parameter is the amount of indexation θ, so we seek the value of θ that minimizes variation. In this model, it is better to stabilize employment rather than output because output *should* fluctuate in response to supply shocks s whereas as long as labor supply is perfectly inelastic the optimal, equilibrium value of ℓ is zero. Standard statistical formulas for the variance of a random variable tell us that if

$$\ell = a_m m + a_s s,$$

where the a coefficients are constants and m and s are random variables whose probability distributions have variances V_m and V_s, respectively, then

$$\text{var}(\ell) = a_m^2 V_m + a_s^2 V_s + 2a_m a_s \,\text{cov}(m, s),$$

where cov(m, s) is the covariance between the two shocks. Assume that the two shocks are independent so their covariance is zero. Find an expression for the variance of log-employment as a function of V_m and V_s and the parameters of the model (including θ). Based on this expression, find the indexing rule (i.e., the value of θ) that minimizes the variance of employment. Discuss optimal indexation in the limiting case of $V_m = 0$ and in the alternative limiting case of $V_s = 0$. Based on this result, what characteristics of an economy make it desirable to index wage contracts?

g. Consider two countries that are equally subject to supply shocks (i.e., where the variance of s is the same in both countries), but one of which, say, Argentina, has a higher variability of monetary shocks than the other, say, Germany. (This is kind of like the experiment in the Lucas paper that is summarized in Romer's Section 6.10: different countries have more or less volatile aggregate demand.) Use your results from part f to explain which country will have the higher degree of indexation. In part e you calculated the elasticity of Y with respect to M, $\partial y/\partial m$, which depended on θ. Use this result to explain which country will have the more elastic aggregate supply curve. How does this result compare with the differences across countries in the slope of the AS curve predicted by Lucas's imperfect-information model?

h. In a country with less-the-full indexation, will the real wage (W/P, or $w - p$ in log terms) be procyclical (positively correlated with output) or countercyclical when business cycles are caused by monetary (demand) shocks? By supply shocks? How do these results correspond to observed, slightly pro-cyclical variation in real wages?