

Instructions: This problem set is due in class on Wednesday, September 8. Each student is to hand in his or her own independent solutions to the problem set. You may work together on the problem sets as long as “working together” means learning together or learning from each other and not simply sharing final answers.

If you get stuck, you are encouraged to ask questions of the instructor or the tutors. Tutors will be available at work sessions Monday and Tuesday evenings in the Dorothy Johansen House. Although Jeff’s office hours are canceled on Monday (due to holiday) and Tuesday (due to medical appointment), he will be available by email both days. Individual tutoring (at no cost for up to one hour per week) can be arranged through the Office of Student Services.

1. **Production Possibilities Frontiers.** Suppose that the tiny nation of Agrizona has 50,000 acres of flat land and 80,000 acres of hills. Each acre of flat land can produce 3 tons of parsnips or 2 tons of avocados. Each acre of hills can produce 1 ton of parsnips or 1 ton of avocados.

(a) How many tons of parsnips can be produced if Agrizona specializes in parsnips?

The 50,000 acres of flat land can produce 3 tons each for a total of 150,000 tons. The 80,000 acres of hills can produce 1 ton each, making a total of $150,000 + 80,000 = 230,000$ tons if all land is used for parsnips.

(b) How many tons of avocados can be produced if they specialize in avocados?

The 50,000 acres of flat land can produce 2 tons each for a total of 100,000 tons. The 80,000 acres of hills can also produce 1 ton each, making a total of $100,000 + 80,000 = 180,000$ tons if all land is used for avocados.

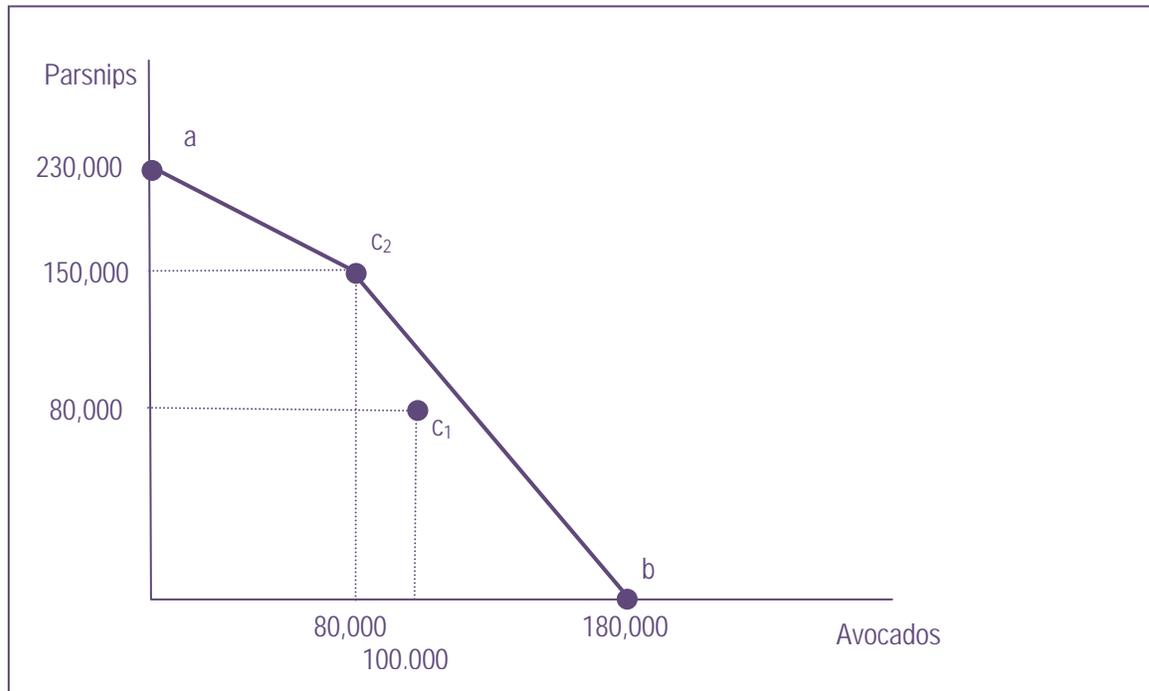
(c) If Agrizona devoted all of its flat land to avocados and all of its hills to parsnips, how much of each could it produce? What if it devoted all hills to avocados and all flat land to parsnips?

If all flat land produced avocados, $2 \times 50,000 = 100,000$ tons of avocados would be produced, while the hills would produce 80,000 tons of parsnips. If all flat land produced parsnips, then $3 \times 50,000 = 150,000$ tons of parsnips would be produced along with 80,000 of avocados.

(d) Based on your answer to (c), if Agrizona is to produce both parsnips and avocados, where should the parsnips be grown and where should they grow avocados? Why?

Flat land is three times more productive at growing parsnips and only twice as productive at growing avocados, so it is more efficient to use flat land to grow parsnips and hills to grow avocados

(e) Draw the production possibilities frontier (PPF) for Agrizona with production of parsnips on the vertical axis and production of avocados on the horizontal axis. Identify the four points discussed in parts (a) through (c). Why is its shape different from the smooth curve in Stiglitz's Figure 2.4? How could we change our simple assumptions to make the PPF a smooth curve?



The points are noted in the diagram. Note that c_1 is inside the PPF because it is more efficient to use the flat land for parsnips (leading to point c_2) and the hills for avocados than the other way around. Of course, if Agrizona is to produce more than 80,000 tons of avocados, then *some* avocados will be grown on flat land and if they want to produce more than 150,000 tons of parsnips then they will need to devote *some* hills to parsnip production.

The PPF is piece-wise linear because of the simple assumptions we have made: only two kinds of land and constant productivity of each product on each kind of land. More realistic and sophisticated assumptions would lead to the bowed-out curve that Stiglitz shows in his figures.

(f) What is the opportunity cost of avocados (in terms of parsnips) on the upper part of the PPF? On the lower part? Explain the relationship between opportunity cost and the slope of the PPF.

On the top part, we sacrifice one ton of parsnips to produce one ton of avocados, so the opportunity cost of each avocado is 1 and the slope of the PPF there is -1 . On the bottom

part, we sacrifice 1.5 tons of parsnips for each additional ton of avocados and the slope of the PPF is -1.5 . The opportunity cost is the absolute value of the slope.

(g) What additional information would we need in order to know the point on the PPF at which Agrizona would (or should) produce?

We would need to know something about Agrizona's preferences for parsnips and avocados. If Agrizona is an open economy, trading with other countries, the "world price" of parsnips in terms of avocados would be important to deciding where Agrizona produces and whether it exports or imports each vegetable.

(h) Show how and explain why Agrizona's PPF would change if it conquered an additional 20,000 acres of hills from a neighboring country.

The shape and slopes would be the same. The vertical intercept would increase to 250,000 and the horizontal intercept to 200,000. The kink would now occur at 100,000 tons of avocados and 150,000 tons of parsnips.

(i) Show how and explain why Agrizona's PPF would change if a new technology of parsnip cultivation raised the flat-land (only) yield of parsnips to 4 tons per acre. (Assume the original land areas.)

The vertical intercept would increase to $4 \times 50,000 + 1 \times 80,000 = 280,000$. The horizontal intercept would be unchanged because avocado technology has not changed. The kink would now occur at (80,000, 200,000) and the lower part of the PPF would now have a slope of -2 .

2. Analytics of a Linear Demand Curve. Suppose that the market demand for asparagus in Astonia is given by $Q_d = 25,000 - 2,000P$, where Q_d is the quantity demanded in pounds and P is the price of asparagus in dollars per pound.

(a) Graph the demand curve with price on the vertical axis and quantity on the horizontal axis.

Straight line with vertical intercept at \$12.5 and horizontal at 25,000.

(b) What is the slope of the demand curve? Is it constant? Explain the relationship between the slope and the coefficients of the demand equation?

Slope is constant at $-1/2,000 = -0.0005$, which is the reciprocal of the price coefficient in the demand equation. We must take the reciprocal because we are putting price on the vertical axis and quantity on the horizontal.

(c) At what values does the demand curve intersect the vertical and horizontal axes? What economic interpretation can be attached to each of these points?

See (a). At a price of \$12.5, no one buys asparagus. If the price were zero, then 25,000 pounds would be demanded.

(d) What happens when the price is above the vertical intercept value or if quantity is greater than the horizontal intercept value? Is this realistic?

If we take the equation literally, then quantity demanded becomes negative at a price above \$12.5. Negative quantity demanded doesn't really make sense. If the price were negative (paying people to take asparagus), then quantity demanded would be greater than 25,000 pounds. Again, this is not a very economically meaningful situation. Thus, if we use a linear demand curve, we should understand that the linear demand curve only applies in the positive quadrant (and perhaps in the area of the positive quadrant away from the axes).

3. Analytics of a Logarithmic Demand Curve. Suppose that the market demand for peanut butter in Legumia is given by $\ln Q_d = 1.3863 - \ln P$, where \ln is the natural (base e) logarithm function.

(a) Take the anti-logs of both sides of the demand equation to express Q_d directly as a function of P . (You should use a scientific calculator or a Web site such as <http://www.1728.com/logrithm.htm> (*sic*) if it is helpful.)

The anti-log of $\ln Q_d$ is just Q_d . The antilog (base e) of 1.3863 is approximately 4. The antilog of $\ln P$ is P . The antilog of a difference is the quotient of the antilogs, so the antilog of the right-hand side is $4/P$. Thus, the equation becomes $Q_d = 4 / P$.

(b) Calculate the values of Q_d corresponding to prices of 0.10, 0.50, 1, 5, 10, and 100. Graph these points. What does the demand curve look like? Is it a straight line?

The points in question are (40, 0.1), (8, 0.5), (4, 1), (0.8, 5), (0.4, 10), and (0.04, 100). The demand curve is a rectangular hyperbola that bends away from both axes, not a straight line.

(c) At what values, if any, does the demand curve intercept the vertical and horizontal axes? Is this more or less realistic than the linear demand curve in the previous problem?

It never intersects either axis. This *may* be more realistic than the linear demand curve because a few people may be willing to buy at an extremely high price and the quantity demanded may get very large as price goes to zero. This is reflected by the curve bending along the axes rather than intersecting.

4. Market Equilibrium, Price Supports, and Subsidies. The U.S. Department of Agriculture is interested in analyzing the domestic market for corn. The USDA's staff economists estimate the following linear equations for the demand and supply curves (in the neighborhood of equilibrium):

$$Q_d = 1,600 - 125P$$

$$Q_s = 440 + 165P$$

Quantities are measured in millions of bushels; prices are measured in dollars per bushel.

(a) Calculate the price and quantity that will prevail in competitive equilibrium. Graph the curves and show the equilibrium.

$P = 4$ and $Q = 1100$.

(b) Suppose that the government imposes a \$4.50 per bushel support price (in other words, it does not allow sales at a price below \$4.50) and commits to buying any surplus that might arise at that price.

(i) What impact will this price floor have on the market?

Quantity demanded will be $1600 - 125 \times 4.5 = 1037.5$. Quantity supplied will be $440 + 165 \times 4.5 = 1182.5$. There will be a surplus of $1182.5 - 1037.5 = 145$.

(ii) Will the government be forced to purchase corn in order to support the price floor? If so, how much and what will be the cost to the Treasury?

Government must purchase 145 million bushels at \$4.5 for a cost of \$652.5 million.

(iii) How much (if any) additional corn (relative to equilibrium) will be produced as a result of the price support? How much (if any) less corn will be consumed?

Compared with the original equilibrium quantity of 1100, quantity supplied increases by 82.5 and quantity consumed falls by 62.5 million bushels.

(c) Now suppose that a new corn hybrid is developed that increases yields so that the quantity supplied increases by 145 million bushels at each level of the corn price. What is the new equation for the supply curve? How will your answers to parts (a) and (b) be different?

The new supply curve is $Q_s = 585 + 165P$. The new equilibrium price is 3.5 and quantity is 1162.5.

With the price support at \$4.50, the quantity demanded is still 1037.5 but quantity supplied increases to 1327.5, doubling the surplus to 290m bushels, which costs the government \$1305m.

(d) Returning to the original values, suppose that instead of a price floor, the government pays corn growers \$0.50 for each bushel produced.

(i) Explain why the supply curve with the subsidy would become $Q_s = 440 + 165(P + 0.50)$.

Each seller gets a \$0.50 payment in addition to the price for each unit he or she sells. This means that sellers should be willing to sell the same amount that they used to sell for \$4.00 at a price of \$3.50, because they will still receive \$4.00 when the subsidy is added in. Representing that in an equation replaces the P in the supply equation by the new amount received: $P + 0.50$.

(ii) Calculate the new market equilibrium quantity and price.

Solving the demand and new supply equations together gives an equilibrium price of about \$3.7155 (so sellers receive \$4.2155 with the subsidy payment) and a quantity exchanged of 1135.6.

(iii) Graph the new supply curve and show the new equilibrium.

The new supply curve is parallel to the old one and lies \$0.50 below it (or 82.5 units to the right).

(iv) How does the outcome under the subsidy compare to the outcome under the price support in terms of quantity bought by consumers, quantity produced by farmers, and cost to the government? Are there any other differences in the outcomes?

It differs in several ways.

	Price floor	Subsidy
Price paid by buyers	\$4.50	\$3.72
Quantity bought by buyers	1037.5m bushels	1135.6m bushels
Price received by sellers	\$4.50	\$4.22
Quantity sold by sellers	1182.5m bushels	1135.6m bushels
Total revenue to sellers	\$5321.25m	\$4787.12m
Cost to government	\$652.5m	\$567.8m
Government acquires:	145m bushels	nothing

Consumers are clearly better off under the subsidy because they buy more at a lower price. Sellers get somewhat less revenue under the subsidy than the price floor, but also produce less so they should have lower costs. The government pays less under the subsidy, but does not get any corn.