

A particular kind of nonstationary time series that is of great importance is the “difference stationary” series. This means a series that can be made stationary by differencing one or more times. Such processes have a “unit root,” the intuition of which is explored in this problem.

1. Consider the second-order polynomial  $P(x) = x^2 - 5x + 6$  in the variable  $x$ . Very specifically, what do we mean by the “roots” of  $P(x)$ ? What are the roots of  $P(x)$ ?
2. Now consider the first-order polynomial  $\alpha(L) = 1 - 0.25L$  in the lag operator  $L$ . What is the root of  $\alpha(L)$ ?
3. In the more general first-order lag polynomial  $\alpha(L) = 1 - \alpha L$ , what is the root?
4. For each of the following second-order autoregressive processes, express the process in the form  $\alpha(L)y_t = v_t$  and find the roots of the lag polynomial  $\alpha(L)$ . Assuming that  $v_t$  is white noise, tell whether the process is stationary (all roots are  $> 1$  in absolute value), difference stationary (one or more roots with absolute value of 1, others  $> 1$  in absolute value), or completely nonstationary (one or more roots with absolute value  $< 1$ ).
  - a.  $y_t = 0.75y_{t-1} - 0.125y_{t-2} + v_t$
  - b.  $y_t = 1.25y_{t-1} - 0.25y_{t-2} + v_t$
  - c.  $y_t = 2y_{t-1} - y_{t-2} + v_t$