Economics 314 — Spring 2013

Project #8 Assignment — Due: 9am, Thursday, April 11

Partner assignments

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Problems

- Work Romer's Problem **6.10**.
  - Notice that $\phi < 1$ is the normal case and $\phi > 1$ is a case of "negative real rigidity." This may help you interpret the results.
  - Show on your graph the values of the gains from price adjustment at $f = 0$ and $f = 1$ in each case.
  - As an addition to part (c): For a given value of Z, find the range of values of m' (if any) for which multiple equilibria may exist (i.e., for which either full price adjustment or non-adjustment is an equilibrium).

- Work Romer's Problem **6.16**.
  - In working part (c), show the equation for y—an aggregate-supply curve.

- Work the following variant of Romer's Problem 6.12 on wage indexation:

The economy is composed of a large number of firms distinguished by subscript i. Firm i has a production function given by $Y_i = SL_i^{\alpha}$, where S is a supply shock and $0 < \alpha \leq 1$. The log of the firm's production is $y_i = s + \alpha \ell_i$, where the lower-case letters refer to the logs of the respective capital letter variables. Aggregation takes the form of averaging the logs of variables, so aggregate output is $y = s + \alpha \ell$. (This form of aggregation makes the aggregate variable equal to the geometric mean of the individual values rather than the sum or the arithmetic mean.) Aggregate demand is given by $y = m - p$. The logs of the supply shock and the aggregate-demand shock (s and m) are random variables whose probability distributions have mean of zero and variances of V_s and V_m, respectively. Firms are price-takers in product markets.

(a) What is the marginal product of labor? Price-taking firms will set the marginal product of labor equal to the real wage. Can you derive Romer's equation

$p_i = w_i + (1-\alpha)\ell_i - s$ from this condition? Why or why not? Does his assumption that we can simply set the constant equal to zero seem reasonable to you? (Making this assumption does not change any of the subsequent results, so we will use it below.)

(b) Suppose that labor supply is inelastic at the level of one unit per person (and per firm), so $L^s = 1$ and $\ell^s = 0$. Let aggregate labor demand be given (in log terms) by the mean across firms of Romer's price equation: $\ell^d = \frac{1}{1-\alpha}(p - w + s)$. Assume initially that the wage is perfectly flexible, in other words, that it adjusts to make aggregate labor supply equal to labor demand. Show that when $s = 0$ and $m = 0$, the equilibrium values of the logs of the endogenous variables are $w = p = y = \ell = 0$.

(c) Now suppose that wages are set in contracts that last one period. At the beginning of the period, firms and workers determine a rule for setting the nominal wage. The rule may set a specific wage for the period, or it may allow the wage to vary depending on how high or low prices turn out to be: an *indexed* contract. Given the wage, the quantity of employment is then determined by firms' labor demand. (As part of the contracting process, workers agree to work as much as firms want them to.) If they were going to set a fixed wage for the period, they would presumably set the log-wage to zero, since that is the level that clears the market when there are no shocks (i.e., when the shocks are at their expected values—zero). However, it may be desirable to index the wage at least partially to changes in prices. Suppose that firms and workers agree on an indexing rule stipulating that $w = \theta p$. We assume that $0 \le \theta \le 1$. If $\theta > 0$, this rule moves the wage away from zero whenever a money or supply shock pushes prices away from zero. Find expressions for p, y, and ℓ as functions of the shocks m and s and the parameters θ and α. Find expressions for the sensitivity of log employment and output to m and s, i.e., find $\partial \ell / \partial m$, $\partial \ell / \partial s$, $\partial y/\partial m$, and $\partial y/\partial s$ using your solutions. How sensitive are log employment and output to each shock when there is no indexing ($\theta = 0$)? How sensitive are log employment and output to each shock when the wage is fully indexed ($\theta = 1$)? Explain the intuition of this result.

(d) In evaluating the desirability of alternative policy rules, we often look for rules that minimize the effects of shocks. We implement this idea by seeking parameter values that minimize the variation of employment or output. In the next part of the problem, we will find the indexing rule (i.e., the value of θ) that minimizes the variance of log employment. The variance of a variable is the expected value of the squared deviation from its mean value. Thus, minimizing variance minimizes the expected squared fluctuations in log employment relative to the equilibrium level. In intuitive economic terms, why is minimizing fluctuations in employment an appropriate welfare criterion? (Hint: Remember what the labor supply curve is and think about the relationship between minimizing deadweight losses in the labor market and minimizing squared deviations of employment from its expected value.) Why is it better to minimize employment fluctuations than output fluctuations? (Does natural output change in response to shocks? Does natural employment change?)

(e) Standard statistical formulas for the variance of a random variable tell us that if

$$\ell = a_m m + a_s s,$$

where the a coefficients are constants and m and s are random variables whose probability distributions have variances V_m and V_s, respectively, then

$$\text{var}(\ell) = a_m^2 V_m + a_s^2 V_s + 2a_m a_s \text{cov}(m, s),$$

where cov(m, s) is the covariance between the two shocks. Assume that the two shocks are independent so their covariance is zero. Find an expression for the variance of employment as a function of V_m and V_s and the parameters of the model (including θ). Based on this expression, find the indexing rule (i.e., the value of θ) that minimizes the variance of employment. Discuss optimal indexation in the limiting case of $V_m = 0$ and in the alternative limiting case of $V_s = 0$. Based on this result, what characteristics of an economy make it desirable to index wage contracts?

(f) Consider two countries that are equally subject to supply shocks (i.e., where the variance of s is the same in both countries), but one of which has a higher variability of monetary shocks. (This is kind of like the experiment in the Lucas paper that you read: different countries have more or less volatile aggregate demand.) Use your results from part (e) to explain which country will have the higher degree of indexation. In part (c) you calculated the elasticity of Y with respect to M, $\partial y / \partial m$, which depended on θ. Use this result to explain which country will have the more elastic aggregate supply curve. How does this result compare with the differences across countries in the slope of the AS curve predicted by Lucas's imperfect-information model?

(g) In a country with less-the-full indexation, will the real wage (W/P, or $w - p$ in log terms) be procyclical (positively correlated with output) or countercyclical when business cycles are caused by monetary (demand) shocks? By supply shocks? How do these results correspond to actual cyclical variation in real wages?