Economics 312 **Spring 2013**
Daily Problem #34 **April 18**

Consider the panel-data regression model $y_{it} = \beta_{1i} + \beta_2 x_{2it} + \beta_3 x_{3it} + e_{it}$. Suppose that we consider the individual-specific intercept coefficient to reflect random variation across individuals around a fixed mean: $\beta_{1i} = \bar{\beta}_1 + u_i$, $E(u_i) = 0$, $\operatorname{cov}(u_i, u_j) = 0$, $i \neq j$, $\operatorname{var}(u_i) = \sigma_u^2$.

1. Show that this "random-effects" model can be written as $y_{it} = \bar{\beta}_1 + \beta_2 x_{2it} + \beta_3 x_{3it} + v_{it}$, where $v_{it} = u_i + e_{it}$.

2. Assuming that $E(e_{it}) = 0$, $\operatorname{var}(e_{it}) = \sigma_e^2$, and $\operatorname{cov}(e_{it}, u_i) = 0$, find

   (a) $E(v_{it})$,

   (b) $\operatorname{var}(v_{it})$,

   (c) $\operatorname{cov}(v_{it}, v_{is})$, $s \neq t$,

   (d) $\operatorname{cov}(v_{it}, v_{js})$, $i \neq j, s \neq t$.

3. Explain intuitively how we could attempt to estimate σ_e^2 and σ_u^2 using OLS residuals $\hat{v}_{it}$. Given the properties of v_{ij}, would you expect these estimates to be consistent? (Hint: They will be consistent if the OLS coefficient estimators are consistent.)

4. If we had estimates of σ_e^2 and σ_u^2, explain in principle how we could use GLS to get asymptotically efficient estimators of the coefficients.