Economics 312 Spring 2010

Practice Problems with Matrix Algebra

1. Suppose that $A = \begin{pmatrix} 1 & 4 \\ 2 & 7 \\ 3 & 9 \end{pmatrix}$ and $B = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

   a. Calculate AB.
   b. Explain why BA and $A'B'$ do not exist.
   c. Show that $(AB)' = B'A'$.

2. Explain why $A'A$ exists for any general $n \times k$ matrix A. Show that this matrix is always square and symmetric.

3. Show that the "inner product" of a column vector $x'x$ is scalar equal to the sum of the squares of the elements of the vector. Show that the "outer product" xx' "explodes" the vector into a matrix of squares and cross-products of all the elements. Use the latter property to explain why $E\left((x - Ex)(x - Ex)'\right)$ is the covariance matrix of any vector of random variables x. If the elements of x are IID with mean 0 and variance σ^2, find the covariance matrix.

4. Show that, with X defined as in class, $X'X = \begin{pmatrix} n & n\bar{X} \\ n\bar{X} & \sum_{i=1}^{n} X_i^2 \end{pmatrix}$. Show that

$$(X'X)^{-1} = \frac{1}{\sum_{i=1}^{n}\left(X_i^2\right) - n\bar{X}^2} \begin{pmatrix} \frac{1}{n}\sum_{i=1}^{n}\left(X_i^2\right) & -\bar{X} \\ -\bar{X} & 1 \end{pmatrix}$$

by showing that the product of this matrix with $X'X$ is the identity matrix. When X is non-random and u is homoskedastic, $\sigma^2 X'X$ is the covariance matrix of the OLS coefficient estimator. What is $\text{var}\left(\hat{\beta}_1\right)$? What is $\text{cov}\left(\hat{\beta}_0, \hat{\beta}_1\right)$? Is the covariance positive or negative? Explain the intuition of this result.