Economics 312 — Spring 2014
Daily Problem #36 — April 16

The variable y_i is one if you decide to come to econometrics class and zero otherwise.

1. Show that $E(y_i) = \Pr[y_i = 1]$.

2. Suppose that your decision whether to come to econometrics class depends on how much sleep you have had the night before S_i. Consider the linear probability model $y_i = \beta_1 + \beta_2 S_i + u_i$, where u_i is a random error term and $\Pr[y_i = 1 | S_i] = \beta_1 + \beta_2 S_i$. Given that y_i is either zero or one, what values can u_i take on? Is it reasonable that u_i could be normally distributed? For extreme values of S_i, will $\Pr[y_i = 1 | S_i]$ always be in the range [0, 1], as all probabilities must be?

3. Since the linear model doesn't work very well, suppose that we choose a nonlinear model $\Pr[y_i = 1 | S_i] = G[\beta_1 + \beta_2 S_i]$. Why would it make sense to choose a G function whose range is restricted to [0, 1]? Given that $\Pr[y_i = 0 | S_i] = 1 - G[\beta_1 + \beta_2 S_i]$, explain why the discrete probability density function of y_i condition on S_i can be written as $f(y_i | S_i) = [G(\beta_1 + \beta_2 S_i)]^{y_i} [1 - G(\beta_1 + \beta_2 S_i)]^{(1-y_i)}$, for $y_i = 0, 1$. (Hint: Remember that y_i is always zero or one, so $1 - y_i$ is always one or zero as well.)

4. (Optional) Derive the likelihood function of (β_1, β_2) for the sample (y_i, S_i), $i = 1, 2, \ldots, N$.