

**Economics 314**  
**Daily Question #6**

**Spring 2013**  
**February 11**

Consider a household that lives  $n$  years. Suppose that the dynamics of the household's capital stock (wealth) in period  $t$  are given by  $K_t = (1 + r_t)K_{t-1} + W_t - C_t$ . Show by repeated substitution for lagged values of  $K$  that, for given initial  $K_0$  and given paths of  $r$  and  $W$ , the household's choice of consumption path  $(C_1, C_2, \dots, C_n)$  must satisfy

$$K_0 + \sum_{t=1}^n \frac{W_t}{\prod_{\tau=1}^t (1+r_\tau)} = \sum_{t=1}^n \frac{C_t}{\prod_{\tau=1}^t (1+r_\tau)} + \frac{K_n}{\prod_{\tau=1}^n (1+r_\tau)}, \text{ where } \prod_{\tau=1}^t (1+r_\tau) \equiv (1+r_1) \times (1+r_2) \times \dots \times (1+r_t).$$