Economics 314 | Spring 2019
Project #1 Assignment | Due: 9am, Wednesday, February 6

Partner assignments

Partners	
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Each team should submit a single paper. You are to work as a team in solving the problems. At a very minimum, both partners must understand everything that's on the paper when you turn it in. You are free to interact with other teams and to seek advice from me as you work on the problems, especially on the mathematical solutions. The idea of problem sets is to learn from them, not to test whether you can do them perfectly without help.

Problems

1. Golden Rule growth path. (Based on Romer's Section 1.4) Economies that follow the Solow growth model converge to a steady-state growth path on which the capital/effective-labor ratio remains constant at some level k^* and the level of output per effective labor unit y^* is also constant, with $y^* = f(k^*)$. Given the (assumed constant) levels of n and g (and the production function and depreciation rate δ), k^* can be thought of as a function of the saving rate s, $k^*(s)$.

a. Differentiate the steady-state equation to find an expression for the effect of s on k^*, $\partial k^* / \partial s$, and determine its sign. Note: Since the steady-state expression defines k^* as an implicit function of s, this will need to be an implicit differentiation of each term with respect to s using the chain rule and other rules of differentiation as appropriate. Any time that k^* appears, you need to include $\partial k^* / \partial s$ via the chain rule. You can then solve the expression for $\partial k^* / \partial s$.
b. Find the corresponding expression for $\partial y^* / \partial s$ and determine its sign.
c. Suppose that an economy wants to choose its saving rate in order to maximize per-capita income in the steady state. What saving rate would it choose? (Remember that $s \in [0,1]$.)

What is the value of per-capita steady-state consumption c^*A, where $c^* = (1-s)y^*$? Does this seem optimal? Explain.

d. Suppose instead that the economy decides to choose the saving rate to maximize steady-state per-capita consumption. What is the first-order condition for this maximization problem? (Note: You won't be able to solve this for *s* without assumption a functional form for *f.* You don't need to.) Show and interpret this condition (in terms of the slopes of the curves) on the Solow-model equilibrium diagram.

e. If our Solow economy had a simple and "perfect" financial market, then the real interest rate on a bond should match the net rate of return on capital: $r = f'(k) - \delta$. (We must subtract the depreciation rate to get the *net* return on capital.) Show what the maximization condition in part d implies for the relationship between the steady-state equilibrium real interest rate and the steady-state real output-growth rate in a "Golden Rule" economy that is maximizing steady-state per-capita consumption. What does this imply for *nominal* interest rates?

f. When we study the Ramsey and Diamond growth models in Chapter 2, we will assume that people always, other things being equal, prefer to consume sooner rather than later. Think about an economy that is on its Golden Rule growth path, maximizing steady-state consumption per person. Why might the people in this economy be better off saving a bit less than the Golden Rule rate? Would they ever be better off saving more than the Golden Rule rate? Explain.

g. We can never truly observe a steady state, but over 2000–07 the U.S. nominal interest rate on 3-month Treasury bills averaged 3.20%, 10-year Treasury bonds averaged 4.71%, and nominal GDP grew at 4.99%. If we take this period to approximate a steady state, what do these data imply about U.S. saving rate relative to the Golden Rule rate? Why?

2. Natural resources in the Solow model. One criticism of traditional growth models is that they do not take account of the use of natural resources—which are not available in unlimited quantities—in assessing the potential for output to grow indefinitely into the future. Romer presents a model in Section 1.8 that incorporates finite land and natural resources.

a. Romer writes the production function as $Y(t) = K(t)^{\alpha} R(t)^{\beta} T(t)^{\gamma} \left[A(t)L(t)\right]^{1-\alpha-\beta-\gamma}$, where $R(t)$ is the amount depletable natural resources used up in production at time *t* and $T(t)$ be the amount of land used at time *t*. Does this Cobb-Douglas function have constant returns to scale? Show this by multiplying each input (not including *A*) by a constant *x* and evaluating the effect on output.

b. Based on this production function, evaluate the model's assumption about each of the following propositions, and assess the extent to which the assumption is reasonable:
    i. Nothing can be produced without some input of land and natural resources.
    ii. With enough labor and capital, we could produce the current U.S. GDP on one square inch of land.
    iii. With enough labor and capital, we could produce the current U.S. GDP with one drop of oil.

c. Given that R in this model refers to the flow use of depletable and non-renewable resources, could we sustain a constant level of R in the steady state? Explain why or why not and how this underpins Romer's equation (1.43).

d. This model follows the standard Solow equations of motion for labor, technology, and capital. In the basic Solow model, the inputs (capital and effective labor) grow at the same rate in the steady state ($n + g$). Given the equations of motion for A, L, and K, and the equations of motion for T and R given in equations (1.42) and (1.43), is this kind of steady-state growth path possible in the enhanced model with resources? Why or why not?

e. An alternative (to finding the conditions for $\dot{k} = 0$) method of finding the steady-state path is to find conditions under which the growth rate of the basic state variable K is constant. First we consider a simpler model with $\beta = \gamma = 0$. Show that this is equivalent to the basic Solow model (with Cobb-Douglas production function). Equation (1.44) applies in this model as well as the model with resources. Explain in detail the logic of Romer's assertion that "for the growth rate of K to be constant, Y/K must be constant ... [so] the growth rates of Y and K must be equal." For the model without resources, show that the steady-state condition derived using this method is identical to the one derived in class from $\dot{k} = 0$.

f. Replicate and explain Romer's derivation of the steady-state growth rates of K, Y, and Y/L. Verify that these growth rates are consistent with the basic Solow model in that they reduce to the usual Solow values if $\beta = \gamma = 0$.

g. What are your conclusions from the model of Section 1.8?

3. Work Romer's Problem 1.3 with additional discussion.

- For each of the changes proposed in Problem 1.3, answer each of the following:
    - i. How, if at all, and why is the actual investment line affected?
    - ii. How, if at all, and why is the breakeven investment line affected?
    - iii. How, if at all, does the steady-state value of k change?
    - iv. How, if at all, does the steady-state growth rate of real output change?
- In part (c), does it matter whether k is greater than, less than, or equal to one? Given that k is on the horizontal axis of your diagram, be careful about how the shape of the production function changes at different points on the curve.

4. Work Romer's Problem 1.4 with additions. Note that this problem is *not* an increase in n; it is a one-time rise in L. At the instant of the increase, $n = \infty$, then it immediately returns to its original value.

- Assume that the new workers bring no capital with them and that their arrival has no effect on A.
- ***Addition*** to part (a): "What happens to total output Y at the time of the jump? Why?"
- ***Addition*** to part (c): "Is *total* output on the new balanced growth path higher, lower, or the same as it would have been if there had been no new workers? Why?"