Economics 314 — Spring 2010

Project #2 Assignment — Due: 9am, Wednesday, February 10

Partner assignments

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Each team should submit a single paper. I strongly encourage you to work as a team in solving the problems. At a very minimum, both partners must understand everything that's on the paper when you turn it in. You are free to interact with other teams and to seek advice from me as you work on the problems, especially on the mathematical solutions. The idea of problem sets is to learn from them, not to test whether you can do them perfectly without help.

Problems

1. **Marginal products in terms of intensive production function.** In the Solow growth model, we write the aggregate production function as $Y = F(K, AL)$, and the intensive production function as $y = f(k)$, where $y \equiv \frac{Y}{AL}$, $k \equiv \frac{K}{AL}$, and $f(\bullet) \equiv F(\bullet, 1)$. Assume constant returns to scale and the usual properties for the partial derivatives of F.

   a. Show that the marginal products of capital and labor are $\frac{\partial Y}{\partial K} = f'(k)$ and $\frac{\partial Y}{\partial L} = A\left[f(k) - kf'(k)\right]$.
   b. Show that the marginal product of capital is decreasing in k and the marginal product of labor is increasing in k.
   c. In competitive equilibrium, each unit of capital and labor will be paid its marginal product. Based on part a, how do the real rental rate of a unit of capital ($r + \delta$ = MPK) and the real wage on labor (w = MPL) behave on a Solow steady-state growth path? Does this seem realistic given the long-run (say, since 1800) behavior of the U.S. economy?
   d. How do total earnings of capital ($K \times$ MPK) and total earnings of labor ($L \times$ MPL) behave along the growth path? Given the behavior of total output, what will happen to capital's share (capital earnings divided by total output) and labor's share in steady-state growth?

2. Work Romer's problem 1.3.

   - ***Addition to each part:*** "Sketch the new steady-state growth path of ln(Y) and the path that ln(Y) will follow as it converges to its new steady-state path. Explain in words what is happening at the moment of the change and afterwards."
   - Be careful on part (c): do you know if $k < 1$ or $k > 1$?

3. Work Romer's problem 1.4.
   - Assume that the new workers bring no capital with them and that their arrival has no effect on A.
   - ***Addition*** to part (a): "What happens to total output Y at the time of the jump? Why?"
   - ***Addition*** to part (c): "Is the total level of output on the new balanced growth path higher, lower, or the same as it would have been if there had been no new workers? Why?"

4. Work Romer's problem 1.6.
   - In part (b), be sure that you think about both the level and the slope of the growth path of Y.

5. Work Romer's problem 1.8.
   - ***Hint***: Romer's equation (1.27) can be useful for this problem.