

*Partner assignments*

Sarah Brauner	Jacob Menick
Alejandro Chavez	Vlad Mankov
Logan Donoughe	Taylor Holdaway
Orphelia Ellogne	Luke Hill
Natalie Pong	Reilly Villanueva
Peter Riehlman	Will Schmid
Nick Fiore	

---

## Project overview

In this project you will use a numerical software package called Dynare to simulate Romer's real-business-cycle model of Chapter 5. In particular, you will construct your own versions of Figure 5.2 through Figure 5.7, and compare the correlations and autocorrelations of the variables in stochastic simulations to those of the actual U.S. economy.

In order to facilitate solution in Dynare, we must simplify the model by expressing it in terms of log-deviations from the model's steady-state values, then calculating a linear approximation of the model's equations near the steady state in terms of these log-deviations. Parts of this process are done below; the remainder are part of the assignment, which may be done on paper.

Once you have achieved the log-linearization, you will use Dynare (operating within the commercial numerical package MATLAB or the open-source substitute Octave) to perform various deterministic and stochastic simulations. This part is done on a computer (either your own or a Reed computer) on which Dynare and MATLAB/Octave have been installed and should be submitted electronically in a Word file. (Both Dynare and Octave can be downloaded without cost.) *This project is still something of a "work in progress," so if you encounter problems with the software, please ask ASAP so that some combination of me and CUS can solve them for you and for others.*

In the following pages, the tasks that you are to perform are listed in sans-serif text with three bold lines on the left margin:

**|||** This is how an assigned task will look.

---

# Preparing the model for simulation with Dynare

## *Some small changes in notation*

Romer's model is a variation of the model discussed by Campbell (1994). You may wish to refer to this paper when doing this assignment. Romer's version makes two essential changes to the Campbell model: (1) Romer adds government spending as a component of output demand and (2) Romer's model restricts the labor/leisure term of the additive utility function to be logarithmic rather than using the CRRA form.

Our focus here is on business cycles, so we approximate the *deviations* of the variables around the steady state and ignore changes in the steady-state values themselves over time. In Romer's model, the population  $N$  grows at a constant rate  $n$  and never deviates from its steady-state path. Thus, the deviations from steady-state never arise from changes in population and we can ignore population growth in the population. Without loss of generality, we set the constant value of the population at  $N = 1$ .

In converting the model to a log-linear approximation to deviations from the steady state, it is useful to follow Campbell's notation convention of using lower-case letters to represent the logs of capital-letter variables. This requires some minor notation changes from Romer's Chapter 5. For example,  $k \equiv \ln K$ , not  $K/AL$  as in the growth models that we have studied. Moreover, in this assignment,  $c \equiv \ln C$  and  $l \equiv \ln L$ , whereas Romer's Chapter 5 uses  $c$  and  $l$  to refer to  $C/N$  and  $L/N$ , respectively. (We don't need the per-capita variables here because we have set  $N = 1$ .) We will try to be careful in the following pages to remind ourselves of the exact definitions of these variables.

In addition, we shall make a couple of trivial modifications to Romer's equations.

- We use  $K_t$  to refer to the capital stock at the *end* of period  $t$  rather than (as Romer uses it) the *beginning*. This is a mere change in timing convention that conforms better to Dynare's requirements; it does not change the structure of the model at all.
- We define  $R_t$  to be the "gross" return on capital, which is one plus the net return that Romer calls  $r_t$ . Intuitively, the gross return on a bond includes the repayment of the bond plus the interest earned. Following our logarithmic-notation convention, we will define  $r_t$  to be  $r_t \equiv \ln R_t \cong R_t - 1$ , so  $r_t$  is (within our approximation) the same as Romer's variable.
- To reserve lower-case letters for logs, we denote the real wage by  $W$  and its log by  $w$ .
- Finally, because we will need  $g$  to be the log of  $G$ , we will denote the growth rate of productivity (Romer's  $g$ ) as  $\gamma$ , with  $e^\gamma \cong 1 + \gamma$ .

## *The Romer model in our notation*

We now proceed to state the equations of the Romer model with these notation changes. To make it convenient to refer back to the full set of equations in a compact location, all eight equations are listed first, with descriptions following:

$$Y_t = K_{t-1}^\alpha (A_t L_t)^{1-\alpha} \quad (1)$$

$$K_t = (1-\delta)K_{t-1} + Y_t - C_t - G_t \quad (2)$$

$$W_t = (1-\alpha) \left( \frac{K_{t-1}}{A_t L_t} \right)^\alpha A_t, \quad (3)$$

$$R_t = \alpha \left( \frac{A_t L_t}{K_{t-1}} \right)^{1-\alpha} + (1-\delta). \quad (4)$$

$$\frac{1}{C_t} = e^{-\rho} E_t \left( \frac{R_{t+1}}{C_{t+1}} \right). \quad (5)$$

$$bC_t = (1-L_t)W_t. \quad (6)$$

$$\begin{aligned} \ln A_t &\equiv a_t = \gamma t + \tilde{a}_t, \\ \tilde{a}_t &= \rho_A \tilde{a}_{t-1} + \varepsilon_{A,t}, \end{aligned} \quad (7)$$

$$\begin{aligned} \ln G_t &\equiv g_t = \bar{g} + \gamma t + \tilde{g}_t, \\ \tilde{g}_t &= \rho_G \tilde{g}_{t-1} + \varepsilon_{G,t}. \end{aligned} \quad (8)$$

Equation (1) is Romer's (5.1), the production function. The only change here is that the beginning-of-period capital stock is now  $K_{t-1}$  rather than  $K_t$ .

Equation (2) is Romer's (5.2), the capital-accumulation equation. Again, the only change is the adjustment of the time subscripts of capital.

Equation (3) is Romer's (5.3), which sets the wage equal to the marginal product of labor. The sole change here is that we are using capital  $W$  for the wage (and reserving lower-case  $w$  for its log).

Equation (4) is Romer's (5.4), which sets the interest rate equal to the marginal product of capital minus the depreciation rate. Note that we have added one to both sides of (5.4) to get (4):  $R_t$  on the left is  $e^{r_t} \cong 1 + r_t$ , and the term  $-\delta$  on the end of the right-hand expression has changed to  $1 - \delta$ .

Equation (5) is Romer's (5.23), the Euler equation for intertemporal consumption. We use capital  $C$  here because our variables are already in per-capita terms (and we want lower-case  $c$  to be its log) and our  $R_{t+1} \equiv 1 + r_{t+1}$ .

Equation (6) is Romer's (5.26), which equates the marginal utility of consumption to that of leisure. Again,  $C$  and  $W$  are in capitals rather than Romer's lower-case, but otherwise the equations are identical.

Equations (7) and (8) are the equations of motion for technology and government spending, corresponding to Romer's (5.8) through (5.11). The equations are identical except for

introducing lower-case  $a$  and  $g$  as the logs of  $A$  and  $G$ , setting  $n = 0$  (because we ignore population growth) and replacing Romer's growth rate  $g$  by  $\gamma$ .

Equations (1) through (8) constitute eight equations in  $Y, K, L, A, W, R, C$ , and  $G$ , including lags of  $K$  and expected future values of  $C$  and  $R$ . To gain some insight into how such a model is solved, suppose that we consider an economy that starts at time zero in a steady state with a given, known value of  $K_0$ . Whatever shocks hit the system at time 0, we expect that the economy will re-converge to a steady state after some large number of periods, say  $T$ . We can iterate the evolution of capital forward through the  $T$  periods from its initial value  $K_0$  using equation (2) (and the other equations of the model). Similarly, because we know that the shock will have died out by period  $T + 1$ , we assume that  $C$  and  $R$  will be back to their steady-state values by then. This allows us to solve the model as  $8 \times T$  equations in the  $8 \times T$  variables corresponding to periods 1 through  $T$ . This solution is much more feasible if the equations of the model are linear (not to mention making handling the expectations feasible), so our next step will be to express the model as a linear approximation of deviations from the steady state.

### ***Calibration of the model***

Recent advances in numerical simulation methods allow one to estimate parameters of the model based on macroeconomic data. (Indeed, Dynare is capable of doing such estimation.) However, the econometrics of this are beyond our scope and the emphasis here is less of achieving an ideal fit than on understanding how the model works, so we will assume a set of parameter values for the model.

In the first paragraph of Romer's Section 5.7, he suggests the following values for the parameters of a model (assuming that each period is one quarter):

- $\alpha = 1/3$ , which corresponds roughly to capital's share of income,
- $\gamma = 0.005$ , which is a 2% per year growth rate of productivity,
- $\delta = 0.025$ , a 10% per year rate of depreciation,
- $\rho_A = \rho_G = 0.95$ , which means that both productivity and government spending return to their steady-state paths at a rate of 5% per quarter.

The other parameters ( $\rho$ ,  $b$ , and  $\bar{g}$ ) will be set in such a way that the steady-state government/output ratio is  $(G/Y)^* = 0.2$ , the steady-state interest rate is  $r^* = 0.015$  (per quarter), and steady-state  $L^* = 1/3$  (meaning that our representative agent works one-third of the time).

### ***Some key steady-state values***

When we do the log-linear approximations to the model, it will be very useful to know some key values in the steady state. The long-run behavior of this model is essentially identical to that of the Ramsey growth model: compare equations (1), (2), and (5) to the corresponding equations from Chapter 2. Thus, we can confidently expect that  $Y, K, C, G, W$ , and  $A$  should all grow at constant rate  $\gamma$  on their steady-state growth paths and that  $R$  and  $L$  should be constant in the steady state. Indeed, we assumed in the calibration discussion above that the steady-state values are  $R^* = 1 + r^* = 1.015$  and  $L^* = 1/3$ .

Because many of the variables are growing in the steady state, it makes sense to think about ratios among them that might be constant, much like the  $K/AL$  variable we used in the Solow and Ramsey growth models. It will be convenient later to know the steady-state values of three ratios:  $A_t/K_{t-1}$ ,  $Y_t/K_{t-1}$ , and  $C_t/Y_t$ , which we denote respectively as  $\left(\frac{A}{K_{-1}}\right)^*$ ,  $\left(\frac{Y}{K_{-1}}\right)^*$ , and  $\left(\frac{C}{Y}\right)^*$ .

We can immediately determine the steady-state value of the interest rate in terms of the model's parameters from (5). In the steady state, consumption is growing at rate  $\gamma$  so  $C_{t+1} = e^\gamma C_t$ . This means that

$$\begin{aligned} \frac{1}{C_t} &= e^{-\rho} \frac{R^*}{e^\gamma C_t}, \\ R^* &= e^{\rho+\gamma}. \end{aligned} \tag{9}$$

Given our calibrations of steady-state  $r$  and  $\gamma$ , this allows us to determine  $\rho$ . (For the values suggested above,  $R^* \cong 1.015 = e^{\rho+0.005}$ , or  $\rho = 0.0099 \cong 0.01$ .)

From equation (4), in the steady state,

$$\begin{aligned} R^* &= e^{\rho+\gamma} = \alpha \left[ \left(\frac{A}{K_{-1}}\right)^* L^* \right]^{1-\alpha} + (1-\delta), \\ \left[ \left(\frac{A}{K_{-1}}\right)^* L^* \right]^{1-\alpha} &= \frac{e^{\rho+\gamma} - (1-\delta)}{\alpha}, \\ \left(\frac{A}{K_{-1}}\right)^* L^* &= \left[ \frac{e^{\rho+\gamma} - (1-\delta)}{\alpha} \right]^{\frac{1}{1-\alpha}}, \end{aligned}$$

or

$$\left(\frac{A}{K_{-1}}\right)^* = \frac{1}{L^*} \left[ \frac{e^{\rho+\gamma} - (1-\delta)}{\alpha} \right]^{\frac{1}{1-\alpha}} \cong \frac{1}{L^*} \left( \frac{\rho + \gamma + \delta}{\alpha} \right)^{\frac{1}{1-\alpha}}. \tag{10}$$

Recall that we are setting  $L^* = 1/3$  as a calibration parameter, so equation (10) gives us a steady-state value for the ratio of  $A$  to last period's  $K$  that depends only on parameters we assume that we know.

Following a similar process, you can

1. Show that the steady-state value of  $Y_t/K_{t-1}$  is  $(\rho + \gamma + \delta)/\alpha$  and that the steady-state value of  $C_t/Y_t$  is  $1 - (G/Y)^* - \alpha(\gamma + \delta)/(\rho + \gamma + \delta)$ .

(Remember that you may turn in the algebraic sections in on paper if you wish.)

### ***Log-linearization of deviations from the steady state***

To log-linearize the model, we introduce an additional bit of notation. For a variable  $X_t$ , the log-deviation is defined as  $\tilde{x}_t \equiv \ln X_t - \ln X_t^* = x_t - x_t^*$ , where  $X_t^*$  is the value that  $X$  would have if the economy were in a steady state at  $t$ . The goal of log-linearization is to express the equations of the model as a linear function of these log-deviation variables. Some equations are already linear in the logs of the variables. Others are not linear in logs and must be approximated using Taylor series.

We will work through equations (1), (2), and (5) here. Equations (7) and (8) are already in terms of log-deviations. Part of your task for the project is to develop log-linear versions of equations (3), (4), and (6).

Equation (1) is an example of an equation that is already linear in logs. Such equations do not require approximation and are easy to handle. Taking the logs of both sides of (1) yields

$$y_t = \alpha k_{t-1} + (1 - \alpha)(a_t + l_t), \quad (11)$$

in terms of the lower-case logged variables. Because this equation must hold in the steady state, we know that

$$y_t^* = \alpha k_{t-1}^* + (1 - \alpha)(a_t^* + l_t^*). \quad (12)$$

Subtracting (12) from (11) gives us

$$\tilde{y}_t = \alpha \tilde{k}_{t-1} + (1 - \alpha)(\tilde{a}_t + \tilde{l}_t). \quad (13)$$

Equation (13) is linear in the log-deviation (tilde) variables, so it is the first equation of our log-linearized system.

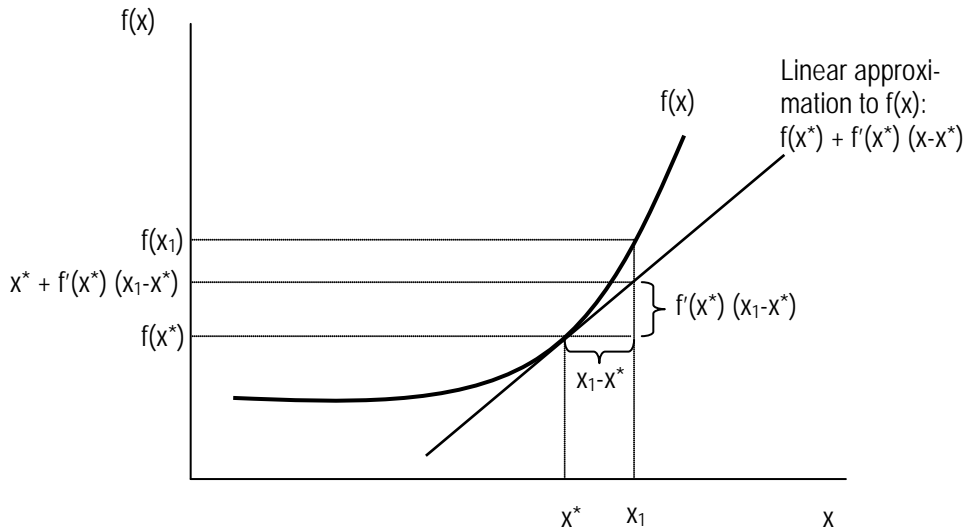
Equation (2) is linear in the *levels* of the variables, not in their *logs*. Thus, while it appears to be a very simple equation it is actually the most challenging to put in log-linear form, requiring us to use a Taylor-series approximation around the steady-state value.

Taylor's Theorem allows us to approximate a function by a straight line around a point at which the function's value is known. We used first-order Taylor series in our analysis of the rate of convergence in the Solow model. (See the related section near the end of Coursebook Chapter 3.) The basic idea of the Taylor series is that a general function  $f(x)$  can be approximated near a specified value  $x^*$  by

$$f(x) \cong f(x^*) + f'(x^*)(x - x^*), \quad (14)$$

where, at the specified value  $x^*$ , the value of the function  $f(x^*)$  and its first derivative  $f'(x^*)$  are known or can be readily calculated.

In the figure below, the straight line is the linear approximation to the nonlinear function  $f(x)$  around the value  $x^*$ . You can see that the first-order (linear) Taylor approximation is just the line that is tangent to the function at  $x^*$ . This approximation will be a good one if  $x$  is very close to  $x^*$  or if  $f(x)$  is nearly linear.



The Taylor approximation of equation (2) will be accomplished by transforming the equation into two additive parts, then taking the approximation of each part. We begin by dividing both sides of (2) by  $K_{t-1}$  to get

$$\frac{K_t}{K_{t-1}} = (1 - \delta) + \frac{Y_t}{K_{t-1}} - \frac{C_t}{K_{t-1}} - \frac{G_t}{K_{t-1}}$$

$$\frac{K_t}{K_{t-1}} - (1 - \delta) = \frac{Y_t}{K_{t-1}} \left( 1 - \frac{C_t}{Y_t} - \frac{G_t}{Y_t} \right).$$

This expression is convenient because we know the steady-state values of  $K_t/K_{t-1}$ ,  $Y_t/K_{t-1}$ ,  $C_t/Y_t$ , and  $G_t/Y_t$ :

- $K$  grows at rate  $\gamma$  in the steady state, so the steady-state value of  $K_t/K_{t-1}$  is  $e^\gamma \cong 1 + \gamma$ ,
- We calibrate the model to a chosen steady-state value of  $G_t/Y_t$ ,
- You will show as part of the project that the steady-state value of  $Y_t/K_{t-1}$  is  $(\rho + \gamma + \delta)/\alpha$ ,

- You will likewise show that the steady-state value of  $C_t/Y_t$  is  $1 - (G/Y)^* - \alpha(\gamma + \delta)/(\rho + \gamma + \delta)$ .

Taking logs of both sides of this equation,

$$\ln[\exp(\Delta k_t) - (1 - \delta)] = y_t - k_{t-1} + \ln[1 - \exp(c_t - y_t) - \exp(g_t - y_t)]. \quad (15)$$

Two parts of equation (15) warrant clarification. First, on the left-hand side,

$$\ln\left(\frac{K_t}{K_{t-1}}\right) = \ln K_t - \ln K_{t-1} = k_t - k_{t-1} \equiv \Delta k_t,$$

so  $K_t/K_{t-1} = \exp(\Delta k_t)$ . Second, on the right-hand side,  $\ln(C_t/Y_t) = c_t - y_t$ , so  $C_t/Y_t = \exp(c_t - y_t)$ , and similarly for  $G$ .

Equation (15) expresses equation (2) in terms of the logs of the variables, but it is highly nonlinear. We therefore approximate the nonlinear parts of (15) with first-order Taylor series, first for the function on the left of the equal sign then for the one on the right.

Working first with the left-hand side, define the function

$$f_1(\Delta k_t) \equiv \ln[\exp(\Delta k_t) - (1 - \delta)].$$

We can approximate  $f_1(\Delta k_t)$  in a neighborhood around the steady state by

$$f_1(\Delta k_t) \cong f_1(\gamma) + f_1'(\gamma)(\Delta k_t - \gamma),$$

where the growth rate  $\gamma$  is known to be the steady-state value of  $\Delta k_t$ . Because we are going to express the model in terms of *deviations* from the steady state, we need worry only about the second term. Using the chain rule,

$$f_1'(\Delta k_t) = \frac{1}{\exp(\Delta k_t) - (1 - \delta)} \exp(\Delta k_t).$$

Evaluating this expression at the steady-state value  $\Delta k_t = \gamma$  and taking advantage of the approximation  $\exp(\gamma) \cong 1 + \gamma$  gives

$$f_1'(\gamma) \cong \frac{1 + \gamma}{(1 + \gamma) - (1 - \delta)} = \frac{1 + \gamma}{\delta + \gamma}. \quad (16)$$

Turning to the right-hand side of (15), we now tackle the nonlinear function

$$f_2[(c_t - y_t), (g_t - y_t)] \equiv \ln[1 - \exp(c_t - y_t) - \exp(g_t - y_t)].$$



For this function of two variables, the first-order Taylor-series approximation is

$$\begin{aligned} f_2[(c_t - y_t), (g_t - y_t)] &\cong f_2[(c - y)^*, (g - y)^*] \\ &+ \frac{\partial f_2}{\partial (c_t - y_t)}[(c - y)^*, (g - y)^*] \cdot [(c_t - y_t) - (c - y)^*] \\ &+ \frac{\partial f_2}{\partial (g_t - y_t)}[(c - y)^*, (g - y)^*] \cdot [(g_t - y_t) - (g - y)^*], \end{aligned}$$

where  $(c - y)^*$  and  $(g - y)^*$  are the steady-state values. The steady-state value of  $(g - y)^*$  is the log of the calibration parameter  $(G/Y)^*$ . From the expression in the bullets above,

$$(c^* - y^*) = \ln\left(\frac{C}{Y}\right)^* = \ln\left\{ \left[ 1 - \left(\frac{G}{Y}\right)^* \right] - \alpha \frac{\gamma + \delta}{\rho + \gamma + \delta} \right\}$$

As before, our interest in deviations around the steady state focuses our attention on the last two terms of the Taylor approximation. Computing the partial derivative with respect to  $(c_t - y_t)$ ,

$$\begin{aligned} \frac{\partial f_2}{\partial (c_t - y_t)}[(c_t - y_t), (g_t - y_t)] &= \frac{1}{1 - \exp(c_t - y_t) - \exp(g_t - y_t)} \cdot (-\exp(c_t - y_t)) \\ &= -\frac{(C_t/Y_t)}{1 - (C_t/Y_t) - (G_t/Y_t)}. \end{aligned}$$

Evaluating this at the steady-state values gives

$$\begin{aligned} \frac{\partial f_2}{\partial (c_t - y_t)}[(c - y)^*, (g - y)^*] &= -\frac{(C/Y)^*}{1 - (C/Y)^* - (G/Y)^*} \\ &= -\frac{\left[ 1 - (G/Y)^* \right] - \alpha \frac{\gamma + \delta}{\rho + \gamma + \delta}}{1 - \left[ \left[ 1 - (G/Y)^* \right] - \alpha \frac{\gamma + \delta}{\rho + \gamma + \delta} \right] - (G/Y)^*} \\ &= 1 - \frac{1 - (G/Y)^*}{\alpha \frac{\gamma + \delta}{\rho + \gamma + \delta}} \cong 1 - \frac{\rho + \gamma + \delta}{\alpha(\gamma + \delta)} \left[ 1 - (G/Y)^* \right]. \end{aligned}$$

Since  $(G/Y)^*$  is a calibrated parameter, this partial derivative can be evaluated numerically at the steady state in terms of the parameters of the model. Following similar logic, it is straightforward to show that

$$\frac{\partial f_2}{\partial (g_t - y_t)} [(c - y)^*, (g - y)^*] = -\frac{\rho + \gamma + \delta}{\alpha(\gamma + \delta)} (G/Y)^*.$$

We are now prepared to express equation (15) in terms of deviations around the steady state. Subtracting the steady-state values yields

$$\begin{aligned} \ln[\exp(\Delta k_t) - (1 - \delta)] - \ln[\exp(\gamma) - (1 - \delta)] &= (y_t - y_t^*) - (k_{t-1} - k_{t-1}^*) \\ &+ \ln[1 - \exp(c_t - y_t) - \exp(g_t - y_t)] - \ln[1 - (C/Y)^* - (G/Y)^*] \end{aligned}$$

or

$$\begin{aligned} f_1(\Delta k_t) - f_1(\gamma) &= (y_t - y_t^*) - (k_{t-1} - k_{t-1}^*) \\ &+ f_2[(c_t - y_t), (g_t - y_t)] - f_2[(c - y)^*, (g - y)^*]. \end{aligned}$$

Substituting from the Taylor-series approximations for the expression on the left and the expression on the second line of the right:

$$\begin{aligned} \frac{1 + \gamma}{\delta + \gamma} (\Delta k_t - \gamma) &= (y_t - y_t^*) - (k_{t-1} - k_{t-1}^*) \\ &+ \left( 1 - \frac{\rho + \gamma + \delta}{\alpha(\gamma + \delta)} [1 - (G/Y)^*] \right) [(c_t - c_t^*) - (y_t - y_t^*)] \\ &+ \left( -\frac{\rho + \gamma + \delta}{\alpha(\gamma + \delta)} (G/Y)^* \right) [(g_t - g_t^*) - (y_t - y_t^*)]. \end{aligned}$$

Or, in terms of the deviations,

$$\begin{aligned} \frac{1 + \gamma}{\delta + \gamma} (\tilde{k}_t - \tilde{k}_{t-1}) &= (\tilde{y}_t - \tilde{k}_{t-1}) \\ &+ \left( 1 - \frac{\rho + \gamma + \delta}{\alpha(\gamma + \delta)} [1 - (G/Y)^*] \right) (\tilde{c}_t - \tilde{y}_t) \\ &- \left( \frac{\rho + \gamma + \delta}{\alpha(\gamma + \delta)} (G/Y)^* \right) (\tilde{g}_t - \tilde{y}_t). \end{aligned}$$

Collecting terms in  $\tilde{y}_t$  and  $\tilde{k}_{t-1}$  simplifies this to

$$\begin{aligned}\frac{1+\gamma}{\delta+\gamma}\tilde{k}_t &= \frac{\rho+\gamma+\delta}{\alpha(\gamma+\delta)}\tilde{y}_t + \frac{1-\delta}{\delta+\gamma}\tilde{k}_{t-1} \\ &+ \left(1 - \frac{\rho+\gamma+\delta}{\alpha(\gamma+\delta)}[1-(G/Y)^*]\right)\tilde{c}_t \\ &- \frac{\rho+\gamma+\delta}{\alpha(\gamma+\delta)}(G/Y)^*\tilde{g}_t,\end{aligned}$$

or

$$\begin{aligned}(1+\gamma)\tilde{k}_t &= (1-\delta)\tilde{k}_{t-1} + \frac{\rho+\gamma+\delta}{\alpha}\tilde{y}_t \\ &+ \left[ (\gamma+\delta) - \frac{(\rho+\gamma+\delta)[1-(G/Y)^*]}{\alpha} \right]\tilde{c}_t \\ &- \frac{\rho+\gamma+\delta}{\alpha}\left(\frac{G}{Y}\right)^*\tilde{g}_t.\end{aligned}\tag{17}$$

Equation (17) is an approximation of equation (2) that is linear in terms of the log-deviations from the steady state, which is what we require in order to solve the model. Note again that all of the coefficients appearing in (17)— $\gamma$ ,  $\delta$ ,  $\lambda$ , and  $(G/Y)^*$ —are parameters that we assume to be known.

Leaving equations (3) and (4) for you to examine as part of the exercise, we now turn to the log-linearization of equation (5), which needs special attention because of the expectation operator on the right-hand side. If we take logs of both sides, we get

$$-c_t = -\rho + \ln\left[E_t(R_{t+1}/C_{t+1})\right].\tag{18}$$

If we could just take the expectation operator outside the log operator, this could easily be reduced to an expression in  $\tilde{c}_t$ ,  $\tilde{r}_{t+1}$ , and  $\tilde{c}_{t+1}$ . However, the log of an expectation is *not* equal to the expectation of the log, so it is not so simple.

In order to simplify (18) we must make a simple assumption about the distribution of the random shocks. If the  $\varepsilon$  shocks both follow the normal probability distribution, then all of the log variables in the linearized system will also be normal random variables and the capital-letter variables will follow the *log-normal* distribution. If  $R$  and  $C$  are distributed log-normally, then

$$\ln\left[E_t\left(\frac{R_{t+1}}{C_{t+1}}\right)\right] = E_t(r_{t+1} - c_{t+1}) + \zeta,$$

where  $\zeta$  (zeta) is a constant. Because  $\zeta$  is the same in the steady state as outside, it will cancel when we take deviations from the steady state, so we need not evaluate it.

Applying this convenient solution to the expectation problem and taking deviations from the steady state in (18) yields

$$-\tilde{c}_t = E_t \tilde{r}_{t+1} - E_t \tilde{c}_{t+1}, \quad (19)$$

which is our log-deviation version of equation (5).

2. Using the methods shown above in the derivation of equations (13) and (17), derive: (a) an expression corresponding to equation (3) in terms of  $\tilde{w}_t$ ,  $\tilde{k}_{t-1}$ ,  $\tilde{a}_t$ , and  $\tilde{l}_t$ ; (b) an expression for equation (4) in  $\tilde{r}_t$ ,  $\tilde{k}_{t-1}$ ,  $\tilde{a}_t$ , and  $\tilde{l}_t$ ; and (c) an expression for equation (6) in  $\tilde{c}_t$ ,  $\tilde{l}_t$ , and  $\tilde{w}_t$ .

## Simulating the model

### *Introduction to Dynare*

Dynare (<http://www.dynare.org/>) was developed as a software tool to facilitate the simulation of dynamic, stochastic, general-equilibrium (DSGE) models. It runs inside the computation platform MATLAB, or the open-source alternative Octave (<http://www.gnu.org/software/octave/>). Both Dynare and Octave are available for free download, the former running under MATLAB or Octave on all platforms and the latter in separate versions for Windows, Mac, or Linux. You will need to run your simulations on a computer on which Dynare and either MATLAB or Octave are installed. CUS has installed Dynare and MATLAB on all public-access computers in the IRCs, PPW, and elsewhere on campus.

You input your model, parameter values, and instructors to Dynare through a text file with a .mod suffix. The Quick Start guide, Tutorial, User Guide, and Manual available at the Dynare Web site (link above) provide details if you want them. We will provide a template for the .mod file you will use for this assignment below, so you won't need to learn the details unless you want to. Note that the .mod file *must be in plain text format*. You can use a simple text editor to create such a file. If you instead use Word or another advanced word processor, you must set the file format to plain text when you save or Dynare won't be able to read your file.

The .mod file consists of five parts:

- The **preamble** section defines your variables and parameters.
- The **model** section contains the equations of your model in a simple algebraic notation similar to standard programming languages.
- The **steady state** section gives the steady-state (initial) values of the model solution.
- The **shocks** section, in which you tell Dynare about the deterministic or random shocks that you want to simulate.

- The **computation** section that tells Dynare to simulate the model and report the results.

### *A template Dynare program*

The easiest way to describe the various sections of the .mod file is by example. The code below is the beginning of a file you can use to run your simulations. Lines preceded by // are comments.

```
// ***Preamble section***
// Define the endogenous variables
var k a l c y w r g;
// Define the exogenous variables (shocks)
varexo epsg epsa;
// Define the parameters of the model
parameters alpha rhoa rhog rho lstar gamma delta gystar;
// Set the values of the parameters to match Romer's calibration
alpha = 1/3;
rhoa = .95;
rhog = .95;
rho = 0.01;
lstar = 1/3;
gamma = 0.005;
delta = 0.025;
gystar = 0.2;

// ***Model section***
model;

// Equation (19) from above. Expected future value of x is x(+1)
-c = r(+1) - c(+1);

// Equation (17) from above. Lags of x denoted by x(-1)
(1+gamma) * k = (rho + gamma + delta)/alpha * y
+ (1-delta) * k(-1)
+ (gamma + delta - (rho + gamma + delta)*(1-gystar)/alpha) * c
- gystar*(rho+gamma+delta)/alpha * g;

// Equation (13) from above.
y = alpha * k(-1) + (1-alpha) * (a + l);

// YOU WILL NEED TO ADD THREE EQUATIONS HERE

// Equations (7) and (8) from above
a = rhoa * a(-1) + epsa;
```

```

g = rhog * g(-1) + epsg;

end;
// *** Steady-state section ***
initval;
c = 0;
l = 0;
k = 0;
y = 0;
g = 0;
a = 0;
w = 0;
r = 0;
end;

steady;
check;

// *** Shocks section ***
shocks;
var epsa;
periods 1;
values .01;
end;

// *** Computation section ***
simul(periods=200);
// *** Some MATLAB plot commands similar to those below
subplot(3,3,1);
plot(k, ";K;");
subplot(3,3,2);
plot(a, ";A;");
subplot(3,3,3);
plot(l, ";L;");
subplot(3,3,4);
plot(c, ";C;");
subplot(3,3,5);
plot(y, ";Y;");
subplot(3,3,6);
plot(w, ";w;");
subplot(3,3,7);
plot(r, ";r;");
subplot(3,3,8);
plot(g, ";g;");

```

Before we get into details, note several features of the Dynare program:

- Variables are case-sensitive.
- All commands end with a semi-colon.
- Blocks of commands such as model equations, steady-state (initial) values, and shocks end with the end; statement.
- Options for any command are entered in parentheses immediate after the name of the command.

### ***Preamble section***

The definitions of the variables are straightforward. I've used lower-case letters corresponding to the notation in the text (but without tildes). Similarly, the parameters are named in ways that correspond to the equations of the model. The parameter values correspond to those at the beginning of Romer's Section 5.7.

The preamble section should not need to be altered from the above in order to run your simulations.

### ***Model section***

The equations are entered in standard computer-algebraic notation, with +, -, \*, /, and ^ as the principal operators and nested sets of parentheses used to indicate the order of operation. Note that it is not necessary to "solve" the individual equations to isolate an endogenous variable alone on the left-hand side. Suffix parentheses are used to indicate leads and lags, with  $x(-1)$  denoting last period's value of  $x$  and  $x(+1)$  indicating today's expectation of next year's value of  $x$ . Equations can span more than one line of the program, but be sure that each equation ends with a semi-colon.

You will need to add Dynare code for the three linearized log-deviation equations that you derive in Problem 2 above based on equations (3), (4), and (6).

### ***Steady-state section***

The variables of our Dynare model are all log-deviations from the steady state, thus their steady-state values are simply zero: when the economy is in the steady state there are no deviations. This makes the steady-state section trivial and it need not be changed for your simulations. The `steady` and `check` commands simply direct Dynare to verify that these are indeed a steady state and that certain dynamic-stability conditions hold.

### ***Shocks section***

This section defines the shocks that you want to simulate. There are two kinds of simulations that you can run in Dynare: deterministic and stochastic. In a deterministic simulation, you set all shocks to zero except those specified in the shocks section and you use the `simul` command in the computation section. The file shown above is set up to run a deterministic simulation in which the value of `epsa` ( $\varepsilon_A$ ) is set to +0.01 (one percent) in period one. All

other shocks (the government spending shock) and values of `epsa` in periods other than period one are set to zero. This command should generate simulations that match those shown in Romer's Figures 5.2 through 5.4. To perform simulations of the government spending shock, you could change `epsa` to `epsg` in the `var` command of the shocks section.

Stochastic simulations involve drawing a random value for each shock in each period from a normal probability distribution with specified standard deviations and correlations for the shocks. For example, to simulate the model with random shocks to both `epsa` and `epsg` each having a standard deviation of 0.005 (one-half percent) and no correlation between them, you would enter the shocks section as follows:

```
// *** Shocks section for stochastic simulation***
shocks;
var epsa; stderr 0.005;
var epsg; stderr 0.005;
end;
```

The fact that you have not entered a correlation value means that Dynare sets it to zero. In fact, any standard deviation or correlation that is not explicitly set in the shocks section is assumed to be zero, so if you left out the `var epsg` line, you would get a simulation with the technology shock varying randomly but the government-spending shock set to zero throughout (because a random variable with a standard deviation of zero is just a constant).

### ***Computation section***

The computation section directs Dynare to simulate the model and create specific output elements that you want. As with the shocks section, the computation section will depend on whether you are doing a deterministic or stochastic simulation.

The program above shows a deterministic simulation using the `simul` command. The `periods=200` option tells Dynare to assume that the model will be back in (or sufficiently close to) the steady-state after 200 periods (quarters). Setting this option to a value that is too small will cause the simulation to be incorrect. Examine your results closely to see if there is a “jump” at the end of the simulations. If there is, then you haven't set this option high enough. There are many other options for the `simul` command that are described in the Dynare documentation.

If you are doing a stochastic simulation, you will use the `stoch_simul` command. Like the `simul` command, it takes a `periods` option that determines the number of periods in the simulation. In a stochastic simulation, you may want to set the number of periods to be much larger (2000?) so that the summary statistics of the simulation can be averaged over a larger number of periods.



The stochastic simulation automatically generates useful output that is printed on your screen. This includes the basic summary statistics of your endogenous variables over the periods of the simulation, the pairwise correlations between them, and the autocorrelations of each variable. This output may be inconvenient to copy and paste from the screen window, but it is replicated in a text file with the suffix .log following the same name that your .mod file has. Dynare creates this .log and places it in the same folder as your .mod file. You can easily open this file in a text editor and copy results into your report. The stochastic simulation also generates impulse-response graphs for each shock, which should be similar to the output of the deterministic simulations of the shocks.

### ***Producing graphic output***

Dynare and Octave/MATLAB are capable of graphing your impulse-response functions (from either a deterministic or stochastic simulation) and your simulated time series from a stochastic simulation. Running Dynare under Octave on the Macintosh platform has sometimes proved troublesome, so there are two practical choices for performing the computations on this project. If you have a Windows computer, you can download both Dynare and Octave onto it and run that way. Or you can use one of the Reed IRC/PPW computers on which MATLAB and Dynare are installed.

### ***Running Dynare under Windows/Octave***

Once you have created your .mod file, you can execute it with Dynare. To do so requires three commands. If you are running Octave under Windows, for example, you first need to start Octave, then, at the Octave command prompt, type

```
addpath c:\dynare\4.2.0\matlab
```

This command tells Octave where to find the Dynare execution file. (If you put the Dynare software in a different folder, you would need to change this command to point to that folder.)

You next need to tell Octave where to find your .mod file. The easiest way to do this is to make it your default Octave directory. If you put your .mod file in a folder called mymacroasgn inside the c:\dynare folder, you would say

```
cd c:\dynare\mymacroasgn
```

Now you should be ready to run your Dynare program by typing into Octave:

```
dynare mymodfile.mod
```

where mymodfile.mod is the name of your .mod file, which is located in the folder pointed to by your cd command above.

### ***Running Dynare under Macintosh/MATLAB***

In order to run your Dynare program on Reed's public computers, you can use MATLAB, which is available in the Application folder. After starting MATLAB, you will get a command prompt. Type in the (case-sensitive) command

```
addpath /Applications/Dynare/4.2.0/matlab
```

This tells MATLAB where to look for the Dynare program.

Next you tell MATLAB where to find your .mod file (and where to put its output files). The easiest option is to put your files in an identifiable (i.e., including your name) folder on the Mac desktop. (Note that on a public Reed computer this folder might be erased during the overnight re-initiation of the computer. Be sure to keep a copy someplace secure.) If you use the name “Jane314” for this folder, then you would type

```
cd ~/Desktop/Jane314/
```

to change MATLAB’s default directory.

Now you are ready to tell MATLAB to run your Dynare mod file. If your file (inside the Jane314 folder) is called mymodfile.mod, then you type

```
dynare mymodfile.mod
```

### ***Running DYNARE under Macintosh / Octave***

Last year’s class had difficulty running Octave on Macs. The magicians at CUS worked in this during the summer and have come up with the following instructions for preparing to run Octave (and the Gnuplot graphics package) on campus Macs:

Octave was created as an open-source alternative to Matlab. Octave runs on Mac, Linux, and Windows systems, but has problems creating graphs on Mac systems. We'll walk you through how to set up Octave, and how to connect Octave to its graphing tools.

All of the IRC machines should have these tools already loaded and functional. If you encounter a problem with the IRC machines, please contact the CUS helpdesk at [cus@reed.edu](mailto:cus@reed.edu) .

Note: This write-up also includes Dynare, a program useful for macroeconomic work.

1. Install Dynare in your Applications folder
2. Install Octave in your Applications folder
3. Download Maxima, then drag the Gnuplot program icon to your Applications folder
4. Open a terminal window (e.g. using the application Terminal)
5. Enter the following text in the terminal window, at the prompt:

```
ln -s /Applications/Gnuplot.app/Contents/Resources/bin/gnuplot gnuplot
```

What just happened? You made a "symlink" between Gnuplot and Octave, which will allow Octave to graph using Gnuplot. (This is similar to an "alias" icon.)

6. You can launch Octave from the terminal and start working!

```
exec '/Applications/Octave.app/Contents/Resources/bin/octave'
```

If you have problems with the installation, contact Kristin Bott in Instructional Technology Services at [bottk@reed.edu](mailto:bottk@reed.edu).

***Your results***

If Dynare runs your program successfully, you will see the graphic output you have requested in new windows and (for stochastic simulations) tables of results in your Octave/MATLAB window. The results are also saved in a file called mymacroasgn.log (where mymacroasgn is the name of your .mod file) in the same folder as your .mod file. To get these results into your report document (or into Excel or another program for further processing) it is easier to copy and paste from the .log file than from the Octave screen itself.

Dynare diagnostics are largely useless, so if your program does not execute successfully you are probably going to be on your own to figure out why. Common errors to watch for include misspelling variable names, mismatched parentheses in expressions, and leaving off the semi-colon at the end of a command.

3. Replicate Romer's Figures 5.2 through 5.7 for a deterministic simulation with A, then G, shocked by 1% (0.01 shock to the log of the variable). Interpret the results. Are they the same?
4. Perform stochastic simulations of the RBC model for each of the following values of the standard deviations of the shocks:
  - (a) Only productivity shocks:  $sd(\varepsilon_A) = 0.01$ ,  $sd(\varepsilon_G) = 0$ ;
  - (b) Both productivity and government spending shocks:  $sd(\varepsilon_A) = 0.005$ ,  $sd(\varepsilon_G) = 0.005$ ;
  - (c) Only government spending shock:  $sd(\varepsilon_A) = 0$ ,  $sd(\varepsilon_G) = 0.01$ .

Create a table to summarize your results. The three columns of the table should contain the results of your three stochastic simulations. The rows should contain the following results captured from your screen or the log file: (i) the standard deviations of log GDP, log consumption, log employment, log real wages, the real interest rate, and log government spending; (ii) the correlation coefficients between log GDP and each of the other variables; (iii) the first-order autocorrelations of all variables. The table below describes these measures for quarterly cyclical fluctuations in the U.S. from 1985 to 2009. Which measures correspond well to the U.S. economy and which do not? (Compare relative sizes, not absolute levels.) Summarize your results briefly with an overall assessment of your RBC simulation results.

Variable	Standard deviation	Correlation with GDP	1st-order autocorrelation
log GDP	0.0113	(1.000)	0.9110
log consumption	0.0099	0.9302	0.9239
log employment	0.0108	0.8369	0.9182
log real wage	0.0048	0.0133	0.9339
real interest rate	0.0025	0.5516	0.9250
log gov't spending	0.0025	-0.4268	0.9059