Economics 201 | Fall 2010
Introduction to Economic Analysis | Jeffrey Parker
Problem Set #6 | Due: Wednesday, November 3

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1. Cournot Duopoly. Bartels and Jaymes are two individuals who one day discover a stream that flows wine cooler instead of water. Bartels and Jaymes decide to bottle the wine cooler and sell it but, depending on the circumstances, they may either cooperate (collude as a monopolist) or compete as Cournot duopolists. The marginal cost of bottling wine cooler and the fixed cost to bottled wine cooler are both zero. The market demand for bottled wine cooler is given as:

$$P = 90 - 0.25Q,$$

where Q is the total quantity of bottled wine cooler produced and P is the market price of bottled wine cooler. Use graphs to illustrate your answers when appropriate.[1]

a. What is the economically efficient price of bottled wine cooler? What is the economically efficient quantity of bottled wine cooler produced?

Economic efficiency occurs where price = marginal cost. Since marginal cost is zero, the efficient price of wine cooler is zero. Using the demand curve, $90 - 0.25Q = 0$, or $0.25Q = 90$, $Q = 360$ units.

b. If Bartels and Jaymes were to collude with one another and produce the profit-maximizing monopoly quantity of bottled wine cooler, how much bottled wine cooler will they collectively produce? At that output level, what price will Bartels and Jaymes charge for bottled wine cooler? At that output level, what is the welfare loss relative to the economically efficient point?

Using the formula in the footnote, MR = $90 - 0.50Q$. Setting MC = MR means MR = 0, so $90 - 0.50Q = 0$, or $Q = 180$. To sell 180 units, they can charge a price of $90 - 0.25 \times 180$ = \$45. They make a profit equal to their revenue, which is 180 × \$45 = \$8,100.

At the efficient point, there is no profit to B&J but consumer surplus is a triangle with height \$90 and width 360, which has area of ½ × \$90 × 360 = \$16,200. Under collusion, consumer surplus shrinks to ½ × (\$90 − \$45) × 180 = \$4,050, but B&J earn a profit of \$8,100. Total surplus (consumer plus producer) as \$4,050 + \$8,100 = \$12,150, which is \$4,050 smaller than at the social optimum. This is the area of the deadweight loss triangle.

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[1] Remember that you can calculate the marginal-revenue curve corresponding to a linear demand curve easily: it has the same vertical intercept and twice the slope as the demand curve. In other words, if the demand curve is $P = \alpha - \beta Q$, then the marginal-revenue curve is $MR = \alpha - 2\beta Q$. You will need to apply this formula more than once in this problem.

c. Now suppose that Bartels and Jaymes act as Cournot duopolists, what are the reaction functions for Bartels and for Jaymes? (Hint: They are symmetric.) In the long run, what level of output will each produce if the two producers act as Cournot duopolists? In the long run, what will be the price of wine coolers be if they act as Cournot duopolists? What will be the welfare loss compared with economic efficiency and with monopoly?

Under Cournot duopoly, Bartels takes Jaymes's output as given and maximizes profit. We know that $Q = Q_B + Q_J$, so substituting this into the demand curve gives $P = (90 - 0.25Q_J) - 0.25Q_B$. Bartels takes Q_J as constant, so the terms in parenthesis are the vertical intercept of his demand curve.

The corresponding marginal revenue curve is $MR_B = (90 - 0.25Q_J) - 0.50Q_B$. Setting MR = MC = 0, $(90 - 0.25Q_J) - 0.50Q_B = 0$, or $Q_B = 180 - 0.50Q_J$. This is Bartles's reaction function.

By symmetry, $Q_J = 180 - 0.50Q_B$. Solving these two equations together yields $Q_B = 180 - 0.50(180 - 0.50Q_B) = 90 + 0.25Q_B$ or $0.75Q_B = 90$, or $Q_B = 120$. Since Jaymes solves the same problem, $Q_J = 120$ as well and total output is 240. The demand curve tells us that 240 units can be sold at a price of $90 - 0.25 \times 240 = \$30$.

This is less than the collusive monopoly price of $45, but more than the efficient price of $0. Selling 120 units for a price of $30, each seller earns revenue (and profit) of 120 × $30 = $3,600, for a total profit of $7,200. The consumer-surplus triangle has height of 90 − 30 = $60 and width of 240 units, so its area is ½ × $60 × 240 = $7,200. Total gains are the sum of the $7,200 profit and the $7,200 consumer surplus, or $14,400. Note that the Cournot solution lies between the efficient point and the monopoly point in terms of price, quantity, consumer surplus, profit, and total gains from exchange.

2. Production, marginal product, and input demand. The table below shows a firm's output per day for zero through six workers.

Q	L
0	0
46	1
84	2
114	3
136	4
150	5
156	6

The firm's demand and marginal revenue curves are:

$$P = 50 - 0.125Q \qquad MR = 50 - 0.25Q,$$

where Q = daily sales, and P = output price.

a. Determine the marginal product of labor for one through six workers.

Marginal product is change in total product from one more worker.

Q	L	MP
0	0	–
46	1	46
84	2	38
114	3	30
136	4	22
150	5	14
156	6	6

b. Determine the firm's marginal revenue product.

MRP = MR • MP_L, so we must determine MR at levels of output corresponding to one through six workers. We can either do this with the formula MR = 50 – 0.25Q or by calculating the difference in total revenue between adjacent values of Q divided by the change in Q.

Q	P	TR	MR = ΔTR/ΔQ	MR = 50 – 0.25Q
0	50.00	0.00		
46	44.25	2035.50	44.25	38.50
84	39.50	3318.00	33.75	29.00
114	35.75	4075.50	25.25	21.50
136	33.00	4488.00	18.75	16.00
150	31.25	4678.50	14.25	12.50
156	30.50	4758.00	11.75	11.00

Now, to calculate MRP = MR • MP_L for each level of labor input:

L	Q	MP_L	By difference		By formula	
			MR	**MRP_L**	MR	**MRP_L**
1	46	46	44.25	**2035.50**	38.50	**1771**
2	84	38	33.75	**1282.50**	29.00	**1102**
3	114	30	25.25	**757.50**	21.50	**645**
4	136	22	18.75	**412.50**	16.00	**352**
5	150	14	14.25	**199.50**	12.50	**175**
6	156	6	11.75	**70.50**	11.00	**66**

c. How many workers should the firm hire if total wage costs including fringe benefits are $30 per hour? (Remember that output is measured daily; each worker is assumed to be employed for eight hours per day.)

The firm would want to hire every worker whose MRP $\geq W$. Thirty dollars per hour is $240 per day. The firm would want to hire four workers regardless of which method we use to calculate marginal revenue.

3. A competitive labor market. The market for production workers in Dudeville, California is highly competitive. The market supply and demand curves for production workers are given as:

$$L_S = -2500 + 1000W \qquad L_D = 10500 - 625W,$$

where L_D = labor demand is full time workers per hour, L_S = labor supply is full time workers per hour, and W = hourly wage. RollerBall Manufacturing Co. employs production workers in the manufacture of bearings for skateboards and roller skates. The firm's production function is given by the expression:

$$Q = 88.8L - 0.5L^2,$$

where Q = output, measured as boxes of bearings per hour, and L = number of workers employed per hour. From this production function, the marginal product and average product of labor are:

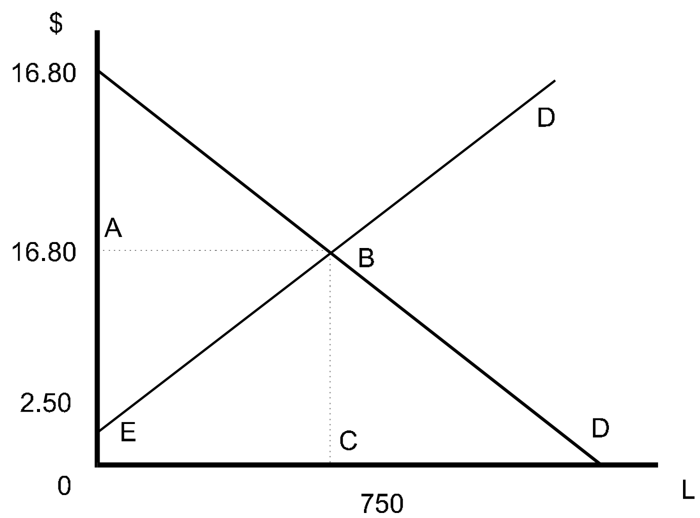
$$\text{MP} = 88.8 - L \qquad \text{AP} = 88.8 - 0.5L$$

RollerBall currently sells bearings for $10 per box and is a price-taker in both output and input markets.

a. Determine the equilibrium wage and level of employment in the market. Calculate the total rent that is being earned by workers.

We are given that $L_S = -2500 + 1000W$, $L_D = 10500 - 625W$. Equate L_S to L_D: $-2{,}500 + 1{,}000W = 10{,}500 - 625W$, $-13{,}000 = -1{,}625W$, or $W = \$8.000$. Plugging this wage back into the labor supply equation yields $L_S = -2{,}500 + 1{,}000(8)$, $L = 5{,}500$.

To calculate rent, it helps to solve the supply and demand curves for W: $L_S = -2{,}500 + 1{,}000W$, $L_S + 2{,}500 = 1{,}000W$, so $W = 2.5 + 0.001L_S$. Similarly, $L_D = 10{,}500 - 625W$, so $W = 16.8 - 0.0016L_D$.



Rent to labor suppliers is the area ABE = (0.5)(5,500)(8−2.50) = 15,125.

b. Determine the number of workers that RollerBall Manufacturing would employ at the wage determined in part (a). What total output will RollerBall produce?

MR is constant at 10, so MRP is just 10 times MP, or $888 - 10L$. Equating MRP to the equilibrium wage (determined above to be \$8.00), $888 - 10L = 8$, so $-10L = -880$, $L = 88$. To determine output, substitute L into production function: $Q = 88.8(88) - 0.5(88)^2$, $Q = 3{,}942.4$.

4. Making an investment decision. The Edgeworth Box Company produces packaging materials. Edgeworth is considering undertaking one or both of two investment projects. The first investment involves a new automated warehouse for the firm's cardboard, foam, and plastic inventory. The warehouse can be expected to have a useful life of ten years, after which it will be obsolete with no scrap value. The warehouse involves \$3,000,000 in capital cost that must be paid immediately. The warehouse will lower the firm's cost \$400,000 for each of the first five years, and \$500,000 per year thereafter. The second project involves the acquisition of a computerized order system that would allow the firm's salespeople to link directly with the computer to place orders. The computerized network will require an initial capital cost of \$1,000,000, but will save the firm \$300,000 per year in support staff costs. Edgeworth's managers believe that the order system will be obsolete after five years. Cash flows for each project will be at year end. Edgeworth uses a 10% discount (interest) rate in evaluating the investment projects. Interest rates and future cash flows are in real terms, net of all tax effects.

a. Calculate the net present value of each investment project. Which project(s) should the firm accept?

The general formula for net present value is

$$NPV_{investment} = -C + \frac{\pi_1}{(1+R)^1} + \frac{\pi_2}{(1+R)^2} + \frac{\pi_3}{(1+R)^3} \cdots \frac{\pi}{(1+R)^n}.$$

For the warehouse:

$$\begin{aligned} NPV_{warehouse} &= -3{,}000{,}000 + \frac{400{,}000}{(1+0.10)^1} + \frac{400{,}000}{(1+0.10)^2} + \frac{400{,}000}{(1+0.10)^3} + \frac{400{,}000}{(1+0.10)^4} \\ &+ \frac{400{,}000}{(1+0.10)^5} + \frac{500{,}000}{(1+0.10)^6} + \frac{500{,}000}{(1+0.10)^7} + \frac{500{,}000}{(1+0.10)^8} \\ &+ \frac{500{,}000}{(1+0.10)^9} + \frac{500{,}000}{(1+0.10)^{10}} \\ &= -3{,}000{,}000 + 2{,}693{,}204.88 \\ &= -306{,}795.12. \end{aligned}$$

Given that the $NPV < 0$, the project should not be accepted.

For the computerized order system:

$$\begin{aligned} NPV_{computer\ system} &= -1{,}000{,}000 + \frac{300{,}000}{(1+0.10)^1} + \frac{300{,}000}{(1+0.10)^2} + \frac{300{,}000}{(1+0.10)^3} + \frac{300{,}000}{(1+0.10)^4} + \frac{300{,}000}{(1+0.10)^5} \\ &= -1{,}000{,}000 + 1{,}137{,}236.03 \\ &= 137{,}236.03. \end{aligned}$$

Given that the $NPV > 0$, the project should be accepted.

b. Comment on the impact of a change in the discount rate on the NPV. (Analyze both an increase and a decrease in the discount rate.)

Raising the discount rate lowers the NPV, lowering the discount rate raises the NPV.