

Consider the log form of the Cobb-Douglas production function with four inputs: capital (K), labor (L), energy (E), and materials (M):

$$\ln(Y_i) = \beta_1 + \beta_2 \ln(K_i) + \beta_3 \ln(L_i) + \beta_4 \ln(E_i) + \beta_5 \ln(M_i) + e_i,$$

where we have observations across N establishments indexed by i .

1. Constant returns to scale imply that $\beta_2 + \beta_3 + \beta_4 + \beta_5 = 1$. Show how you could impose this condition on the model and use OLS to estimate a restricted (or constrained) model in which the dependent variable is $\ln\left(\frac{Y_i}{M_i}\right)$ and the regressors include $\ln\left(\frac{K_i}{M_i}\right)$ and similar terms.

2. In light of yesterday's daily problem, how could you use the restricted and unrestricted regressions to test whether constant returns to scale holds in this data set?