1. Present vs. future consumption. Consider a consumer Felicia's choice between "goods this year" (C_1) on the horizontal axis and "goods next year" (C_2) on the vertical axis. Both sets of goods are measured in dollar terms. We assume no inflation so that we don't need to worry about the distinction between real and nominal income and expenditure. We also assume that Felicia has no previous savings and does not want to have any savings at the end of next year, so these are the only two goods relevant to her consumption behavior. Both current and future consumption are normal goods. Felicia's preferences are such that she tends to like reasonably smooth consumption (close to equal in the two periods) rather than a very uneven consumption path.

Felicia earns \$20,000 this year and \$30,000 next year. She has the option of borrowing or lending at a 10% interest rate. She can borrow or lend as much as she wants at 10%, as long as she repays any borrowing next year. One consumption option is to consume the same amount as she earns in each period, so this "endowment point" (C_1 = 20,000, C_2 = 30,000) must lie on her budget constraint.

a. What is the "relative price" of goods this year in terms of goods next year: the amount of next year's goods that must be given up to consume an additional unit of goods this year? How is this related to Felicia's budget constraint? Graph the budget constraint, showing the values at which it intersects each axis. Is it a straight line?

Each dollar of goods consumed today means a dollar not lent. If the dollar had been lent, it would have earned interest and become $\$1.00 \times (1 + r) = \$1.10$ next year. Since the price level is assumed to be stable, this would buy 1.1 times as many goods next year as the amount given up today, so the relative price of one current good is 1.1 future goods. The slope of Felicia's budget constraint is $-(1 + r) = -1.1$. The budget constraint is a downward-sloping line that passes through the endowment point (20000, 30000). If she consumes nothing now and lends her entire current income of 20,000, she will be able to consume 30,000 + 1.1 × 20,000 = 52,000 next year, so that is the vertical intercept. With an interest rate of 10%, if she has no consumption next year she can borrow 30000/1.1 = 27272.73 this year and repay it out of her future income, so the horizontal intercept is 20000 + 27272.73 = 47272.73. (30000/1.1 is called the "present value" of 30000 one year from now, a concept that we shall study in detail later in the course.)

b. Suppose that Felicia's current-year income increases by \$1,000 with no change in future income. How will her consumption in the current year and the future year be affected?

The endowment point, and therefore the budget constraint, would shift to the right by 1000. The horizontal intercept increases by 1000 and the vertical intercept increases by 1100, since she could consume 1100 more next year by saving the entire income increase. Assuming that both current and future goods are normal (which is assured by the assumption that she prefers relatively smooth consumption), she will increase consumption in both periods, saving some of the increased current income to be consumed in the future. This is often called "consumption smoothing."

c. Now show the effects on consumption in the two periods of a $1,000 increase in next-year's income (with no change in current income). How (if at all) are they different from those in part (b)? Explain.

If future income increases by 1000, then the endowment point and the budget constraint shift upward by 1000. The vertical intercept increases by 1000 and the horizontal intercept increases by 1000/1.1 = 909.09, the amount she could borrow today and repay next year with the additional 1000 of future income. Note that because the increase happens in the future, the budget constraint shifts by less than in part b. Again, Felicia will increase consumption in both periods, but by slightly less than if the increase in income were current.

d. Show the effects on consumption in the two periods of a $1,000 increase in income in each year ($2,000 total).

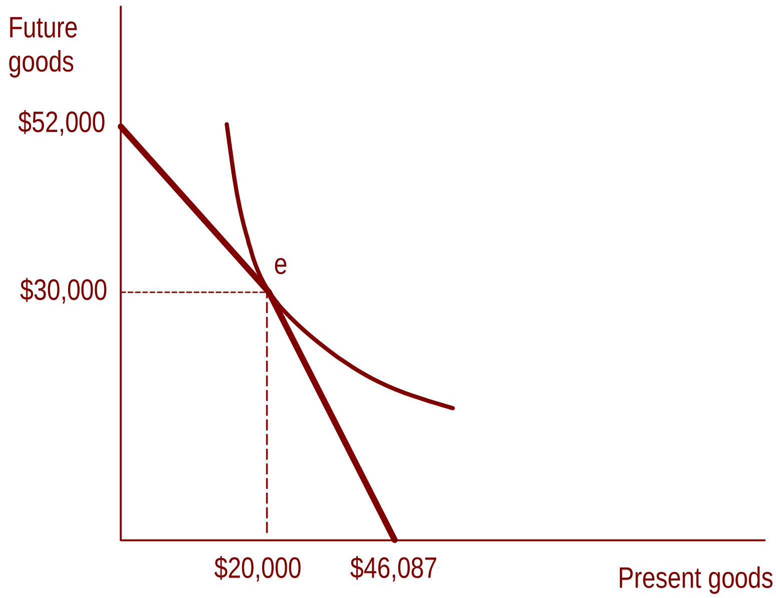
This would combine the effects of b and c. The vertical intercept increases by 2100 and the horizontal intercept by 1909.09. Consumption increases in both periods, by approximately twice as much as in the cases above.

e. Based on your analysis above, does an increase in someone's income have a larger effect on current consumption if it is believed to be permanent or if it is believed to be temporary? Why?

For a consumer with the ability to borrow and lend, current consumption is based on lifetime income, not just current income. A permanent increase in income has a larger effect on lifetime income, so it shifts the budget constraint out further and leads to a larger increase in current (and future) consumption.

f. We have assumed so far that Felicia is able to borrow or lend at the same interest rate. Now suppose, more realistically, that Felicia has to pay a higher interest rate (15%) on her borrowing than the rate that she receives if she lends (10%). What does the budget constraint look like in this case (at the original levels of income)? Does this imperfection in the credit market make it more likely that Felicia would choose to consume at exactly the endowment point? Why?

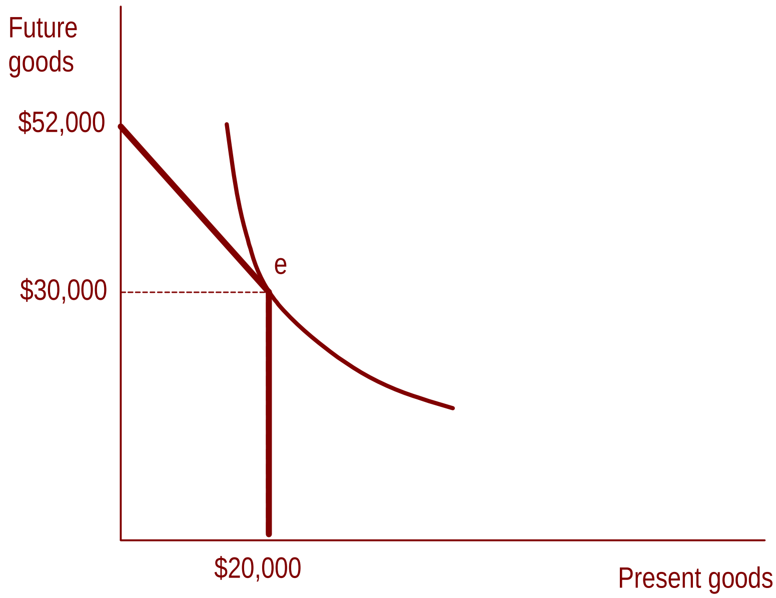
First of all, the endowment point is still on the budget constraint because she still has the option of consuming her income in each period. Lending is moving upward along the constraint (so that consumption in period one is less than period-one income). Since she faces a 10% interest rate when she lends, the segment of the budget constraint above the endowment point has a slope of −1.1 as before. Borrowing occurs as she moves to points to the right of and below the endowment point. Since she now pays 15% interest, this segment of the budget constraint has a slope of −1.15, which is steeper than the upper section. Thus, the budget constraint is kinked as shown below with a horizontal intercept of 20000 + (30000/1.15) = 46087. Because the kink in the budget constraint will touch the highest achievable indifference curve for a whole range of MRS values, it is much more likely that she will consume at the endowment point (neither borrow nor lend) when there is a kink in the constraint.



g. Finally, suppose that Felicia cannot borrow at all, but is able to lend at 10%. What does her budget constraint look like in this case? If she begins at a point of no lending, how (if at all) is the answer to part (e) different in this case?

If she is completely "liquidity constrained" and cannot borrow, then she cannot move to the right of the endowment point at all. The right-hand segment of the budget constraint is vertical as below.

This makes it almost certain that she will choose to consume at *e* if she prefers smooth consumption since her present income is lower than her future income. In this case, an increase if $1000 in current income would shift the budget constraint to the right, which would almost certainly cause her to increase current consumption by the full $1000. Since she cannot do the borrowing that she would like to do, consuming the entire $1000 of additional income is the optimal strategy. However, if her future income increases as well, she does not have the option of borrowing against it. Thus, if the budget constraint shifts upward as well as to the right, only the shift to the right affects current consumption. If Felicia is perfectly liquidity constrained, then permanent and temporary changes in income have the same effect on current consumption.



2. Consumer Equilibrium. George has a given amount of income and can afford at most 9 units of Y if he spends his entire income on Y. Alternatively, if he spends all his income on X, he can afford at most 6 units of X.

a. What is the relative price of X in terms of Y? (How much Y must George give up to get a unit of X?)

The relative price of X is P_X/P_Y. From the information given, $I/P_Y = 9$ and $I/P_X = 6$, so $P_X/P_Y = 9/6 = 1.5$.

b. Draw George's budget line and an indifference curve such that George chooses to buy 4 units of X. (Put X on the horizontal axis.)

This is a standard indifference curve diagram with X = 4 and Y = 3.

c. In equilibrium, what is George's marginal rate of substitution between Y and X?

At the point of equilibrium, the indifference curve is tangent to the budget line, so they have equal slopes (−1.5). The MRS is the absolute value of the slope of the indifference curve, so at the point of equilibrium the MRS is 1.5.

d. Martha faces the same prices as George, yet she chooses to buy 2 units of X. Is Martha's income higher, lower, or the same as George's, or can we tell for sure?

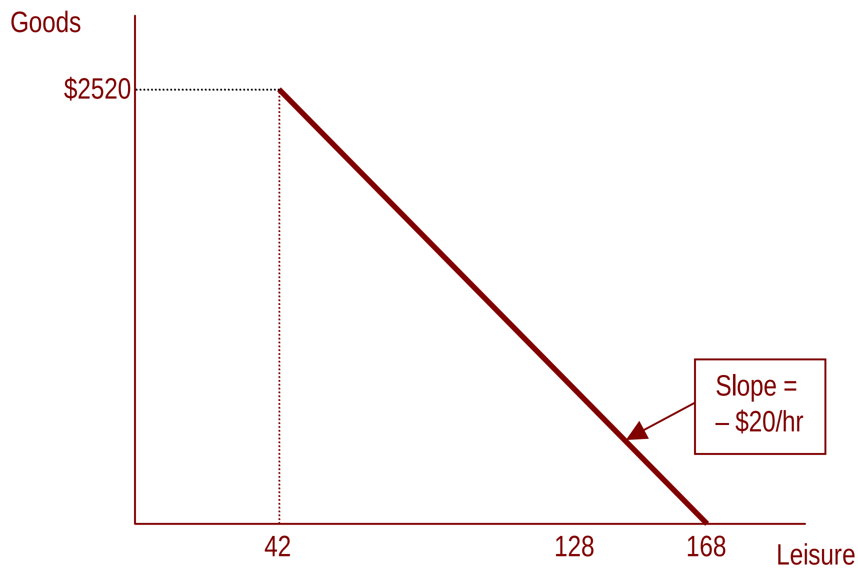
Since we don't know whether Martha's preferences are similar to George's, we can't tell if Martha consumes less X because her income is lower or because she just doesn't like X as much.

e. In equilibrium, is Martha's marginal rate of substitution between Y and X higher, lower, or the same as George's, or can we tell for sure?

Since Martha faces the same prices as George, her budget constraint has the same slope as his, which means that at their respective points of equilibrium their MRSs are the same.

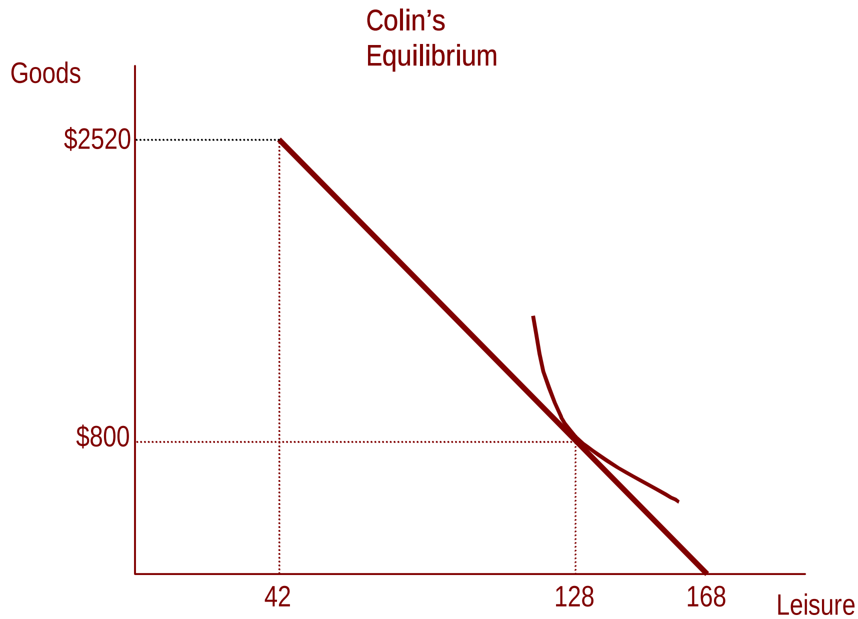
3. Choosing between leisure and consumption. The consumer-equilibrium model can be used to examine the tradeoff between leisure and labor (or leisure and the goods that can be bought with one's labor earnings). To do this, we put weekly hours of leisure (non-work time) on the horizontal axis and the generic commodity "goods" on the vertical axis (measured in dollars). There are 168 hours in a week, so the number of leisure hours equals 168 minus the number of hours worked.

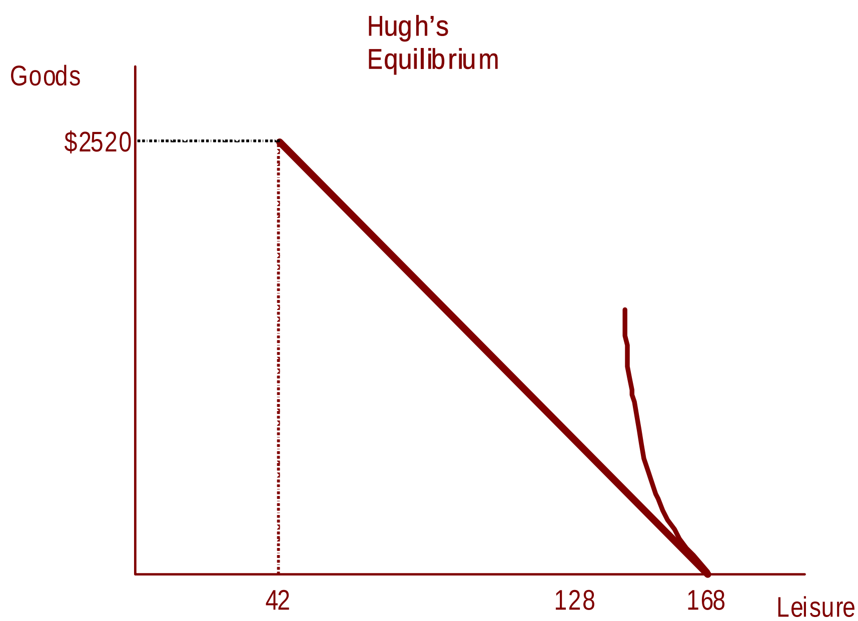
a. Suppose that a consumer, Colin, can work as many hours as he wishes during the week for a wage of \$20/hour, but that he needs to sleep at least 42 hours per week (which counts as leisure). Show the graph of his budget constraint.



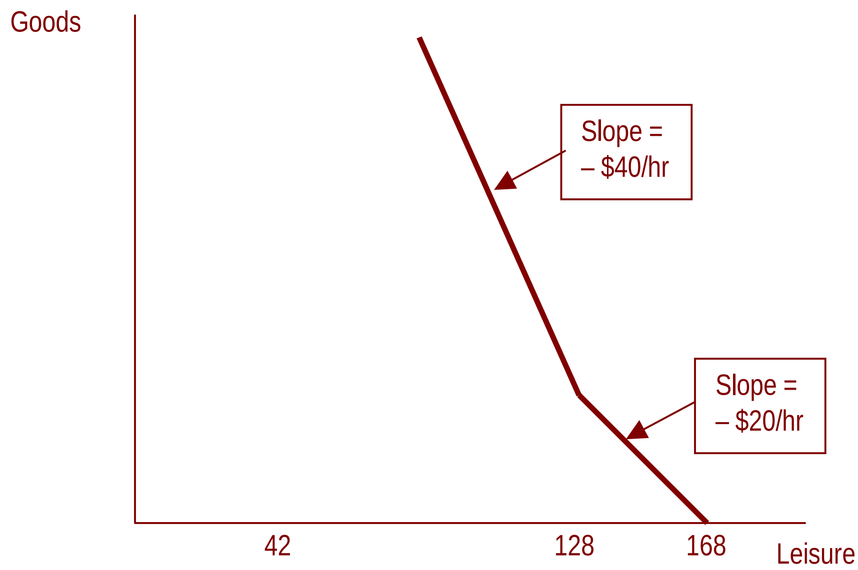
The absolute value of the slope of the budget constraint is the amount of goods he gives up in order to get one more hour of leisure, which is his wage, or \$20.00/hour. The budget constraint effectively stops at 42 hours of leisure because he cannot consume less than that. If he works 168 – 42 = 126 hours, he earns 126 × \$20 = \$2,520.

b. Colin and his friend Hugh face the same budget constraint, but they have very different preferences: Colin chooses to work 40 hours per week, but Hugh chooses zero. Show (on separate graphs) Colin and Hugh's indifference maps and their respective points of utility-maximizing equilibrium.





c. Suppose that overtime work (all hours above 40 per week) earns a doubled wage ($40/hour). Show how the budget constraint changes. Can you be sure that Colin will work overtime? Might Hugh choose to work now, and if so, will he work overtime?



Colin will surely choose to work overtime. If an indifference curve is tangent to the original budget line at 128 hours of leisure, then that indifference curve surely lies partially below the upper segment of the new, kinked line. Thus, he can achieve higher utility by moving upward to the left and increasing his hours. Colin will always work overtime. Hugh would never choose more than zero but fewer than 40 hours; he had that choice before and demonstrated that he preferred zero to any positive amount of work at a $20 wage. His indifference curve through the point (168, 0) lies above the lower segment of the budget constraint. However, that indifference curve *could be* sufficiently flat that it crosses the upper branch of the budget constraint. If so, he can achieve higher utility by working more than 40 hours than by working zero. He will either work overtime or not at all in this situation!

d. Now suppose that (with no overtime wage), anyone who doesn't work at all gets $200 per week in welfare payments, but that these payments are reduced by $0.50 for each dollar of income earned, so that workers earning $400 or more per week get none. Might Hugh work? Might Colin quit working altogether? Might Colin choose to work fewer (but still positive) hours?

They get $200 if they work zero hours (168 hours of leisure), so the right end-point of the budget constraint is above the axis at (168, $200). For the first 20 hours worked, they lose half of their earnings in lost benefits, so their wage is effectively $10 per hours. Once they have reached 20 hours of work (148 hours of leisure), their total earnings are $400 and they have lost all of their welfare benefit, so their wage increases to $20 again along the original budget constraint.

Hugh would never work in this situation if he doesn't work without the welfare payment. His original indifference curve through (168, $0) lies above and is steeper than the original budget line (except at that point). The one through (168, $200), which is now attainable, lies above the original one, and thus it would also lie above and be steeper than the new budget constraint. Thus, his highest utility is at (168, $200) and he does not work.

Colin's initial position is on an indifference curve that is tangent to the original budget line at (128, $800). If this indifference curve lies above the lower (flatter) portion of the budget constraint, then he will continue to work 40 hours. If it is flat enough that it passes below the lower portion of the kinked budget constraint, he could either find it optimal to choose zero hours (a corner solution at (168, $200)) or possibly to work less than 20 hours (if a higher indifference curve than his original one is tangent to the lower portion of the constraint). Colin would never choose a number of hours between 20 and 40 or more than 40, because those parts of the budget constraint have not changed and he preferred 40 hours before.