**Economics 312** **Spring 2012**
**Daily Problem #34** **April 5**

Suppose that we have a two-variable dynamic system involving two stationary variables *x* and *y*. There is contemporaneous causality running from *x* to *y*, but not vice versa. The structural model is, therefore, given by

$$\begin{aligned} x_t &= \alpha_0 + \alpha_1 x_{t-1} + \theta_1 y_{t-1} + \varepsilon_t^x \\ y_t &= \phi_0 + \phi_1 y_{t-1} + \delta_0 x_t + \delta_1 x_{t-1} + \varepsilon_t^y \end{aligned}$$

where the ε error terms are (homoskedastic) white noise and $\text{var}\left(\varepsilon_t^x\right) = \sigma_x^2$, $\text{var}\left(\varepsilon_t^y\right) = \sigma_y^2$, and $\text{cov}\left(\varepsilon_t^x, \varepsilon_t^y\right) = 0$. These ε terms are the "pure shocks" to *x* and *y* that are unrelated to anything in the past or anything having to do with the other variable.

1. Show that the solution of this system of equations is a VAR system that can be written

$$\begin{aligned} x_t &= \beta_{x,0} + \beta_{x,1} x_{t-1} + \gamma_{x,1} y_{t-1} + v_t^x \\ y_t &= \beta_{y,0} + \beta_{y,1} x_{t-1} + \gamma_{y,1} y_{t-1} + v_t^y. \end{aligned}$$

What are the β and γ coefficients in terms of the α, θ, ϕ, and δ parameters?

2. Calculate the VAR error terms *v* in terms of the structural shocks ε. What is the variance of each of the *v* error terms and what is the covariance between them, expressed in terms of $\sigma_x^2$, $\sigma_y^2$, and the parameters of the model?

3. Suppose that we estimate the VAR and use the residuals $\hat{v}_t^x$ and $\hat{v}_t^y$ to calculate $\widehat{\text{var}}\left(v_t^x\right)$, $\widehat{\text{var}}\left(v_t^y\right)$, and $\widehat{\text{cov}}\left(v_t^x, v_t^y\right)$. Show how we can identify $\sigma_x^2$, $\sigma_y^2$, and $\delta_0$ from these three parameters, and that given these identifications we can identify all of the α, θ, ϕ, and δ in the structural system from the reduced-form coefficients.

4. Without doing too much actual calculation, why would it be impossible to identify the parameters of the model if $y_t$ appeared in the equation for $x_t$?