Economics 312 — Spring 2014
Daily Problem #2 — January 31

There are two options for today's daily problem: a basic one and one that is more challenging. Both deal with the principle of maximum-likelihood estimation. Choose whichever one suits you; you are not expected to do both.

Basic problem

Suppose that we have *N* observations that are assumed to be independent draws from a normal distribution with known variance of one and unknown mean μ. The density function of the *i*th observation is thus assumed to be

$$f\left(x_i \mid \mu\right) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(x_i - \mu\right)^2}.$$

Write the joint density function of the sample of *N* observations, given the value of μ. (Hint: Remember that the observations are assumed to be independent and use equation (P.7) on page 24 of the text.) Write the likelihood function for μ given the sample values. Take the log of the likelihood function and show that the value of μ at which it is maximized is $\hat{\mu}_{ML} = \bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$. You have just proved that the maximum-likelihood estimator of the population mean in a normal distribution is the sample mean.

Challenging problem

Suppose that we have a sample of *N* independent observations drawn from a uniform distribution with lower limit α and upper limit β, so the density function of each observation is

$$f\left(x_i \mid \alpha, \beta\right) = \begin{cases} \dfrac{1}{\beta - \alpha}, \text{ if } \alpha \le x_i \le \beta, \\ 0, \text{ if } x_i < \alpha \text{ or } x_i > \beta. \end{cases}.$$

Write the joint density function of the sample of *N* observations given α and β. (Hint above applies here as well.) Write the likelihood function for (α, β) given the sample values. (Note that the joint density and likelihood functions will have "branches" as in the formula immediately above rather than a single formula that applies for all values as you would have in distributions such as the normal.) Find the values of α and β at which the likelihood function is maximized—the maximum-likelihood estimators $\hat{\alpha}_{ML}$ and $\hat{\beta}_{ML}$.