Economics 312 **Spring 2013**
Daily Problem #23 **March 27**

Suppose that $y_{t,R}$ is the "yield rate" at Reed College in year t, the share of admitted students who choose to attend, and that $y_{t,LC}$ is the yield rate at Lewis & Clark College. We hypothesize the following model for yield at each school:

$$y_{t,i} = \beta_{1,i} + \beta_{2,i} p_{t,i} + \beta_{3,i} x_{t,i} + e_{t,i},\; i = R, LC,$$

where $p_{t,i}$ is inflation-adjusted tuition at college i and $x_{t,i}$ is a measure of perceived college quality (perhaps its *US News & World Report* rating from the previous year).

We assume that p and x are exogenous to the yield rate and ignore any possible correlation over time in $e_{t,i}$. We are interested in two hypotheses:

i. $H_0 : \beta_{2,R} = \beta_{2,LC}$ because we want to know if the tuition sensitivity of enrollment varies between the colleges, and
ii. $H_0 : \beta_{3,R} = \beta_{3,LC}$ because we think that Reed applicants might be less sensitive to *USNWR* rankings than those applying to Lewis & Clark due to Reed's famous non-participation in the *USNWR* survey.

In order to test these hypotheses, we must estimate the Reed and Lewis & Clark equations jointly using the method of "seemingly unrelated regressions" or SUR.

1. Let $\mathbf{y}_R$ be the $T \times 1$ vector of observations $y_{t,R}$ for $t = 1, 2, \ldots, T$ and $\mathbf{y}_{LC}$ be the corresponding vector for Lewis & Clark. Let $\mathbf{e}_i$ be the $T \times 1$ vector of the error term for school i and $\mathbf{X}_i$ be the $T \times 3$ matrix with the first column being ones, the second column being the vector of $p_{t,i}$ values, and the third column being the vector of $x_{t,i}$ values for the school. Show that we can write a "stacked" regression for both schools as $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$, where

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_R \\ \mathbf{y}_{LC} \end{bmatrix}, \mathbf{X} = \begin{bmatrix} \mathbf{X}_R & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_{LC} \end{bmatrix}, \boldsymbol{\beta} = \begin{bmatrix} \boldsymbol{\beta}_R \\ \boldsymbol{\beta}_{LC} \end{bmatrix}, \mathbf{e} = \begin{bmatrix} \mathbf{e}_R \\ \mathbf{e}_{LC} \end{bmatrix}.$$

2. Why might we worry, even if $\sigma_R^2 = \operatorname{var}(e_{t,R})$ and $\sigma_{LC}^2 = \operatorname{var}(e_{t,LC})$ are constant over time, that $\sigma_R^2 \neq \sigma_{LC}^2$? How would that affect the OLS estimator of the stacked regression?

3. Why might we worry that $\operatorname{cov}(e_{t,R}, e_{t,LC}) \equiv \sigma_{R,LC} \neq 0$? How would that affect the OLS estimator of the stacked regression?

4. Still ignoring correlation across time, suppose that $\operatorname{cov}\left(e_{t,R}, e_{t,LC}\right) = \sigma_{R,LC}$ and that $\operatorname{var}\left(e_{t,R}\right) = \sigma_R^2$ and $\operatorname{var}\left(e_{t,LC}\right) = \sigma_{LC}^2$ for every *t*. Explain why the covariance matrix of the error term in the stacked regression is $\mathbf{V} = \begin{bmatrix} \sigma_R^2 \mathbf{I}_T & \sigma_{R,LC} \mathbf{I}_T \\ \sigma_{R,LC} \mathbf{I}_T & \sigma_{LC}^2 \mathbf{I}_T \end{bmatrix}$, where $\mathbf{I}_T$ is the $T \times T$ identity matrix. (This matrix is an example of a "Kronecker product" that can be written $\mathbf{V} = \begin{bmatrix} \sigma_R^2 & \sigma_{R,LC} \\ \sigma_{R,LC} & \sigma_{LC}^2 \end{bmatrix} \otimes \mathbf{I}_T$.)