Economics 312 Spring 2014
Daily Problem #26 March 27

A particular kind of nonstationary time series that is of great importance is the "difference stationary" series. This means a series that can be made stationary by differencing one or more times. Such processes have a "unit root," the intuition of which is explored in this problem.

1. Consider the second-order polynomial $P(x) = x^2 - 5x + 6$ in the variable *x*. Very specifically, what do we mean by the "roots" of $P(x)$? What are the roots of $P(x)$?

2. Now consider the first-order polynomial $\alpha(L) = 1 - 0.25L$ in the lag operator *L*. What is the root of $\alpha(L)$?

3. In the more general first-order lag polynomial $\alpha(L) = 1 - \alpha L$, what is the root?

4. For each of the following second-order autoregressive processes, express the process in the form $\alpha(L) y_t = \varepsilon_t$ and find the roots of the lag polynomial $\alpha(L)$. Assuming that ε_t is white noise, tell whether the process is stationary (all roots are > 1 in absolute value), difference stationary (one or more roots with absolute value of 1, others > 1 in absolute value), or completely nonstationary (one or more roots with absolute value < 1).

a. $y_t = 0.75 y_{t-1} - 0.125 y_{t-2} + \varepsilon_t$
b. $y_t = 1.25 y_{t-1} - 0.25 y_{t-2} + \varepsilon_t$
c. $y_t = 2 y_{t-1} - y_{t-2} + \varepsilon_t$