Economics 304 **Fall 2013**
Problem Set #1 **Solutions**

1. This is an algebraic problem pertaining to the economy of Reedistan, which behaves according to the Solow growth model. Suppose that the production function in Reedistan is

$$Y = F(K, AL) = K^{1/3}(AL)^{2/3}.$$

a. Show that the intensive form of this production function is $y = k^{1/3}$.

$$y = \frac{Y}{AL} = \frac{K^{1/3}(AL)^{2/3}}{AL} = K^{1/3}(AL)^{(2/3-1)} = K^{1/3}(AL)^{-1/3} = k^{1/3}.$$

b. Solve for the steady-state values $\bar{k}$ and $\bar{y}$ as functions of s, δ, n, and a.

The steady state occurs when $\Delta k = 0$. Using the equation of motion for the model,

$$\Delta k = sk^{1/3} - (n + a + \delta)k = 0$$
$$sk^{1/3} = (n + a + \delta)k$$
$$s = \frac{(n + a + \delta)k}{k^{1/3}} = (n + a + \delta)k^{2/3}$$
$$k^{2/3} = \frac{s}{n + a + \delta}$$
$$\bar{k} = \left(\frac{s}{n + a + \delta}\right)^{3/2}$$
$$\bar{y} = \bar{k}^{1/3} = \left(\frac{s}{n + a + \delta}\right)^{1/2}.$$

c. Suppose that $\delta = 0.06$, $n = 0.02$, and $a = 0.01$. Calculate the values of $\bar{k}$, $\bar{y}$, and $\bar{c}$ for s = 0.04, 0.09, 0.16, 0.25, 0.36, and 0.49. Which of these values of s is the "best"?

s	$\bar{k}$	$\bar{y}$	$\bar{c}$
0.04	0.296	0.667	0.64
0.09	1.000	1.000	0.91
0.16	2.370	1.333	1.12
0.25	4.630	1.667	1.25
0.36	8.000	2.000	**1.28**
0.49	12.704	2.333	1.19

If we adopt the Golden Rule criterion, then the best saving rate is 0.36, because this leads to the largest value of steady-state consumption.

d. The laws of calculus tell us that the marginal product of capital for this production function depends on *k* according to $MPK = \frac{1}{3} k^{-2/3}$. Using this formula, calculate the Golden Rule values of *s*, $\bar{k}$, and $\bar{c}$ for the parameters given in part c. Is this consistent with the evidence from part c?

The Golden Rule saving rate occurs where the slope of the production function (MPK) equals the slope of the capital-broadening line ($n + a + \delta$). Setting these equal yields

$$\frac{1}{3} k^{-2/3} = n + a + \delta = 0.09$$
$$k^{-2/3} = 0.27$$
$$\bar{k}_{GR} = (0.27)^{-3/2} = 7.128,$$
$$\bar{y}_{GR} = \left(\bar{k}_{GR}\right)^{1/3} = 1.925,$$
$$s_{GR} = (n + a + \delta)\left(\bar{k}_{GR}\right)^{2/3} = \frac{0.09}{0.27} = \frac{1}{3}$$
$$\bar{c}_{GR} = (1 - s_{GR})\bar{y}_{GR} = \frac{2}{3}(1.925) = 1.283.$$

This is indeed consistent with part c because the highest value of steady-state consumption per effective labor unit in the table is 1.28 for $s = 0.36$, which is the close to the true maximum value of 1.283 when *s* is the true maximizing value of 1/3.

e. Suppose that (with the other parameters as in part c) the saving rate is initially 0.09 and Reedistan is in a steady-state equilibrium. Shortly after a revolution brings him to power in Year 1, Grand Ayatollah Mohammed Al-Kroger decrees convincingly that excessive consumption is sinful and as a result the saving rate increases to 0.36. Assume that the capital stock in any year is determined by the *previous* year's saving, so that the change in saving in year one does not affect the capital stock until year 2:

$$k_2 - k_1 = \Delta k_2 = s_1 f(k_1) - (\delta + n + a)k_1 .$$

How much will *k* and *y* increase in year 2 as a result of the increase in the saving rate that occurred in year 1? What percentage of the gap between the old and new steady state values of *y* has been made up in year 2? (This percentage is called the "rate of convergence.") About how many years will it take for *y* to travel halfway to the new steady state? [You might find a spreadsheet helpful in doing the calculations for this problem.]

The initial capital stock per effective labor unit in period one is 1, which is the steady-state value under the old saving rate of 0.09. We also know that the new steady-state value of k is going to be 8 (from the table in part c). In period 2, the change in k will be

$$k_2 - k_1 = \Delta k_2 = s_1 f(k_1) - (\delta + n + a)k_1 = 0.36(1)^{1/3} - 0.09(1) = 0.27.$$

Thus, $k_2 = 1 + 0.27 = 1.27$. Then, $y_2 = (1.27)^{1/3} = 1.083$. By the same formula,

$$k_3 - k_2 = \Delta k_3 = s_2 f(k_2) - (\delta + n + a)k_2 = 0.36(1.27)^{1/3} - 0.09(1.27) = 0.276.$$

$$k_3 = 1.27 + 0.276 = 1.546.$$

We can continue this series forever and the value of k_t will eventually get closer and closer to 8. The gap between the old and new steady states is 8 – 1 = 7, and the amount of the gap closed in the first period is 0.27, so the fraction is 0.27 / 7 or about 4%. Halfway from 1 to 8 is 4.5, which is achieved after just over 15 years. After 100 years, k is 7.98, or almost to the new steady-state value. For y, 8.3% of the gap from 1 to 2 is closed in the first year, and 1.5 (halfway) is reached by year 10.

2. Endogenous growth models incorporate an expanded concept of capital that has non-diminishing marginal returns, often justified as knowledge capital or human capital. Consider a simple production function $Y = AK$, in which K is the stock of this expanded capital, A is a constant, and Y is output. We assume that the labor force is constant, so we don't need to include it in and production function and we can do the familiar Solow diagram in terms of capital letters rather than the intensive form. There is no "exogenous" growth in this model. Suppose that the accumulation of capital is as in the Solow model: $\Delta K = sY - \delta K$. You should assume that $sA > \delta$.

a. Find an expression for ΔK in terms of K and the parameters of the model.

$$\Delta K = sAK - \delta K = (sA - \delta)K.$$

b. Show a graph analogous to the one we used for the Solow model and use it to describe the dynamic behavior of K.

The graph would look like Figure 4.8 in the text.

c. What is the growth rate of K in this model?

$$\frac{\Delta K}{K} = sA - \delta > 0.$$

d. Is there a process of "convergence" to a steady-state path or is the economy always growing at the same rate regardless of the initial level of K?

The growth rate is constant and does not depend on whether the initial value of K is high or low. There is no "convergence" in the sense that poorer countries will grow faster than richer countries with the same parameters.

e. How, if at all, is the growth rate affected by a change in the saving rate?

The growth rate depends positively on the saving rate.

f. Contrast these conclusions with those of the version of the Solow model in which the labor force and exogenous technology do not grow.

This model is able to generate growth even when the labor force and technology are stagnant, hence the label "endogenous growth." In the Solow model, the long-run growth rate is independent of the saving rate (saving has only "level effects") whereas here an increase in the saving rate increases growth forever (a "growth effect").