Economics 314 Spring 2014
Daily Question #6 February 7

Consider a household that lives n years. Suppose that the dynamics of the household's capital stock (wealth) in period t are given by $K_t = (1+r_t)K_{t-1} + W_t - C_t$. Show by repeated substitution for lagged values of K that, for given initial K_0 and given paths of r and W, the household's choice of consumption path (C_1, C_2, …, C_n) must satisfy

$$K_0 + \sum_{t=1}^{n} \frac{W_t}{\prod_{\tau=1}^{t}(1+r_\tau)} = \sum_{t=1}^{n} \frac{C_t}{\prod_{\tau=1}^{t}(1+r_\tau)} + \frac{K_n}{\prod_{\tau=1}^{n}(1+r_\tau)}, \text{ where } \prod_{\tau=1}^{t}(1+r_\tau) \equiv (1+r_1)\times(1+r_2)\times\ldots\times(1+r_t).$$