Economics 314   Spring 2014
Project #2 Assignment   Due: 9am, Wednesday, February 12

Partner assignments

Sarah Brauner	Reilly Villanueva
Alejandro Chavez	Will Schmid
Logan Donoughe	Peter Riehlman
Orphelia Ellogne	Natalie Pong
Nick Fiore	Jacob Menick
Luke Hill	Vlad Mankov
Taylor Holdaway	

Each team should submit a single paper. I strongly encourage you to work as a team in solving the problems. At a very minimum, both partners must understand everything that's on the paper when you turn it in. You are free to interact with other teams and to seek advice from me as you work on the problems, especially on the mathematical solutions. The idea of problem sets is to learn from them, not to test whether you can do them perfectly without help.

Problems

1. **Marginal products in terms of the intensive production function.** In the Solow growth model, we write the aggregate production function as $Y = F(K, AL)$, and the intensive production function as $y = f(k)$, where $y \equiv \dfrac{Y}{AL}$, $k \equiv \dfrac{K}{AL}$, and $f(\bullet) \equiv F(\bullet, 1)$. Assume constant returns to scale and the usual properties for the partial derivatives of *F*.

   a. Show that the marginal products of capital and labor are $\dfrac{\partial Y}{\partial K} = f'(k)$ and

   $$\frac{\partial Y}{\partial L} = A\left[f(k) - kf'(k) \right].$$

   b. Show that the marginal product of capital is decreasing in *k* and the marginal product of labor is increasing in *k*. What do these results mean in economic terms?

   c. What is happening to *k* over time on a steady-state growth path? (Is it constant, increasing, or decreasing?) What is happening to *A* on a steady-state growth path? Based on part a and your answers to these questions, how do the real rental rate of a unit of capital ($r + \delta$ = MPK) and the real wage on labor (w = MPL) behave on a Solow steady-state growth path?

   d. In competitive equilibrium, each unit of capital and labor will be paid its marginal product. Two of Nicholas Kaldor's six famous "stylized facts" about modern growth are that the rate of return to a unit of capital is stable in the long run and that the real wage earned by a unit of labor (not effective labor) grows over time. Is the Solow model consistent with these stylized facts?

   e. How do total earnings of capital ($K \times$ MPK) and total earnings of labor ($L \times$ MPL) behave along the growth path? Given the behavior of total output, what will happen to capital's share (capital earnings divided by total output) and labor's share in steady-state growth?

2. Work **Romer's problem 1.3**.

- ***Addition to each part:*** Sketch the new steady-state growth path of $\ln(Y)$ and the path that $\ln(Y)$ will follow as it converges to its new steady-state path. Explain in words what is happening both at the moment of the change and afterwards.
- Be careful on part (c): Do you know if $k < 1$ or $k > 1$?

3. Work **Romer's problem 1.4**.
   - Assume that the new workers bring no capital with them and that their arrival has no effect on A.
   - ***Addition*** to part (a): "What happens to total output Y at the time of the jump? Why?"
   - ***Addition*** to part (c): "Is *total* output on the new balanced growth path higher, lower, or the same as it would have been if there had been no new workers? Why?"

4. Work **Romer's problem 1.6**.