

A particular kind of nonstationary time series that is of great importance is the “difference stationary” series. This means a series that can be made stationary by differencing one or more times. Such processes have a “unit root,” the intuition of which is explored in this problem.

1. Consider the second-order polynomial $P(x) = x^2 - 5x + 6$ in the variable x . Very specifically, what do we mean by the “roots” of $P(x)$? What are the roots of $P(x)$?
2. Now consider the first-order polynomial $\alpha(L) = 1 - 0.25L$ in the lag operator L . What is the root of $\alpha(L)$?
3. In the more general first-order lag polynomial $\alpha(L) = 1 - \alpha L$, what is the root?
4. For each of the following second-order autoregressive processes, express the process in the form $\alpha(L)y_t = v_t$ and find the roots of the lag polynomial $\alpha(L)$. Assuming that v_t is white noise, tell whether the process is stationary (all roots are > 1 in absolute value), difference stationary (one or more roots with absolute value of 1, others > 1 in absolute value), or completely nonstationary (one or more roots with absolute value < 1).
 - a. $y_t = 0.75y_{t-1} - 0.125y_{t-2} + v_t$
 - b. $y_t = 1.25y_{t-1} - 0.25y_{t-2} + v_t$
 - c. $y_t = 2y_{t-1} - y_{t-2} + v_t$